

ECE 313: Exam II Conflict 2

Name: (in BLOCK CAPITALS) _____

NetID: _____

Signature: _____

Section: A, MWF at 10 am (Milenkovic) B, MWF at 11 am (Katselis)
 C, MWF at 1 pm (Shanbhag) D, MWF at 2pm (Bastopcu) CSP, Chicago (Shanbhag)

Instructions

WRITE YOUR NAME AND NETID ON TOP OF EVERY PAGE. This exam is closed book and closed notes except that one 8.5"×11" sheet of notes is permitted: both sides may be used. Calculators, laptop computers, PDAs, iPods, cellphones, e-mail pagers, headphones, etc. are not allowed.

The exam consists of four problems worth a total of 100 points. The problems are not necessarily weighted equally, so it is best for you to pace yourself accordingly. Write your answers in the spaces provided, and reduce common fractions to lowest terms, but DO NOT convert them to decimal fractions (for example, write $\frac{3}{4}$ instead of $\frac{24}{32}$ or 0.75).

SHOW YOUR WORK; BOX YOUR ANSWERS. Answers without appropriate justification will receive very little credit. You may use the back of the previous page as scratch paper, but we will only grade answers in the space provided. Draw a small box around each of your final numerical answers.

Grading	
1. 25 points	_____
2. 25 points	_____
3. 25 points	_____
4. 25 points	_____
Total (100 points)	_____

1. [25 points] A soft drink company claims that the probability of observing sugar content higher than 35 grams per can is $1/3$. A consumer advocacy group believes that the same probability is higher and is equal to $1/2$. So, the consumer advocacy group independently sampled 3 cans of soda from the production line and noted the number of cans of soda that has sugar content higher than 35 grams.

(a) (8 points) Write out the likelihood matrix (table) for this problem. Find the ML decision rule using the likelihood matrix (table).

(b) (5 points) Find $p_{false\ alarm}$, p_{miss} , and p_e for the ML rule. To calculate p_e , assume that the prior probability of the company's claim being true is equal to $1/4$.

(c) (7 points) Find the MAP rule, assuming that the prior probability of the company's claim being true is equal to $1/4$.

(d) (5 points) Find $p_{false\ alarm}$, p_{miss} , and p_e for the MAP rule.

2. **[25 points]** An Internet server has the task to register the arrivals of data packets, modeled by a Poisson process of rate equal to $\lambda = 1$ packet/second.

(a) (5 points) Find the probability that the server does not encounter any packets in the first minute.

(b) (10 points) Given that the server saw 10 packets arrive during the first minute, what is the probability that the server will see only one packet arrive in the next minute?

- (c) (10 points) What is the probability that the server encountered exactly 1 arrival in both of the two time intervals $[0, 2]$ and $[1, 4]$ seconds?

3. [25 points] Suppose a *fair* die is flipped 100 times. Let the event A be defined as:

$$A = \{(\text{number of times even number shows}) \leq (\text{number of times odd number shows}) + 4\}.$$

(a) (4 points) Let X be the random variable denoting the number of times an even number shows. Express A in terms of X . What is the probability mass function of X ?

(b) (5 points) Write down an expression for the exact value of $P(A)$.

(c) (8 points) Using the Gaussian approximation with the continuity correction, approximate the value of $P(A)$ via the Φ function. Leave the expression in terms of the Φ function.

(d) (8 points) Let B denote the event $\{X = 55\}$. Using the Gaussian approximation with the continuity correction, approximate the value of $P(B)$ via the Q function. Leave the expression in terms of the Q function.

4. [25 points] Suppose that X is a random variable with pdf

$$f_X(x) = cx + c^2, \quad x \in [0, |c|],$$

and $f_X(x) = 0$, otherwise.

(a) (10 points) Find $c > 0$ such that f_X is a valid pdf.

(b) (15 points) For $c > 0$, find $P(\frac{c}{4} < X^2 < \frac{c}{2})$ in terms of c .