

1. (i) B
- (ii) B
- (iii) D
- (iv) either C or D due to typeset error
- (v) B
- (vi) D

2. Part (a): (i) Approach 1:

$$\begin{aligned}
 \oint \mathbf{E} \cdot d\mathbf{l} &= \int_{-1}^1 2du + \int_{-1}^1 -3dy + \int_1^{-1} 2du + \int_1^{-1} -3dy \\
 &= 4 - 6 - 4 + 6 \\
 &= 0.
 \end{aligned}$$

Approach 2: $\nabla \times \mathbf{E} = 0$ so $\oint \mathbf{E} \cdot d\mathbf{l} = 0$.

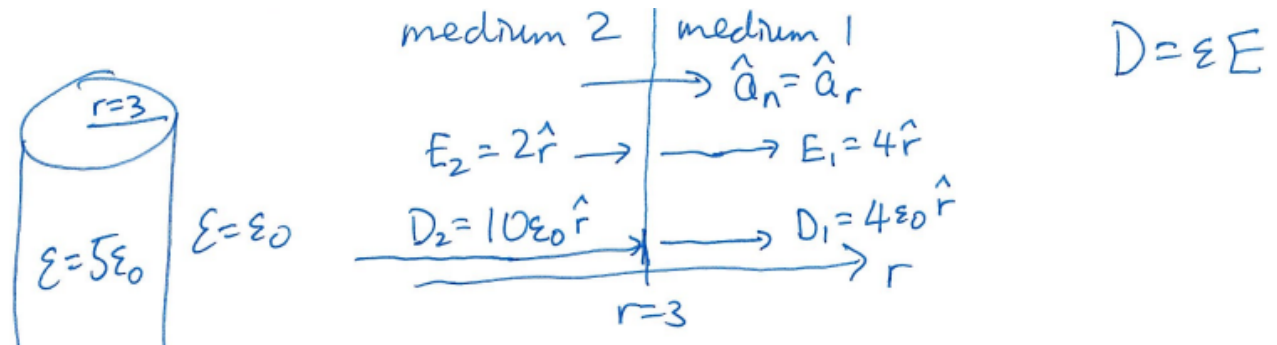
(ii)

$$\begin{aligned}
 V(a) - V(b) &= - \int_b^a \mathbf{E} \cdot d\mathbf{l} \\
 &= - \int_{-1}^1 2du - \int_{-1}^1 -3dy \\
 &= -4 + 6 \\
 &= 2V
 \end{aligned}$$

Part (b): (i) By Stoke's theorem,

$$\begin{aligned}
 \oint \mathbf{E} \cdot d\mathbf{l} &= \iint_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} \\
 &= \iint_S (2\hat{x} - 3\hat{y} + 5\hat{z}) \cdot (dxdy\hat{z}) \\
 &= \int_{-1}^1 \int_{-1}^1 5dxdy \\
 &= 5 \times 4 \\
 &= 20V
 \end{aligned}$$

(ii) E since we have a nonconservative field



3. (a)

We have $D_{1n} - D_{2n} = \rho_s$ so $\rho_s = 4\epsilon_0 - 10\epsilon_0 = -6\epsilon_0 [C/m^2]$

(b) Use Gauss's law with cylinder of radius 3 and length L .

$$\iint_S \mathbf{D} \cdot d\mathbf{S} = Q_{enc}$$

$$D_{2n} 2\pi r L = Q_{enc}$$

$$Q_{enc}/L = D_{2n} 2\pi r = 60\pi\epsilon_0 [C/m]$$

4. (a) False

(b) False

(c) $\frac{-Q}{4\pi\epsilon r^2} \hat{r} [V/m]$

(d) Note flow is inward and constant on a sphere of radius r_o and $d\mathbf{S}$ points outwards by the following:

$$\mathbf{J} = \frac{-\sigma Q}{4\pi\epsilon r^2} \hat{r}$$

We have

$$\begin{aligned} I &= \iint \mathbf{J} \cdot d\mathbf{S} \\ &= \iint |J| |dS| (-1) \\ &= -|J| (\text{area of sphere}) \\ &= -\left| \frac{-\sigma Q}{4\pi\epsilon r^2} \right| 4\pi r^2 \\ &= -\frac{\sigma Q}{\epsilon} [A] \end{aligned}$$

(e) Note $d\mathbf{l} = dr\hat{r}$ below

$$\begin{aligned}
 V(b) - V(a) &= - \int_a^b \mathbf{E} \cdot d\mathbf{l} \\
 &= - \int_a^b \frac{-Q}{4\pi\epsilon r^2} dr\hat{r} \\
 &= \int_a^b \frac{Q}{4\pi\epsilon r^2} dr\hat{r} \\
 &= \left. \frac{-Q}{4\pi\epsilon r} \right|_a^b \\
 &= \frac{Q}{4\pi\epsilon} \left(\frac{1}{a} - \frac{1}{b} \right)
 \end{aligned}$$

Hence

$$\begin{aligned}
 V_o &= \frac{Q}{4\pi\epsilon} \left(\frac{b-a}{ab} \right) [V] \\
 Q &= 4\pi\epsilon V_o \left(\frac{ab}{b-a} \right) [C]
 \end{aligned}$$

(f)

$$\begin{aligned}
 G &= \frac{|I|}{V_o} \\
 &= \frac{\sigma Q}{\epsilon V_o} \\
 &= \frac{\sigma}{\epsilon} 4\pi\epsilon \left(\frac{ab}{b-a} \right) \\
 &= \sigma 4\pi \left(\frac{ab}{b-a} \right) [Siemens]
 \end{aligned}$$