University of Illinois at Urbana-Champaign
ECE 329 Fields and Waves I

Midterm Exam 2 Solutions

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Name: ______________________________

UIN: ______________________________

Section: ______________________________

This exam contains 10 pages (including this cover page) and 5 questions. Total of points is 0.
This is a closed book exam and calculators/electronic devices are not allowed. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate. The exam is double sided. You may use the back of the exam as scratch paper. Good luck!

We will only grade work that is in the designated work box and final answer box.

PART I: MULTIPLE CHOICE QUESTIONS

Fill in the appropriate circle(s). More than one may apply! Note that some of these questions require some calculations to get the right answers, but you do not need to show your work for credit. No partial credit will be given on the multiple choice questions.

1. A transverse electromagnetic wave has an electric field $E = E_0 cos(2\pi 10^8 t + 2\pi z)\mathbf{a_x}$ A/m

   (a) 4 points The directions of (1) **E** field at $t = 1ns, z = 0$, (2) the pointing vector direction, and (3) **H** field at $t = 1ns, z = 0$ are:

   ○ $-\mathbf{a_x}$, $\mathbf{a_z}$, $-\mathbf{a_y}$
   ○ $-\mathbf{a_x}$, $-\mathbf{a_z}$, $\mathbf{a_y}$
   ○ $\mathbf{a_x}$, $-\mathbf{a_z}$, $\mathbf{a_y}$
   √ **$\mathbf{a_x}$, $-\mathbf{a_z}$, $-\mathbf{a_y}$**
   ○ $\mathbf{a_x}$, $\mathbf{a_z}$, $\mathbf{a_y}$

   (b) 4 points The frequency f, propagation constant, and phase velocity are:

   √ **10^8 Hz, 2π m^{-1}, 10^8 m/s**
   ○ 10^8 Hz, 1 m^{-1}, 10^8 m/s
   ○ $2\pi \times 10^8$ Hz, 2π m^{-1}, 3×10^8 m/s
   ○ $2\pi \times 10^8$ Hz, 1 m^{-1}, 3×10^8 m/s

2. (a) 3 points The vector field **E** generated by a time-varying current sheet is an example of a conservative field.

   ○ True
   √ **False**

   (b) 4 points For a plane TEM wave with $\mathbf{E}(\mathbf{r}, t) = -\hat{\mathbf{y}} f(t + \frac{x}{v})$, what is the accompanying magnetic field vector $\mathbf{H}(\mathbf{r}, t)$?

   ○ $\mathbf{H}(\mathbf{r}, t) = \hat{\mathbf{x}} \frac{1}{\eta} f(t + \frac{x}{v})$
   ○ $\mathbf{H}(\mathbf{r}, t) = -\hat{\mathbf{x}} \frac{1}{\eta} f(t + \frac{x}{v})$
   ○ $\mathbf{H}(\mathbf{r}, t) = \hat{\mathbf{y}} \frac{1}{\eta} f(t + \frac{x}{v})$
   ○ $\mathbf{H}(\mathbf{r}, t) = -\hat{\mathbf{y}} \frac{1}{\eta} f(t + \frac{x}{v})$
   √ **$\mathbf{H}(\mathbf{r}, t) = \hat{\mathbf{z}} \frac{1}{\eta} f(t + \frac{x}{v})$**
   ○ $\mathbf{H}(\mathbf{r}, t) = -\hat{\mathbf{z}} \frac{1}{\eta} f(t + \frac{x}{v})$

(c) 3 points A time-varying current sheet flows on the zy-plane according to $\mathbf{J_s} = -\hat{\mathbf{z}}J_z(t)$. The electric field direction of the resulting propagating waves is:

- ◯ $\hat{\mathbf{x}}$
- ◯ $-\hat{\mathbf{x}}$
- ◯ $\hat{\mathbf{y}}$
- ◯ $-\hat{\mathbf{y}}$
- √ $\boxed{\hat{\mathbf{z}}}$
- ◯ $-\hat{\mathbf{z}}$
- ◯ Cannot answer; it depends on the direction of propagation.

(d) 3 points For a particular coil carrying a current I = 4 [A], the flux linkages are 8×10^{-3} [Wb]. What EMF is induces in the coil when the current is changing at the rate of 2A/s?

- ◯ 0 mV
- ◯ 2 mV
- √ **4 mV**
- ◯ 8 mV

(e) 3 points Which of the following is the appropriate unit for inductance?

- ◯ volt ampere/second
- ◯ weber/second
- √ **ohm second**
- ◯ weber meter second

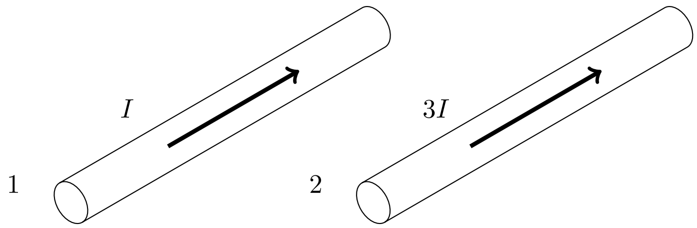
(f) 2 points Permeability of a diamagnetic substance is

- √ **Slightly less than that of vacuum.**
- ◯ Slightly more than that of vacuum.
- ◯ The same as that of vacuum.

(g) 2 points Poynting theorem is a statement of

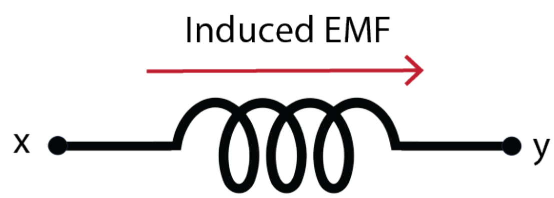
- √ **Conservation of electromagnetic energy**
- ◯ Conservation of electromagnetic power
- ◯ All of the above

(h) [3 points] Two parallel wires 1 and 2 that are near each other carry currents I and $3I$ both in the same direction. Compare the forces that the two wires exert on each other.



- ◯ Wire 1 exerts a stronger force on wire 2.
- ◯ Wire 2 exerts a stronger force on wire 1.
- √ **The wires exert equal magnitude attractive force on each other.**
- ◯ The wires exert equal magnitude repulsive force on each other.
- ◯ The wires exert no forces on each other.

(i) [3 points] Under which of the following conditions will an EMF be induced in the coil in the direction indicated by the arrow? The coil carries:

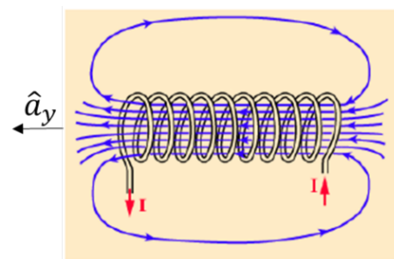


- ◯ a constant current in the y → x direction
- ◯ an increasing current in the direction x → y
- √ **a decreasing current in the direction x → y**
- ◯ a decreasing current in the direction y → x

PART II: DETAILED QUESTIONS

Please show your work within the provided space (box) wherever applicable.

3. A time-varying current $I(t) = I_0 cos(\omega t)[A]$ flows through a solenoid (a coil of wire) with n turns per unit length producing a magnetic field inside given by $\mathbf{H} = nI(t)\mathbf{a_y}[A/m]$ along the positive y-direction as shown in the illustration. The radius of the coil is R.



(a) 15 points If a second solenoid of radius $r = 2[m]$, turns per unit length $n_2 = 3[m^{-1}]$, and length $L = 2[m]$ is placed inside this loop with its normal vector along $\mathbf{a_y}$, find the emf generated in the second solenoid if the parameters for the first solenoid are: $I_0 = 7[A]$, $\omega = 2\pi[rad/s]$, $n = 3[m^{-1}]$, and $R = 5[m]$.

$$\Phi = \mu_0 \mathbf{H} \cdot Area = \mu_0 n I_0 \cos(\omega t)(\pi r^2) \rightarrow \frac{d\Phi}{dt} = -\mu_0 n I_0 \pi r^2 \omega \sin(\omega t)$$

$$\mathcal{E}_2 = -n_2 L \frac{d\Phi}{dt}$$
$$= n_2 L \omega \mu_0 n I_0 \pi r^2 \sin(\omega t)$$
$$= (3)(2)(2\pi)(3)(7)\pi(2)^2 \mu_0 \sin(\omega t)$$

$$\boxed{\mathcal{E}_2 = 1008\pi^2 \mu_0 \sin(\omega t)[V]}$$

(b) 5 points What is the ratio of emf generated if the direction of the normal vector of the second solenoid were parallel to the vector $4\mathbf{a_x} - 2\mathbf{a_y} + 4\mathbf{a_z}$ to the emf generated in part (a). *Note: you can answer this without needing to get part (a).*

Since the inductive coupling between the two solenoids is directly proportional to the dot product of the normal vectors of the solenoids

$$\mathbf{n_1} \cdot \mathbf{n_2}$$

Here,

$$\mathbf{n_1} = \mathbf{a_y} = \hat{y}$$

$$\mathbf{n_2} = \frac{4\mathbf{a_x} - 2\mathbf{a_y} + 4\mathbf{a_z}}{||4\mathbf{a_x} - 2\mathbf{a_y} + 4\mathbf{a_z}||} = \frac{1}{3}[2\hat{x} - \hat{y} + 2\hat{z}]$$

$$\mathbf{n_1} \cdot \mathbf{n_2} = \hat{y} \cdot \frac{1}{3}[2\hat{x} - \hat{y} + 2\hat{z}]$$

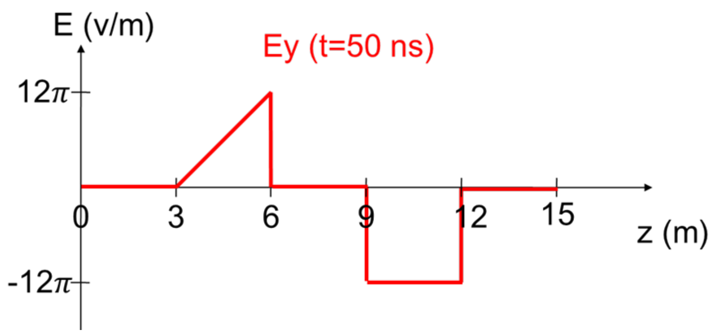
$$\boxed{= -\frac{1}{3}}$$

Therefore, the ratio of emf generated in the second solenoid at this orientation is $-\frac{1}{3}$.

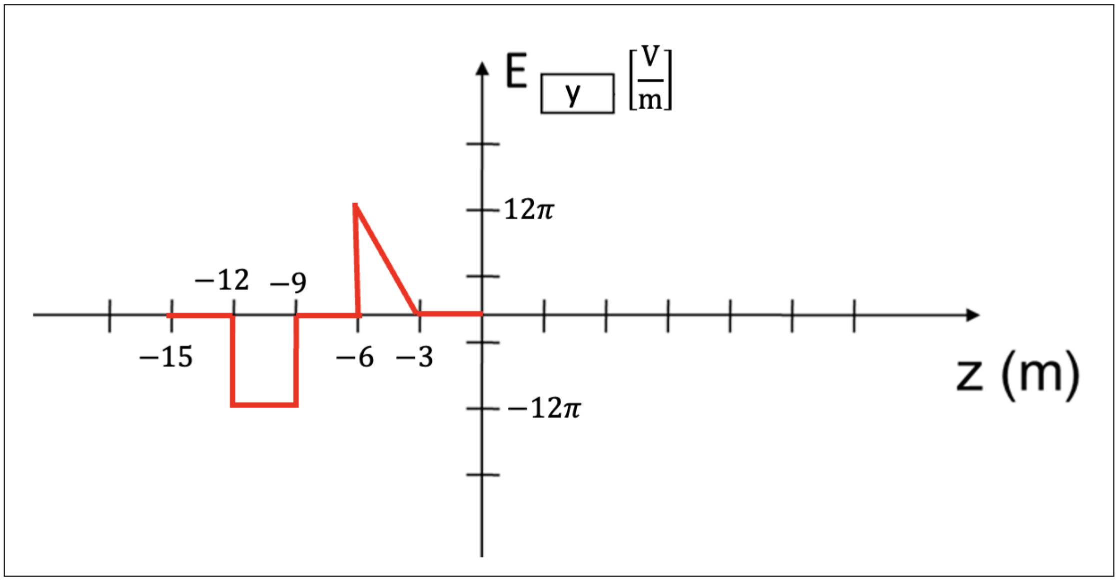
(c) 6 points Assuming everything else is kept constant, the magnitude of the generated emf will *(a) increase* *(b) decrease* or *(c) remain constant* <u>if we</u> :

1. increase the radius of the coil R.
   ○ increase ○ decrease √ **remain constant**
2. decrease the resistance of the second solenoid.
   ○ increase ○ decrease √ **remain constant**
3. increase the radius of the second solenoid.
   √ **increase** ○ decrease ○ remain constant
4. decrease the drive frequency ω.
   ○ increase √ **decrease** ○ remain constant
5. move the second solenoid to a position outside the first solenoid.
   ○ increase √ **decrease** ○ remain constant
6. add an iron core of radius 2m inside the second solenoid.
   √ **increase** ○ decrease ○ remain constant

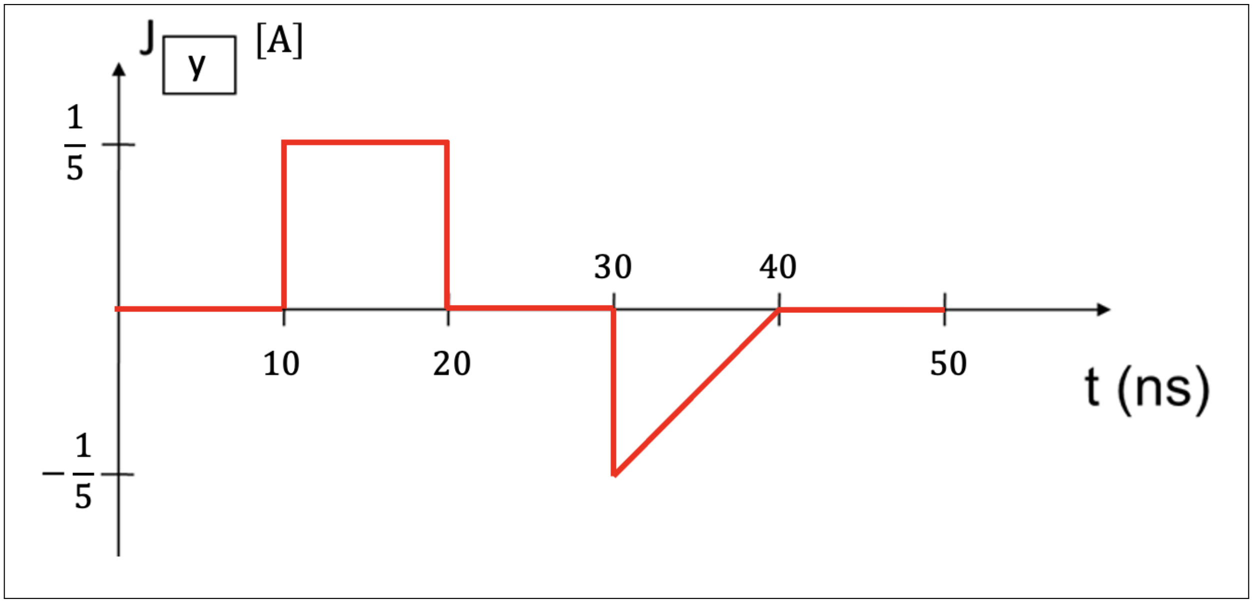
4. The transverse electromagnetic wave is generated by a sheet current J_s located at $z = 0$. The speed of propagation is $3 \times 10^8 (m/s)$. The electric field of the forward propagating wave is measured at $t = 50(ns)$ which only has the y component. Note that in free space the wave impedance η_0 is $120\pi(\Omega)$.



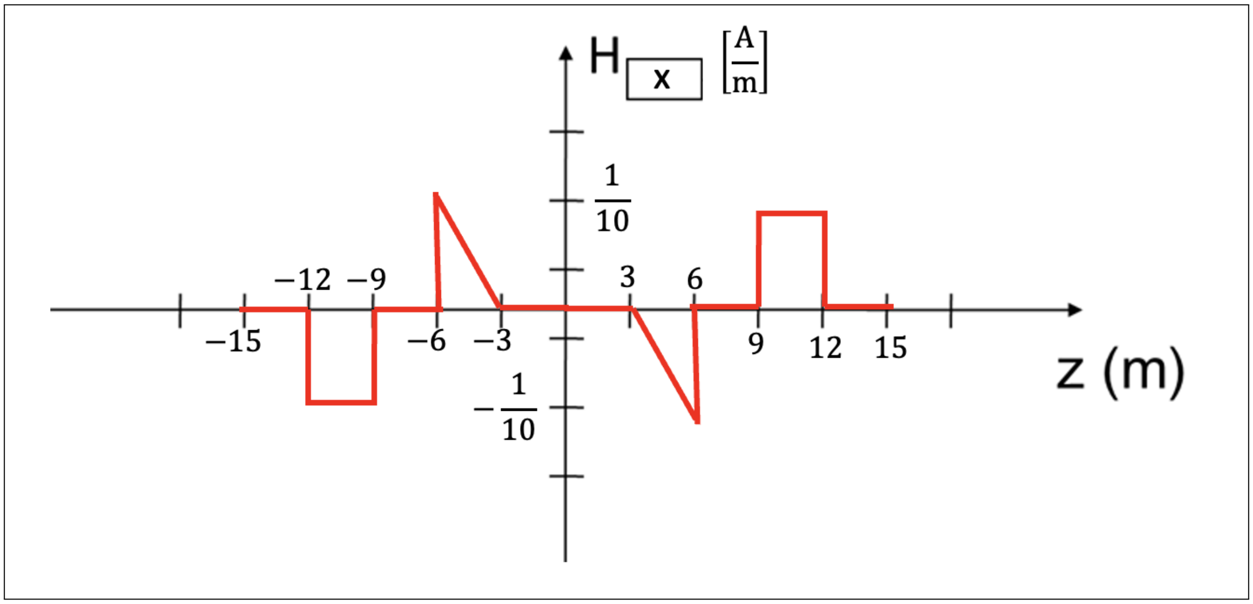
(a) 10 points Sketch the electric field of the backward propagating wave at $t = 50(ns)$. Please make sure to label the direction of the field and put appropriate numerical labels on the vertical axis.



(b) 10 points Sketch the current source J_s as a function time. Please make sure to label the direction of the current and put appropriate numerical labels on the vertical axis.



(c) 10 points Sketch the magnetic field at $t = 50$ (ns) as a function of location z. Please make sure to label the direction of the magnetic field and put appropriate numerical labels on the vertical axis.



5. 10 points Given $\rho(x, y, z, t) = 3sin(x)/(1+t^2)$ find the outward flow of current for a box bounded by $(0, 0, 0)$ and $(1, 1, 1)$ assuming there are no fields other than the one generated by ρ.

Using the divergence theorem:

$$\oiint_S \mathbf{J} \cdot d\mathbf{S} = \iiint_V \nabla \cdot \mathbf{J} dV$$

and the continuity equation

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$
$$= -\frac{\partial}{\partial t}\left(\frac{3\sin(x)}{1+t^2}\right)$$
$$= -\frac{6t\sin(x)}{(1+t^2)^2}$$

Therefore,

$$\oiint_S \mathbf{J} \cdot dS = \iiint_V \left(\frac{6t\sin(x)}{(1+t^2)^2}\right) dV$$
$$= -\frac{6t}{(1+t^2)^2}\int_0^1\int_0^1\int_0^1 \sin(x)dxdydz$$
$$\boxed{= -\frac{6t}{(1+t^2)^2}(\cos(1)-1)[A]}$$

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.