University of Illinois at Urbana-Champaign
ECE 329 Fields and Waves I

Midterm Exam 3 – Solution

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November 15, 2023

Name: ______________________________

UIN: ______________________________

Section: ______________________________

This exam contains 13 pages (including this cover page) and 7 questions. Total of points is 100. This is a closed-book exam, and calculators/electronic devices are not allowed. Please show all your work and make sure to include your reasoning for each answer. All answers should include units wherever appropriate. The exam is double-sided. You may use the back of the exam as scratch paper. Good luck!

Distribution of Points

Question	Points	Score
1	11	
2	9	
3	12	
4	18	
5	24	
6	10	
7	16	
Total:	100	

PART I: MULTIPLE CHOICE QUESTIONS

Circle all that apply! Note that some of these questions require some calculations to get the right answers, but you do not need to show your work for credit. However, partial credit can be given if the work is provided and partially correct.

1. (a) [3 points] Which of the following statements are true:

   ○ Given the direction of propagation, a field of arbitrary polarization can be expressed as a linear superposition of orthogonally polarized fields.

   ○ Given the direction of propagation, a linearly polarized field can be expressed as a superposition of circularly polarized fields.

   ● **All of the above.**

   ○ None of the above.

   (b) [3 points] The loss tangent represents:

   ○ The tangent of the angle by which the displacement current density leads the total current density

   ○ The ratio between the conduction and displacement current

   ● **All of the above**

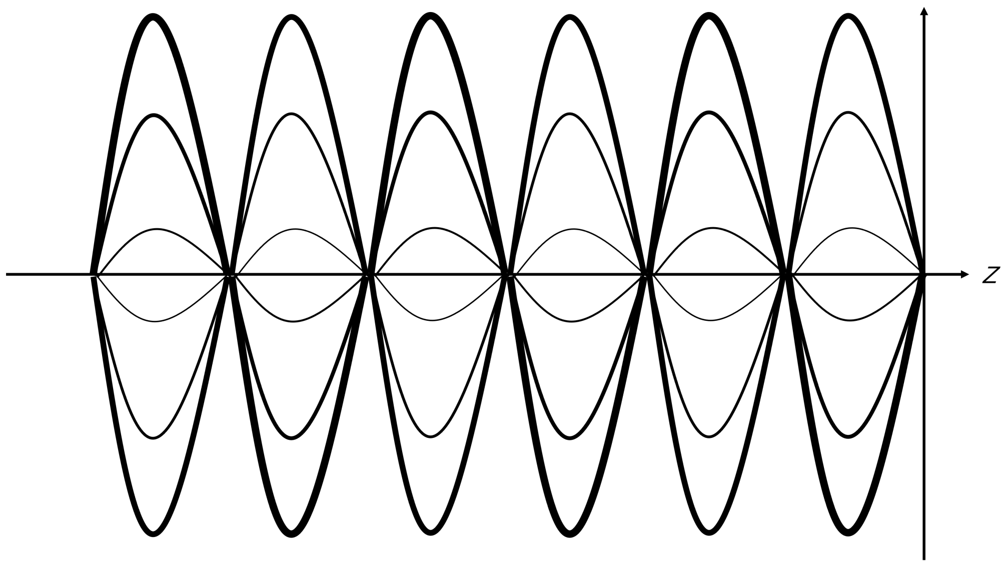
   ○ None of the above

   (c) [2 points] In a region of free space, where a standing wave condition exists, the term $\frac{1}{2}\epsilon_0\mathbf{E}\cdot\mathbf{E}+\frac{1}{2}\mu_0\mathbf{H}\cdot\mathbf{H}=0$.

   ○ True

   ● **False**

(d) 3 points The diagram below with different line thicknesses representing different times could represent:



- ○ the electric field of a standing wave
- ○ the magnetic field of a standing wave
- ○ the electric field of a traveling wave
- ○ the magnetic field of a traveling wave
- ● **either the electric field of a standing wave or the magnetic field of a standing wave**
- ○ either the electric field of a standing wave or the electric field of a traveling wave

2. For a wave carrying the magnetic intensity field,

$$\mathbf{H}(z,t) = \hat{\mathbf{x}}Acos(\omega t - kz) + \hat{\mathbf{y}}Bcos(\omega t - kz + \phi)$$

identify the polarization of the wave for each of the following cases:

(a) 3 points $A = 2$ [V/m], $B = 4$ [V/m], $\phi = 0$

- ● **linear**
- ○ right circular
- ○ left circular
- ○ elliptical

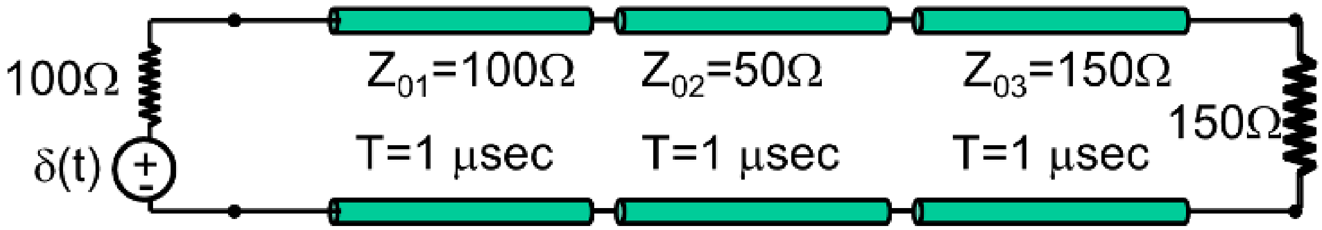
(b) 3 points $A = 4$ [V/m], $B = 2$ [V/m], $\phi = \pi$

- ● **linear**
- ○ right circular
- ○ left circular
- ○ elliptical

(c) 3 points $A = 2$ [V/m], $B = 2$ [V/m], $\phi = \pi/4$

- ○ linear
- ○ right circular
- ○ left circular
- ● **elliptical**

3. (a) 12 points A discontinued transmission line is displayed below, at t=0, the pulse source is turned on, and a wave starts to propagate through the transmission line.



(i) The injection coefficient τ_g is:

- ○ 1
- ● **1/2**
- ○ 2/3
- ○ 3/4

(ii) The reflection coefficient of voltage Γ between the first and second segments of the transmission line for a wave propagating to the right is:

- ○ 1/3
- ● **-1/3**
- ○ 1/2
- ○ -1/2

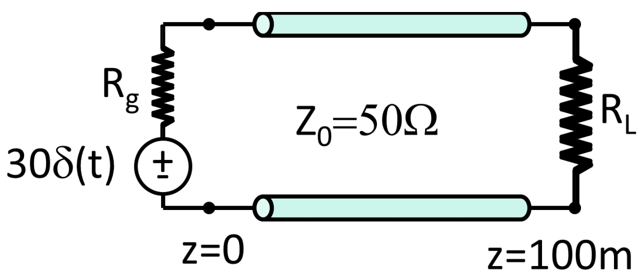
(iii) The reflection coefficient of current Γ_c between the second and third segments of the transmission line for a wave propagating to the right is:

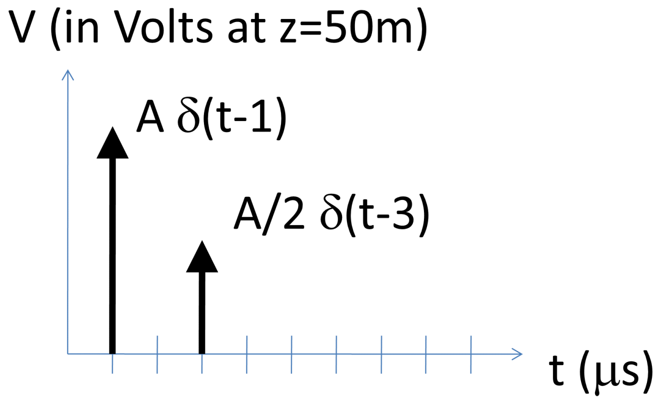
- ○ 1/3
- ○ -1/3
- ○ 1/2
- ● **-1/2**

(iv) It takes a minimum of __________ microseconds starting from t=0 for the reflected wave between the second and third segments of the transmission line to reach the generator.

- ○ 2
- ○ 3
- ● **4**
- ○ 5

4. A pulse generator with unknown internal resistance sends a delta function pulse at t = 0s down a lossless transmission line circuit ($Z_0 = 50\Omega$) that is terminated by an unknown resistive load R_L. The voltage is measured at z = 50m (i.e., the midpoint of the line), and two pulses are observed: one at 1µs and the other at 3µs, as shown in the figure. The amplitudes of these pulses in volts are A and A/2 for some constant A. No other pulses are observed.





(a) 5 points The internal resistance of the generator in ohms is:

- ○ 0
- ○ 1/2
- ○ 50/3
- ● **50**
- ○ 100
- ○ 150
- ○ ∞
- ○ none of these

(b) 5 points The load resistance in ohms is:

- ○ 0
- ○ 1/2
- ○ 50/3
- ○ 50
- ○ 100
- ● **150**
- ○ ∞
- ○ none of these

(c) 4 points The amplitude A in volts is:

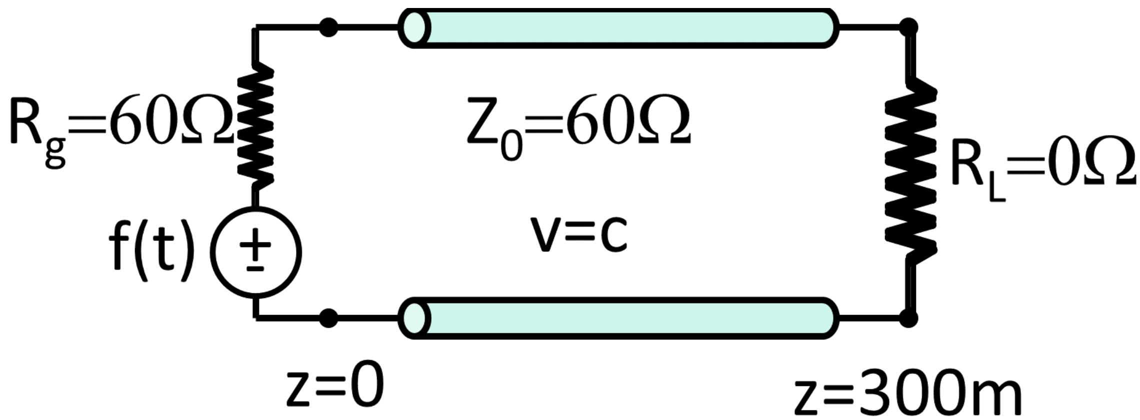
- ○ 0
- ○ 10
- ● **15**
- ○ 20
- ○ 22.5
- ○ 30
- ○ 60
- ○ none of these

(d) 4 points Assuming the transmission line is non-magnetic, i.e., $\mu = \mu_0$, what is the relative dielectric constant ϵ_r :

- ○ 1
- ○ 1.5
- ○ 2
- ○ 3
- ○ 4.5
- ○ 9
- ● **36**
- ○ none of these

PART II: DETAILED QUESTIONS

5. A pulse generator with an internal resistance of 60 [Ω] outputs a voltage $f(t)$. The generator is connected to a lossless transmission line with an impedance of $Z_0 = 60$ [Ω] and propagation speed c = 300 [m/μs]. The load is **short circuited**: $R_L = 0$ [Ω].



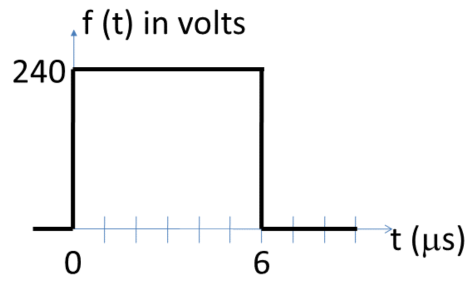
(a) 2 points Assuming that $f(t) = 240\ u(t)$, where $u(t)$ is the unit step function, what is the steady state **voltage** in [V] at $z = 0$ [m]?

- ○ -4
- ● **0**
- ○ 2
- ○ 4
- ○ 60
- ○ 120
- ○ 240
- ○ none of these

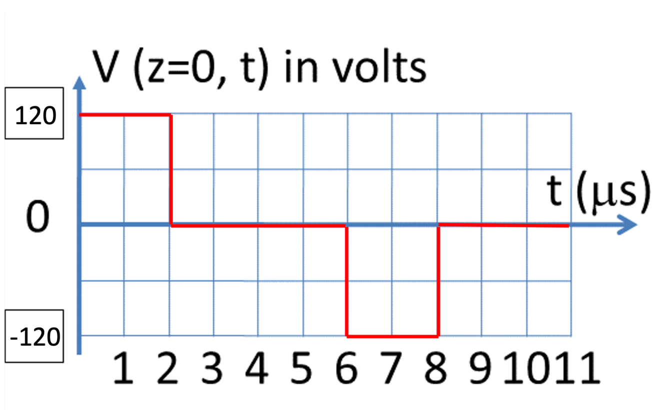
(b) 2 points Assuming that $f(t) = 240\ u(t)$, where $u(t)$ is the unit step function, what is the steady state **current** in [A] at $z = 300$ [m]?

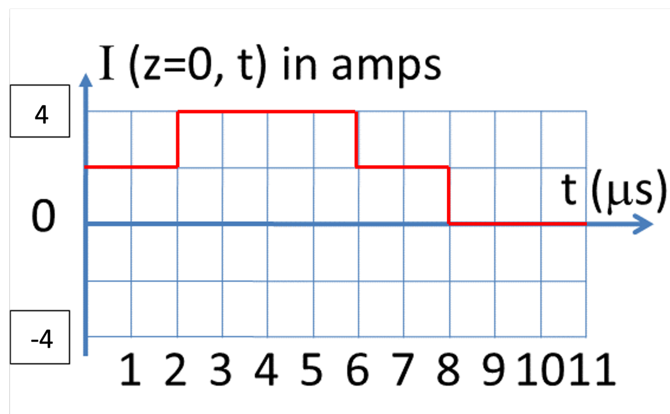
- ○ -4
- ○ 0
- ○ 2
- ● **4**
- ○ 60
- ○ 120
- ○ 240
- ○ none of these

(c) 20 points Consider now that $f(t)$ is instead given as a 240 [V] rectangular pulse, as seen below.

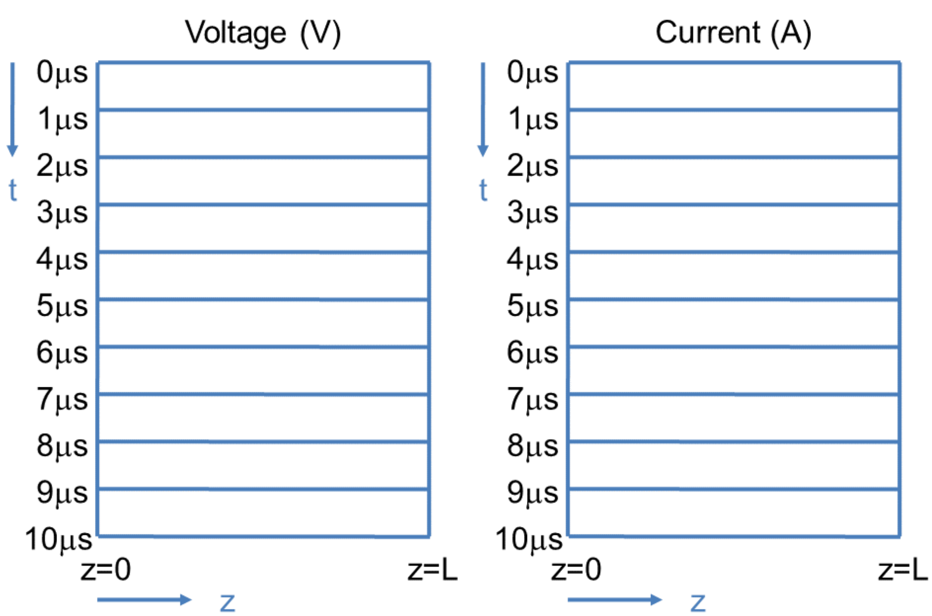


Sketch both the **voltage** and the **current** at $\mathbf{z = 0}$ [m] versus time using the plots below. Label the value in the box for the scale you are using on the y-axis. (optional: you can use the bounce diagram chart as scratch paper.) Only the plots of the waveforms at z = 0 versus time will be graded.





(Optional but helpful)



6. The electric field phasor of a uniform plane wave traveling downward in water is given by

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}5e^{-0.2z}e^{-j0.2z} \ [V/m]$$

where $\hat{\mathbf{z}}$ is the downward direction and z=0 is the water surface. Use η for the water impedance.

(a) [5 points] Find the time-averaged power density as a function of depth.

$$\begin{aligned} < S > &= \frac{1}{2}Re[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^\star] \\ &= \frac{1}{2}Re[\hat{\mathbf{x}}5e^{-0.2z}e^{-j0.2z} \times \hat{\mathbf{y}}\frac{5}{\boldsymbol{\eta}}e^{-0.2z}e^{+j0.2z}] \\ &= \frac{1}{2}Re[\hat{\mathbf{x}}5e^{-0.2z}e^{-j0.2z} \times \hat{\mathbf{y}}\frac{5}{|\eta|}e^{-0.2z}e^{+j0.2z}e^{+j\tau}](\tau = 45^\circ \text{ for good conductors}) \\ &= \hat{\mathbf{z}}\frac{25}{2|\eta|\sqrt{2}}e^{-0.4z} \end{aligned}$$

Your answer: $\hat{\mathbf{z}}\frac{25}{2|\eta|\sqrt{2}}e^{-0.4z}$[W/$m^2$]

(b) [5 points] Determine the attenuation rate for power

$S = S_0e^{-0.4z}$

Power Attenuation $= 10\log_{10}(\frac{S_0}{S}) = 10\log_{10}(e^{0.4z}) = 4z\log_{10}(e)[dB]$

Attenuation Rate for Power $= 0.4[1/m] = 4log(e)[dB/m] = 1.737[dB/m]$

Your answer:0.4 [1/m] or 1.737[dB/m]

7. A lossless transmission line of length $l = 1000$ [m] and velocity $v = 1 \times 10^8$ [m/s] supports resonant oscillations. In a co-sinusoidal steady state, the current phasor of these oscillations at the input (i.e., l away from the load) is measured to be $\tilde{I_{in}} = 0$ [A], and the voltage phasor at the load is measured to be $\tilde{V_L} = 0$ [V]

   (a) 4 points What are the input impedance Z_{in} at location $d = l$, and the load impedance Z_L for these resonances?

   Z_{in}= $\infty[\Omega]$ Z_L= $0[\Omega]$

   (b) 9 points Determine the first two lowest resonances of these oscillations and choose the correct graph for both $\tilde{I}$ and $\tilde{V}$ as a function of d.

   (i) (3 pts) The first lowest resonance ω is: $5\pi 10^4[rad/s]$

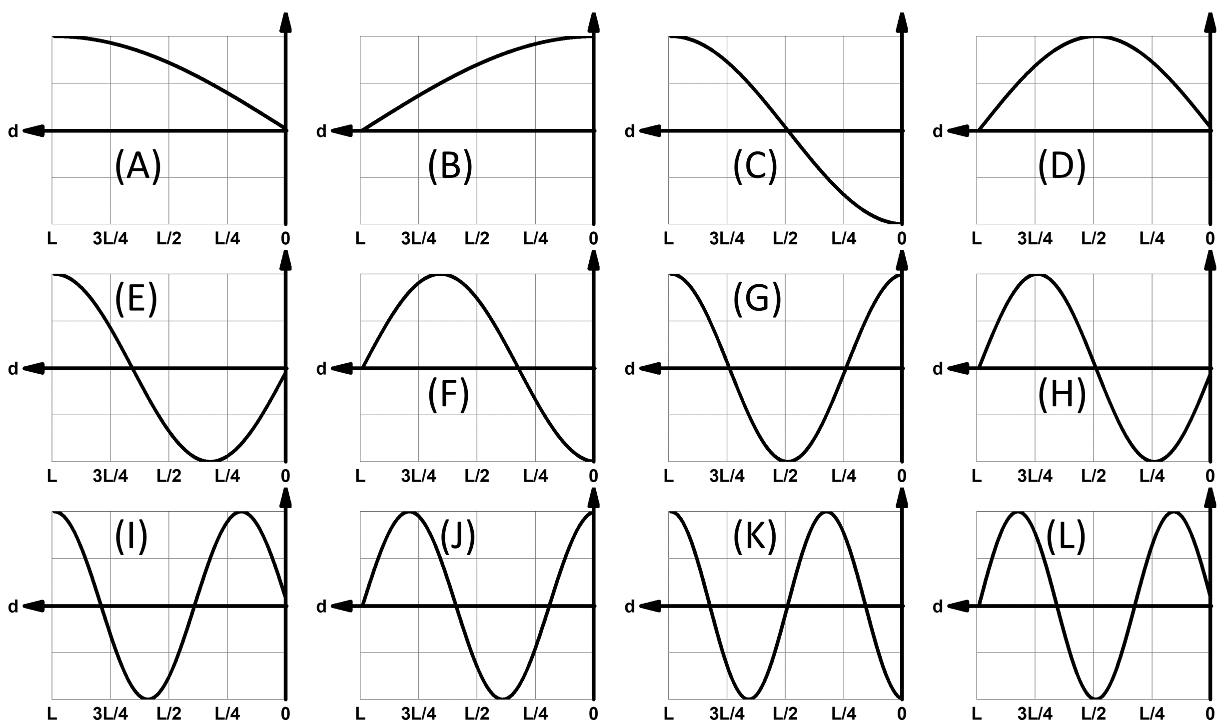
   At the first lowest resonance, $\lambda = 4L$. $\omega = 2\pi f = 2\pi \frac{v}{\lambda} = 2\pi \times \frac{1\times 10^8}{4\times 1000} = 5\pi 10^4$

   $\tilde{V}$(choose from A to L) :A $\tilde{I}$(choose from A to L) : B

   (ii) (6 pts) The second lowest resonance ω is: $1.5\pi 10^5[rad/s]$

   At the second lowest resonance, $\lambda = \frac{4}{3}L$. $\omega = 2\pi f = 2\pi \frac{v}{\lambda} = 2\pi \times \frac{1\times 10^8}{\frac{4}{3}\times 1000} = 1.5\pi 10^5$

   $\tilde{V}$(choose from A to L) :E $\tilde{I}$(choose from A to L) : F



(c) 3 points At the second lowest resonance, what is the shortest distance away from the load where the voltage acts as a short-circuit input? Express d in terms of λ .

d=0.5 ________________ λ

This page is intentionally left blank to accommodate work that wouldn't fit elsewhere and/or scratch work.