NAME ____

MIDTERM EXAM **SOLUTIONS**

ECE 451

October 15, 2014

12:00 – 12:50 p.m.

Instructions: Write your name and section where indicated. Show all work. Indicate the units of your answers.

Mason's non-touching loop rule:

$$T = \frac{P_1\left[1-\sum L(1)^{(1)}+\sum L(2)^{(1)}-\ldots\right]+P_2\left[1-\sum L(1)^{(2)}+\sum L(2)^{(2)}-\ldots\right]+\ldots}{1-\sum L(1)+\sum L(2)-\sum L(3)+\ldots}$$

Problem 1 (25 pts)	Problem 2 (25 pts)	Problem 2 (25 pts)	Problem 2 (25 pts)	Total (100 pts)

1. A transmission line of characteristic impedance Z_o, length d and propagation constant β is terminated with an open.

   (a) Find the input impedance.

   (b) Draw a rough sketch of Z_{in}/Z_o for βd ranging from 0 to π and label the frequency bands where the transmission line looks capacitive and where it looks inductive.

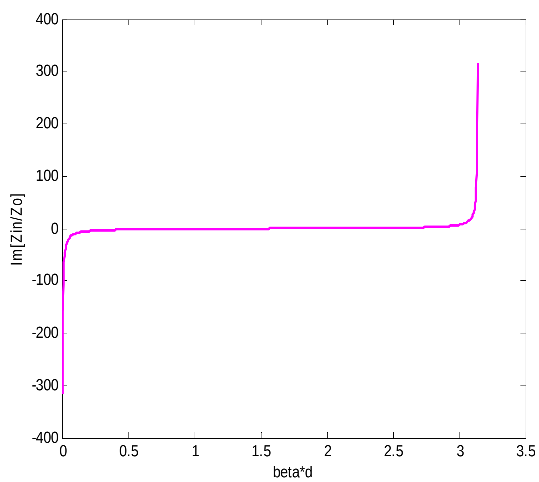
   (c) At what frequencies does this open transmission line look like a short circuit?

(a) For a transmission line of length d, we have:

$$Z_{in} = Z_o \left\{ \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} \right\}$$

If $Z_L \to \infty$, then $Z_{in} = -jZ_o \text{ cotan } \beta d$

(b)

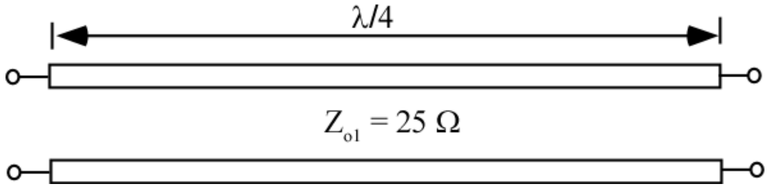


(c) The TL looks like a short for

$$\beta d = \frac{(2n+1)\pi}{2}, n = 0,1,2,\ldots \text{ or } \frac{2\pi fd}{v} = \frac{(2n+1)\pi}{2}, n = 0,1,2,\ldots$$

where v is the propagation velocity in the TL. This leads to: $f = \frac{(2n+1)v}{4d}, n = 0,1,2,\ldots$

2. For the transmission line shown below, write the scattering parameter matrix as measured on a 50-Ω network analyzer.



$$S_{11} = \frac{\left(1 - X^2\right)\Gamma}{1 - X^2\Gamma^2} \quad \text{and} \quad S_{21} = \frac{\left(1 - \Gamma^2\right)X}{1 - X^2\Gamma^2}$$

with $\Gamma = \dfrac{Z_{o1} - Z_o}{Z_{o1} + Z_o}$ and $X = e^{-j\frac{2\pi}{\lambda}l}$

$$X = e^{-j\frac{2\pi}{\lambda}\frac{\lambda}{4}} = e^{-j\frac{\pi}{2}} = -j$$

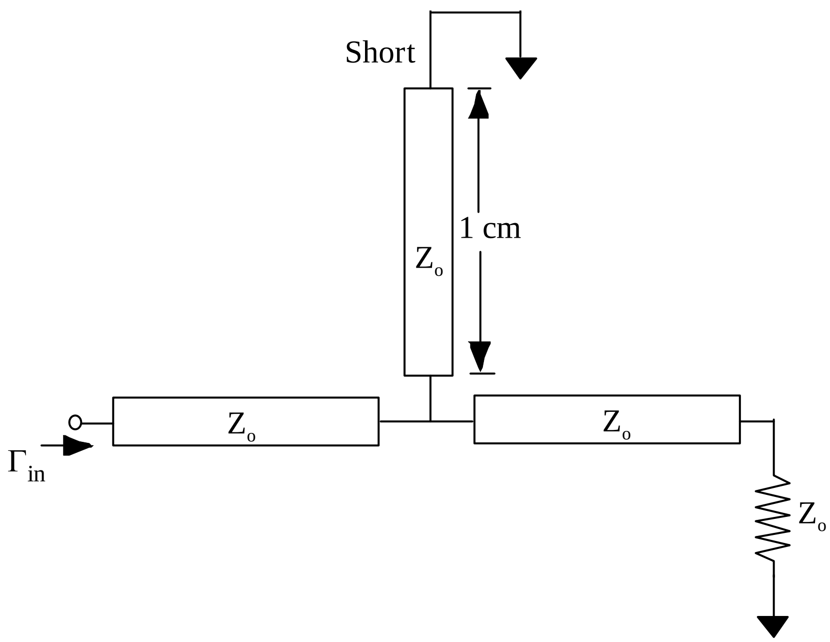
$$\Gamma = \frac{25 - 50}{25 + 50} = \frac{-25}{75} = -\frac{1}{3}$$

$$S_{11} = \frac{\left(1 - j^2\right)\left(-1/3\right)}{1 - \left(j^2\right)\left(1/9\right)} = \frac{\left(2\right)\left(-1/3\right)}{1 + 1/9} = \frac{\left(-2/3\right)}{10/9} = -3/5 = -0.6$$

$$S_{21} = \frac{\left(1 - 1/9\right)\left(-j\right)}{1 - \left(j^2\right)\left(1/9\right)} = \frac{\left(-j8/9\right)}{10/9} = -\frac{4j}{5} = -0.8j$$

$$S = \begin{bmatrix} -0.6 & -0.8j \\ -0.8j & -0.6 \end{bmatrix}$$

3. For the circuit shown below (lab student unknown), the transmission lines use air as dielectric. What is the lowest frequency for which $\Gamma_{in} = 0$? (Use Z_o as your reference impedance).



Solution

Γ_{in}=0 when shorted stub length=$\lambda/4$, or

$$\lambda = \frac{c}{f} = 4cm$$

$$f = \frac{c}{\lambda} = \frac{0.3 \times 10^9}{4 \times 10^{-2}} = 7.5\ GHz$$

4. A lossless transmission line has the following per unit length parameters: L = 80 nH-m^{-1} , C = 200 pF·m^{-1} Consider a traveling wave on the transmission line with a frequency of 1 GHz.

(a) What is the attenuation constant?

(b) What is the phase constant?

(c) What is the phase velocity?

(d) What is the characteristic impedance of the line?

(e) Now consider that the dielectric is replaced by a dielectric with ε_r = l (or air). The capacitance per unit length of the line is now C(air) =50 pF.m^{-1} . What is the effective relative dielectric constant of the line?

(a) $\alpha = 0$

(b) $\beta = \omega\sqrt{LC} = 25.13\, radians/m$

(c) $v_p = \omega/\beta = \dfrac{1}{\sqrt{LC}} = 2.5\times10^8\ m/s$

(d) $Z_o = \sqrt{L/C} = 20\,\Omega$

(e) $\varepsilon_r = \dfrac{200}{50} = 4$