NAME **Solutions** NETID ____________

MIDTERM EXAM

ECE 451

March 10, 2025

12:00 – 12:50 pm

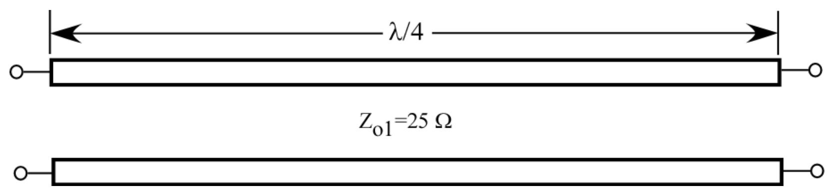
Instructions: Write your name and NetID where indicated. This examination consists of 3 problems. This is an open-book and open-notes exam. Use 50 Ω as the reference impedance for all measurement systems.

Problem 1 (40 pts)	Problem 2 (40 pts)	Problem 3 (20 pts)	Total (100 pts)

Mason's non-touching loop rule:

$$T = \frac{P_1\left[1-\sum L(1)^{(1)} + \sum L(2)^{(1)} - \ldots\right] + P_2\left[1-\sum L(1)^{(2)} + \sum L(2)^{(2)} - \ldots\right] + \ldots}{1-\sum L(1) + \sum L(2) - \sum L(3) + \ldots}$$

1. For the transmission line shown below, write the scattering parameter matrix as measured on a 50-Ω network analyzer.



Solutions

$$S_{11} = \frac{\left(1-X^2\right)\Gamma}{1-X^2\Gamma^2} \quad \text{and} \quad S_{21} = \frac{\left(1-\Gamma^2\right)X}{1-X^2\Gamma^2}$$

with $\Gamma = \dfrac{Z_{o1}-Z_o}{Z_{o1}+Z_o}$ and $X = e^{-j\frac{2\pi}{\lambda}l}$

$$X = e^{-j\frac{2\pi}{\lambda}\frac{\lambda}{4}} = e^{-j\pi/2} = -j$$

$$\Gamma = \frac{25-50}{25+50} = \frac{-25}{75} = -\frac{1}{3}$$

$$S_{11} = \frac{\left(1-(-j)^2\right)(-1/3)}{1-(-j)^2(1/9)} = \frac{-2(1/3)}{1+1/9} = -0.6$$

$$S_{21} = \frac{(1-1/9)(-j)}{1-(-j)^2(1/9)} = \frac{-8j/9}{1+1/9} = -j0.8$$

$$S = \begin{bmatrix} -0.6 & -j0.8 \\ -j0.8 & -0.6 \end{bmatrix}$$

2. A transmission line of characteristic impedance Z_o, length d and propagation constant β is terminated with an open.

   (a) Find the input impedance.

   (b) Draw a rough sketch of Z_{in}/Z_o for βd ranging from 0 to π and label the frequency bands where the transmission line looks capacitive and where it looks inductive.

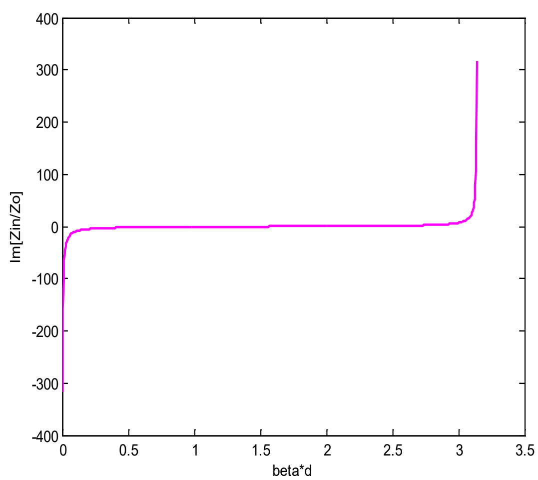
   (c) At what frequencies does this open transmission line look like a short circuit?

(a) For a transmission line of length d, we have:

$$Z_{in} = Z_o \left\{ \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d} \right\}$$

If $Z_L \to \infty$, then $Z_{in} = -jZ_o \text{cotan}\, \beta d$

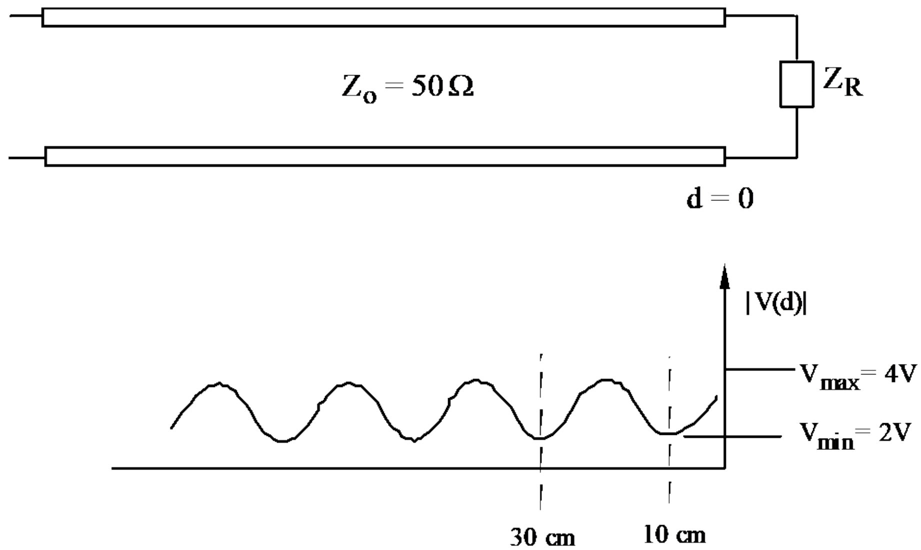
(b)



(c) The TL looks like a short for

$$\beta d = \frac{(2n+1)\pi}{2}, n = 0,1,2,... \text{ or } \frac{2\pi f d}{v} = \frac{(2n+1)\pi}{2}, n = 0,1,2,...$$

where v is the propagation velocity in the TL. This leads to: $f = \frac{(2n+1)v}{4d}, n = 0,1,2,...$



2. A slotted line is made of coaxial conductors with air as the dielectric. The characteristic impedance is 50 Ω. When a load Z_R is connected to the slotted line, the voltage magnitude is that of the plot shown in the figure.

   (a) What is the frequency of the signal ?

$$\frac{\lambda}{2} = 20 \text{ cm} \Rightarrow \lambda = 40 \text{ cm}$$

$$f = \frac{c}{\lambda} = \frac{3\times 10^{8}}{40\times 10^{-2}} = 750\ MHz$$

   (b) What is the value of Z_R ?

$$d_{\min} = 10 \text{ cm} = \frac{\lambda}{4}$$

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{4}{2} = 2$$

$$\Gamma_R = -\left(\frac{VSWR-1}{VSWR+1}\right)e^{+2j\beta d_{min}}$$

$$\Gamma_R = -\left(\frac{2-1}{2+1}\right)e^{+2j\frac{2\pi}{\lambda}\frac{\lambda}{4}} = \frac{1}{3}$$

$$Z_R = Z_o\left(\frac{1+\Gamma_R}{1-\Gamma_R}\right) = 50\left(\frac{1+1/3}{1-1/3}\right) = 100\ \Omega$$