1. (10 points) Let $f(x) = 4x^2 - 9$. Use the definition of a derivative as a limit to show that $f'(x) = 8x$. Show each step in your calculation and be sure to use proper terminology.

2. (10 points) Evaluate the following derivatives.

   (a) $\dfrac{d}{dx}\left(\sec x\right) =$

   (b) $\dfrac{d}{dx}\left(\tan x\right) =$

   (c) $\dfrac{d}{dx}\left(\tan^{-1} x\right) =$

   (d) $\dfrac{d}{dx}\left(\sin^{-1} x\right) =$

   (e) $\dfrac{d}{dx}\left(e^x\right) =$

3. (6 points) Find $g'(t)$ given that $g(t) = 12t^3 - 8t^2 + 13t - 32$

4. (6 points) Find $f'(x)$ given that $f(x) = x^6 \ln x$

5. (6 points) Find $P'(t)$ given that $P(t) = 30\left(t^6 - 5t^3 + 8\right)^4$

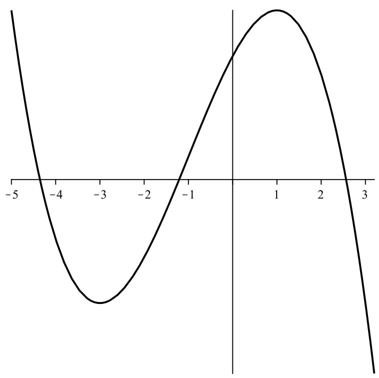
6. (4 points) Find $\dfrac{dy}{dx}$ given that $y = x^{4x}$

7. (4 points) Find $\dfrac{dy}{dx}$ given that $\sin y = \dfrac{x}{y}$

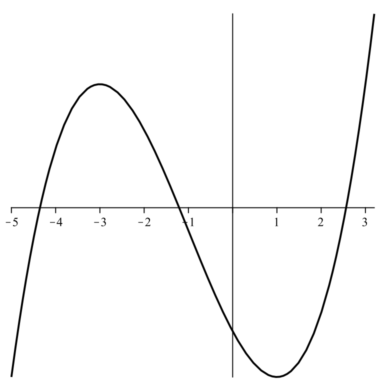
8. (4 points) The graph of a function $y = f(x)$ has the property that the slope of the tangent line at each point on this graph is equal to twice its y-coordinate. If the graph goes through the point $(0, 5)$, then find a formula for $f(x)$.

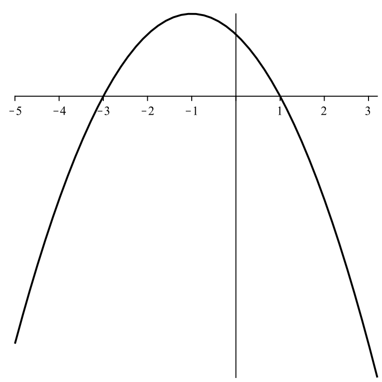
9. (10 points) A spherical balloon is inflated at a rate of 150 cubic feet per minute. How quickly is the balloon's radius increasing at the instant the radius is 5 feet?

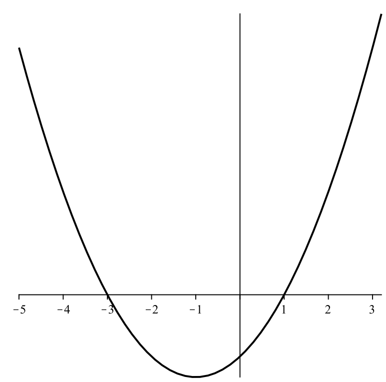
10. (5 points) The graph of $f(x)$ is shown below.

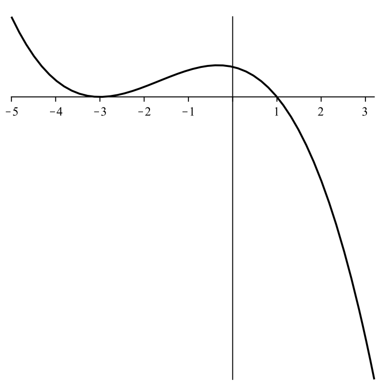


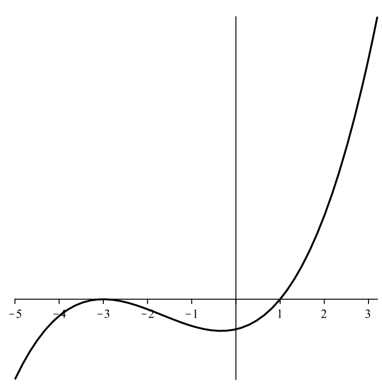
Circle the graph of $f'(x)$, given that it is one of the six choices below.

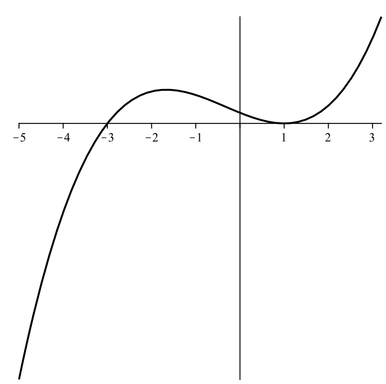












11. (8 points) Find each value of x on the interval $[0, 2\pi]$ at which the graph of $y = \sin x + \cos^2 x$ has a horizontal tangent line.

12. (7 points) What are the coordinates (x, y) for the highest point on the graph of the function $g(x) = \dfrac{\ln x}{x}$. Be sure each coordinate is in simplified form.

13. (5 points) A function $f(x)$ is given below along with its first and second derivatives in factored and unfactored forms.

- $f(x) = x^4 - 4x^3 + 16x - 16 = (x+2)(x-2)^3$
- $f'(x) = 4x^3 - 12x^2 + 16 = 4(x+1)(x-2)^2$
- $f''(x) = 12x^2 - 24x = 12x(x-2)$

The graph of $f(x)$ is concave down upon which one of the following intervals?

(a) $(-2, 2)$ (b) $(-1, 2)$

(c) $(0, 2)$ (d) $(-\infty, -2)$

(e) $(-\infty, -1)$ (f) $(-\infty, 0)$

(g) $(-2, \infty)$ (h) $(-1, \infty)$

(i) $(0, \infty)$ (j) $(-\infty, \infty)$

14. (5 points) A function $g(x)$ has the following derivative.

$$g'(x) = 5e^x(x-1)^2(x-2)^3(x-3)^4$$

Which one of the following statements is true about the graph of $g(x)$?

(a) There is a local minimum at $x = -1$

(b) There is a local minimum at $x = 0$

(c) There is a local minimum at $x = 1$

(d) There is a local minimum at $x = 2$

(e) There is a local minimum at $x = 3$

(f) There is a local maximum at $x = -1$

(g) There is a local maximum at $x = 0$

(h) There is a local maximum at $x = 1$

(i) There is a local maximum at $x = 2$

(j) There is a local maximum at $x = 3$

15. (5 points) From a height of 8 feet, a ball is thrown straight up with an initial velocity of 16 feet per second. Until it hits the ground, the ball's height in feet above ground level is given by $h = -16t^2 + 16t + 8$ where t is the number of seconds after the ball is thrown. What is the maximum height above ground level attained by the ball?

    (a) 5 feet

    (b) 6 feet

    (c) 8 feet

    (d) 9 feet

    (e) 12 feet

    (f) 16 feet

    (g) 24 feet

    (h) 32 feet

16. (5 points) If f is an even function which is differentiable everywhere, then show very clearly how Rolle's Theorem can be used to prove the existence of a real number c for which $f'(c) = 0$.