MATH 220 **Test 3** **Fall 2010**

Name *SOLUTIONS*

(circle your TA discussion section)

- ▷ **BD1**, MW 11:00-12:50, Noel DeJarnette
- ▷ **BD2**, TR 9:00-9:50, Kyle Knee
- ▷ **BD3**, TR 10:00-10:50, Scott Tichenor
- ▷ **BD4**, TR 2:00-2:50, Dan Schultz
- ▷ **BD5**, TR 11:00-11:50, Scott Tichenor
- ▷ **BD7**, MW 1:00-2:50, Scott Wetter
- ▷ **BD8**, TR 12:00-12:50, Hong Liu
- ▷ **BD9**, TR 1:00-1:50, Dan Schultz
- ▷ **DD1**, TR 11:00-11:50, Sarah Son
- ▷ **DD2**, TR 10:00-10:50, Sarah Son
- ▷ **DD4**, TR 12:00-12:50, Donghoon Jang
- ▷ **DD5**, TR 1:00-1:50, Donghoon Jang
- ▷ **DD6**, MW 1:00-2:50, Jesse Beder
- ▷ **DD7**, TR 8:00-8:50, Mu-Tsun Tsai
- ▷ **DD8**, MW 9:00-10:50, Joseph Vandehey
- ▷ **DD9**, TR 9:00-9:50, Mu-Tsun Tsai
- ▷ **AD1**, TR 11:00-11:50, Stephen Longfield
- ▷ **AD2**, TR 2:00-2:50, Lisa Hickok
- ▷ **AD3**, TR 1:00-1:50, Stephen Longfield
- ▷ **AD6**, TR 9:00-9:50, Lisa Hickok
- ▷ **AD7**, TR 3:00-3:50, Sarah Yeakel
- ▷ **AD8**, TR 4:00-4:50, Daniel Hockensmith

- Sit in your assigned seat (shown below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

263	264	265	266	267	268	269	270	•	271	272	273					278	279	•	280	281	282	283	284	285	286	287
	240	241	242	243	244	245	246	•	247	248	249	250	251	252	253	254	255	•	256	257	258	259	260	261	262	
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FRONT OF ROOM – 314 Altgeld Hall

1. (6 points) The function $f(x) = 10x^3 - 20x + 1$ has one root in the interval $[1, 2]$. In order to approximate this root, begin with an initial estimate of $x_1 = 2$ and use Newton's Method to obtain a second estimate x_2. Write the value of x_2 in decimal form.

$f'(x) = 30x^2 - 20$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 2$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2 - \frac{f(2)}{f'(2)}$

$= 2 - \frac{41}{100} = 2 - 0.41 = 1.59$

2. (4 points) Precisely state *The Mean Value Theorem.*

If f is continuous on $[a,b]$
and differentiable on (a,b)
then there is a number c in (a,b)
such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

3. (6 points) A function $f(x)$ has derivative $f'(x) = 6x^2 + 5$. Find a formula for $f(x)$ given that its graph goes through the point $(1, 15)$.

$f(x) = 2x^3 + 5x + c$

$f(1) = 15$ so

$15 = 2 + 5 + c$

$c = 8$

$f(x) = 2x^3 + 5x + 8$

4. (6 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{14k}{n^2}-\frac{4}{n}\right) = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{14k}{n^2}-\sum_{k=1}^{n}\frac{4}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{14}{n^2}\sum_{k=1}^{n}k-\frac{4}{n}\sum_{k=1}^{n}1\right)$$

$$= \lim_{n\to\infty}\left(\frac{14}{n^2}\cdot\frac{n(n+1)}{2}-\frac{4}{n}\cdot n\right)$$

$$= \lim_{n\to\infty}\left(\frac{14(n+1)}{2n}-4\right)$$

$$= \lim_{n\to\infty}\left(\frac{14n}{2n}+\frac{14}{2n}-4\right)$$

$$= \lim_{n\to\infty}\left(7+\frac{7}{n}-4\right)$$

$$= 7+0-4 = 3$$

5. (6 points) The height of a tree is currently 100 inches. It is predicted that over the next 4 years the tree's height will increase by $10-3\sqrt{t}$ inches per year where t represents the number of years from now. What will the tree's height be 4 years from now? Simplify your answer.

height in 4 years = current height + change in height

$$= 100 + \int_0^4 (10-3\sqrt{t})\,dt$$

$$= 100 + \left(10t-2t^{3/2}\right)\Big|_0^4$$

$$= 100 + \left(40-2(4)^{3/2}\right)-(0)$$

$$= 100+40-16$$

$$= 124 \text{ inches}$$

6. (6 points each) Evaluate the following ~~definite~~ and indefinite integrals.

(a) $\int \left(\frac{8}{x} + 4\csc^2 x + 3\right)\,dx$ $= 8\ln|x| - 4\cot x + 3x + C$

(b) $\int_{\pi/2}^{\pi} (10 + 3\cos x)\,dx$ $= (10x + 3\sin x)\Big|_{\pi/2}^{\pi}$

$= (10\pi + 3\sin\pi) - (10\cdot\frac{\pi}{2} + 3\sin(\frac{\pi}{2}))$

$= (10\pi + 3\cdot 0) - (5\pi + 3\cdot 1)$

$= 5\pi - 3$

(c) $\int_0^2 (6x + 2e^{-x})\,dx$ $= (3x^2 - 2e^{-x})\Big|_0^2$

$= (12 - 2e^{-2}) - (0 - 2)$

$= 14 - 2e^{-2}$

(d) $\int x^3 \left(x^4+7\right)^5 dx$ $= \int \frac{1}{4} u^5 du$

$u = x^4 + 7$
$du = 4x^3 dx$
$\frac{1}{4} du = x^3 dx$

$= \frac{1}{24} u^6 + C$

$= \frac{1}{24}(x^4+7)^6 + C$

(e) $\int \sin^3 x \cos^5 x \, dx$ $= \int \sin^2 x \cos^5 x \sin x \, dx$

$u = \cos x$
$du = -\sin x \, dx$

$= \int (1-\cos^2 x)\cos^5 x \sin x \, dx$

$= \int (1-u^2)u^5(-du)$

$= \int (-u^5 + u^7) du$

$= -\frac{1}{6}u^6 + \frac{1}{8}u^8 + C$

$= -\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + C$

an alternate approach

$\int \sin^3 x \cos^5 x \, dx =$
$\int \sin^3 x \cos^4 x \cos x \, dx =$
$\int \sin^3 x (1-\sin^2 x)^2 \cos x \, dx =$
$\int u^3(1-u^2)^2 du$ where $u = \sin x$, $du = \cos x \, dx$
$= \int (u^3 - 2u^5 + u^7) du$
$= \frac{1}{4}u^4 - \frac{1}{3}u^6 + \frac{1}{8}u^8 + C$
$= \frac{1}{4}\sin^4 x - \frac{1}{3}\sin^6 x + \frac{1}{8}\sin^8 x + C$

(f) $\int \left(5 - 3\tan^2 x\right)\, dx = \int (5 - 3(\sec^2 x - 1))\,dx$

$= \int (8 - 3\sec^2 x)\,dx$

$= 8x - 3\tan x + C$

7. (4 points each) Suppose that f is integrable on the interval $[2, 12]$. Given that $\int_2^{12} f(x)\,dx = 25$, $\int_2^8 f(x)\,dx = 10$ and $\int_4^{12} f(x)\,dx = 22$, evaluate the following definite integrals.

(a) $\int_8^2 f(x)\,dx = -\int_2^8 f(x)\,dx = -10$

(b) $\int_2^4 f(x)\,dx = \int_2^{12} f(x)\,dx + \int_{12}^4 f(x)\,dx$

$= \int_2^{12} f(x)\,dx - \int_4^{12} f(x)\,dx$

$= 25 - 22 = 3$

(c) $\int_4^8 f(x)\,dx = \int_4^{12} f(x)\,dx + \int_{12}^8 f(x)\,dx$

$= \int_4^{12} f(x)\,dx + \int_{12}^2 f(x)\,dx + \int_2^8 f(x)\,dx$

$= \int_4^{12} f(x)\,dx - \int_2^{12} f(x)\,dx + \int_2^8 f(x)\,dx$

$= 22 - 25 + 10$

$= 7$

8. (6 points each) Let **R** be the region bounded above by graph of $y = \dfrac{\sin x}{x}$ and bounded below by the x-axis on the interval $[2\pi, 3\pi]$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

(a) The area of **R**.

$$A = \int_{2\pi}^{3\pi} \frac{\sin x}{x}\, dx$$

(b) The volume of the solid obtained when **R** is revolved around the x-axis.

$$V = \int_{2\pi}^{3\pi} \underbrace{\pi \left(\frac{\sin x}{x}\right)^2}_{\pi (\text{radius})^2} dx$$

(c) The volume of the solid obtained when **R** is revolved around the vertical line $x = 3$.

$$V = \int_{2\pi}^{3\pi} \underbrace{2\pi (x-3) \frac{\sin x}{x}}_{2\pi \cdot \text{radius} \cdot \text{height}} dx$$

9. (6 points) Suppose $F(x)$ is a polynomial with $F'(x) = f(x)$. Given that $F(0) = 2$, $F(2) = 8$, $F(4) = 28$, $F(6) = 68$ and $F(8) = 42$, find the average value of $f(x)$ on the interval $[2, 6]$.

$$f_{ave} = \frac{1}{6-2}\int_2^6 f(x)\,dx$$

$$= \frac{1}{4}\left(F(6) - F(2)\right)$$

$$= \frac{1}{4}(68 - 8)$$

$$= 15$$

Students – do not write on this page!

1 (6 points) ______________________

2 (4 points) ______________________

3 (6 points) ______________________

4 (6 points) ______________________

5 (6 points) ______________________

6a (6 points) ______________________

6b (6 points) ______________________

6c (6 points) ______________________

6d (6 points) ______________________

6e (6 points) ______________________

6f (6 points) ______________________

7 (12 points) ______________________

8a (6 points) ______________________

8b (6 points) ______________________

8c (6 points) ______________________

9 (6 points) ______________________

TOTAL (100 points) ____________