MATH 220 (sections AL1 and BL1) **Test 3** **Spring 2010**

1. (8 points) Find a formula for $f(x)$ given that $f''(x) = 5\sin x + 3\cos x$, $f(0) = 10$, and $f'(0) = 10$.

2. (6 points) The population of a town is currently 400, but it is expected to increase at a rate of $200e^{0.5t}$ people per year where t represents the number of years from now. What is the population of this town expected to be in 10 years?

3. (6 points) Evaluate the following limit.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{5k}{n^3} + \frac{7}{n} \right)$$

4. (6 points) The definite integral $\int_2^6 e^{t^2}\, dt$ can be written as a limit. Fill in the missing information in this limit.

$$\int_2^6 e^{t^2}\, dt = \lim_{n\to\infty} \sum_{k=1}^{n} \Big[\qquad\qquad\qquad\qquad \Big]$$

5. (12 points) Suppose that f is an odd function and g is an even function which are each integrable on the interval $[-5, 5]$. Given that $\int_0^5 f(x)\, dx = 8$ and $\int_0^5 g(x)\, dx = 3$, evaluate the following definite integrals.

   (a) $\int_5^0 g(x)\, dx$

   (b) $\int_5^5 f(x)\, dx$

   (c) $\int_{-5}^5 (2f(x) + 4g(x))\, dx$

   (d) $\int_{-5}^5 \left(4 + (f(x))^3\right) dx$

6. (6 points each) Evaluate the following definite and indefinite integrals.

(a) $\int \left(e^x + \frac{1}{3x} + 5\right)\, dx$

(b) $\int_1^2 (10x + 5)\, dx$

(c) $\int_0^2 \left(3 + 2e^{-x}\right)\, dx$

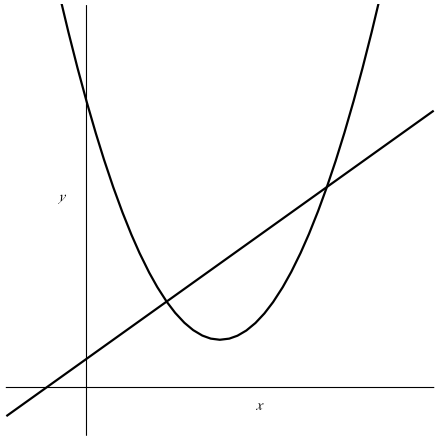
(d) $\int x^2\sqrt{x^3 + 4}\, dx$

7. (5 points each) Evaluate the following indefinite integrals.

(a) $\int x^2(x+4)^{10}\,dx$

(b) $\int \sec^6 x \tan^3 x\,dx$

8. (6 points each) Let $\mathbf{R}$ be the region bounded by the graphs of $f(x) = x^2 - 10x + 30$ and $g(x) = 2x + 3$ as shown below. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.



(a) The area of $\mathbf{R}$.

(b) The volume of the solid obtained when $\mathbf{R}$ is revolved around the y-axis.

(c) The volume of the solid obtained when $\mathbf{R}$ is revolved around the horizontal line $y = -10$.

9. (5 points each) Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

   (a) The average value of $f(x) = \dfrac{e^{3x}}{\pi - 2}$ on the interval $[3, \pi]$.

   (b) The length of the curve $f(x) = 3\sin 2x$ for $0 \leq x \leq 2\pi$.