MATH 220 **Test 2** **Fall 2011**

Name SOLUTIONS NetID ________________

- Sit in your assigned seat (circled below).
- Circle your TA discussion section.
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

- ▷ **BD1**, TR 11:00-12:50, Vicki Reuter
- ▷ **BD2**, TR 9:00-9:50, Tom Mahoney
- ▷ **BD3**, TR 10:00-10:50, Kyle Knee
- ▷ **BD4**, TR 2:00-2:50, Neha Gupta
- ▷ **BD5**, TR 12:00-12:50, Nate Orlow
- ▷ **BD6**, TR 9:00-10:50, Ser-Wei Fu
- ▷ **BD7**, TR 3:00-3:50, Chayapa Darayon
- ▷ **BD8**, TR 1:00-1:50, Eliana Duarte

- ▷ **DD1**, TR 11:00-11:50, Nate Orlow
- ▷ **DD2**, TR 10:00-10:50, Santiago Camacho
- ▷ **DD3**, TR 9:00-9:50, Sarah Loeb
- ▷ **DD4**, TR 12:00-12:50, Lisa Hickok
- ▷ **DD5**, TR 1:00-1:50, Lisa Hickok
- ▷ **DD6**, TR 1:00-2:50, Jennifer Wise
- ▷ **DD7**, TR 8:00-8:50, Sarah Loeb
- ▷ **DD8**, TR 1:00-1:50, Abdulla Eid

- ▷ **AD1**, TR 11:00-11:50, Abdulla Eid
- ▷ **AD2**, TR 2:00-2:50, Ilkyoo Choi
- ▷ **AD3**, TR 1:00-1:50, Ilkyoo Choi
- ▷ **AD4**, TR 9:00-9:50, Michael Santana
- ▷ **AD5**, TR 3:00-3:50, Santiago Camacho
- ▷ **AD6**, TR 4:00-4:50, Joe Nance
- ▷ **AD7**, TR 3:00-3:50, Neha Gupta

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FRONT OF ROOM – 314 Altgeld Hall

1. (8 points) Find $g'(t)$ given that $g(t) = 5t^6 - 4t^3 + 10t - e^2$

$$g'(t) = 5(6t^5) - 4(3t^2) + 10 - 0$$

$$g'(t) = 30t^5 - 12t^2 + 10$$

note: $(e^2)' = 0$ since e^2 is a constant

2. (8 points) Find $\dfrac{dv}{dt}$ given that $v = 5t^4 \tan^{-1} t$

$$\frac{dv}{dt} = \frac{d}{dt}(5t^4) \cdot \tan^{-1} t + 5t^4 \cdot \frac{d}{dt}(\tan^{-1} t)$$

$$\frac{dv}{dt} = 20t^3 \tan^{-1} t + 5t^4 \cdot \frac{1}{1+t^2}$$

$$\frac{dv}{dt} = 20t^3 \tan^{-1} t + \frac{5t^4}{1+t^2}$$

3. (8 points) Find $f'(x)$ given that $f(x) = \dfrac{\ln x}{x^3 + 4}$

$$f'(x) = \frac{(\ln x)'(x^3+4) - (\ln x)(x^3+4)'}{(x^3+4)^2}$$

$$f'(x) = \frac{\frac{1}{x}(x^3+4) - \ln x\,(3x^2)}{(x^3+4)^2}$$

4. (8 points) Find $h'(t)$ given that $h(t) = \sin\left(e^{2t}\right)$

$$h'(t) = \cos(e^{2t}) \cdot (e^{2t})'$$

$$h'(t) = \cos(e^{2t}) \cdot e^{2t} \cdot 2$$

5. (8 points) Find the slope of the line tangent to the curve $x^2y^3 = 3x - 2y$ at the point $(2, 1)$.

$$\frac{d}{dx}(x^2y^3) = \frac{d}{dx}(3x - 2y)$$

$$\frac{d}{dx}(x^2)\,y^3 + x^2\frac{d}{dx}(y^3) = 3 - 2\frac{dy}{dx}$$

$$2xy^3 + x^2 \cdot 3y^2\frac{dy}{dx} = 3 - 2\frac{dy}{dx}$$

At $(x,y) = (2,1)$,

$$2\cdot 2\cdot 1^3 + 2^2\cdot 3\cdot 1^2\cdot\frac{dy}{dx} = 3 - 2\frac{dy}{dx}$$

$$4 + 12\frac{dy}{dx} = 3 - 2\frac{dy}{dx}$$

$$14\frac{dy}{dx} = -1$$

$$\frac{dy}{dx} = \frac{-1}{14}$$

6. (8 points) A function $f(x)$ has the following second derivative.

$$f''(x) = (x+5)^2 - 4 \quad = (x+5-2)(x+5+2) = (x+3)(x+7)$$

What is the largest open interval upon which the graph of $f(x)$ is concave down?

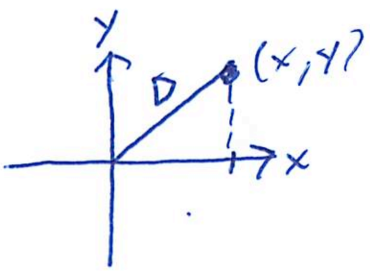
values of $f''(x)$

$+ + + \quad 0 \quad - - - - \quad 0 \quad + + +$

x: -7, -3

f is concave down on $(-7, -3)$

7. (8 points) A particle moves along the curve given below. As the particle passes through the point $(3, 4)$, its x-coordinate increases at a rate of 15 cm/s. How fast is the distance from the particle to the origin changing at this instant?



$$y = \frac{4}{9}x^2$$

$$\frac{dy}{dt} = \frac{8}{9}x\frac{dx}{dt}$$

$$D^2 = x^2 + y^2$$

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

At $(x,y) = (3,4)$, $D^2 = 3^2 + 4^2 \Rightarrow D = 5$

and $2 \cdot 5\frac{dD}{dt} = 2 \cdot 3 \cdot 15 + 2 \cdot 4 \cdot \left(\frac{8}{9} \cdot 3 \cdot 15\right)$

$$10\frac{dD}{dt} = 90 + 320$$

$$\frac{dD}{dt} = 41 \text{ cm/s}$$

note $e^x > 0$ for all x

8. (8 points) A function $g(x)$ has the following derivative.

$$g'(x) = 5e^x(x-1)^2(x-2)^3(x-3)^4$$

Determine the x-value for each local maximum and local minimum on the graph of $g(x)$.

values of $g'(x)$

$$\begin{array}{ccccccc} - - & 0 & - - & 0 & + + & 0 & + + \\ & 1 & & 2 & & 3 & \longrightarrow x \\ g \text{ decr.} & & g \text{ decr.} & & g \text{ incr.} & & g \text{ incr.} \end{array}$$

There is a local minimum for g at $x=2$

9. (12 points) A ball is tossed straight up with an initial velocity of 16 feet per second. The ball is 5 feet above the ground when it is released. Its height at time t is given by

$$h = -16t^2 + 16t + 5$$

What is the ball's maximum height?

height: $h = -16t^2 + 16t + 5$

velocity: $h' = -32t + 16$

$h' = -16(2t-1)$

$h' = 0$ when $2t - 1 = 0$ (i.e. $t = \frac{1}{2}$)

values of h'

$$\begin{array}{ccc} + + + & 0 & - - - \\ & \frac{1}{2} & \longrightarrow t \end{array}$$

max. height at $t = 1/2$ is

$h(\frac{1}{2}) = -16(\frac{1}{2})^2 + 16(\frac{1}{2}) + 5$

$= -4 + 8 + 5 = 9 \text{ ft}$

10. (8 points each) Evaluate the following limits.

(a) $\lim_{x\to 1} \dfrac{x^2+3x-4}{\sin(x-1)}$ $\xrightarrow{} 0$, $\to 0$

$$= \lim_{x\to 1} \frac{2x+3}{\cos(x-1)} \quad \text{by l'Hospital's Rule}$$

$$= \frac{5}{1}$$

$$= 5$$

(b) $\lim_{x\to \pi/4} \dfrac{4x-\pi}{4\tan x}$ $\to 0$, $\to 4$

$$= 0$$

(c) $\lim_{x\to\infty} \left(1-\dfrac{2}{x}\right)^{3x}$

$$= \lim_{x\to\infty} e^{\ln\left(1-\frac{2}{x}\right)^{3x}}$$

(of indeterminate form 1^∞)

$$= e^{\lim_{x\to\infty} \ln\left(1-\frac{2}{x}\right)^{3x}}$$

$$= e^{\lim_{x\to\infty} 3x\ln\left(1-\frac{2}{x}\right)}$$

$$= e^{\lim_{x\to\infty} \frac{3\ln(1-2/x)}{1/x}} \quad \to 0,\ \to 0$$

$$= e^{\lim_{x\to\infty} \frac{\frac{3}{1-2/x}\cdot 2x^{-2}}{-x^{-2}}} \quad \text{by l'Hospital's Rule}$$

$$= e^{\lim_{x\to\infty} \frac{-6}{1-2/x}}$$

$$= e^{-6}$$

alternate approach

$$\lim_{x\to\infty}\left(1-\frac{2}{x}\right)^{3x} = \lim_{x\to\infty}\left(\left(1+\frac{-2}{x}\right)^{x}\right)^{3} = \left(e^{-2}\right)^{3} = e^{-6}$$

Students – do not write on this page!

1. (8 points) ______________________

2. (8 points) ______________________

3. (8 points) ______________________

4. (8 points) ______________________

5. (8 points) ______________________

6. (8 points) ______________________

7. (8 points) ______________________

8. (8 points) ______________________

9. (12 points) ______________________

10a. (8 points) ______________________

10b. (8 points) ______________________

10c. (8 points) ______________________

TOTAL (100 points) ____________