MATH 220 **Test 3** **Fall 2011**

Name SOLUTIONS **NetID** ________________

- Sit in your assigned seat (circled below).
- Circle your TA discussion section.
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- You must show sufficient work to justify each answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

▷ **BD1**, TR 11:00-12:50, Vicki Reuter
▷ **BD2**, TR 9:00-9:50, Tom Mahoney
▷ **BD3**, TR 10:00-10:50, Kyle Knee
▷ **BD4**, TR 2:00-2:50, Neha Gupta
▷ **BD5**, TR 12:00-12:50, Nate Orlow
▷ **BD6**, TR 9:00-10:50, Ser-Wei Fu
▷ **BD7**, TR 3:00-3:50, Chayapa Darayon
▷ **BD8**, TR 1:00-1:50, Eliana Duarte

▷ **DD1**, TR 11:00-11:50, Nate Orlow
▷ **DD2**, TR 10:00-10:50, Santiago Camacho
▷ **DD3**, TR 9:00-9:50, Sarah Loeb
▷ **DD4**, TR 12:00-12:50, Lisa Hickok
▷ **DD5**, TR 1:00-1:50, Lisa Hickok
▷ **DD6**, TR 1:00-2:50, Jennifer Wise
▷ **DD7**, TR 8:00-8:50, Sarah Loeb
▷ **DD8**, TR 1:00-1:50, Abdulla Eid

▷ **AD1**, TR 11:00-11:50, Abdulla Eid
▷ **AD2**, TR 2:00-2:50, Ilkyoo Choi
▷ **AD3**, TR 1:00-1:50, Ilkyoo Choi
▷ **AD4**, TR 9:00-9:50, Michael Santana
▷ **AD5**, TR 3:00-3:50, Santiago Camacho
▷ **AD6**, TR 4:00-4:50, Joe Nance
▷ **AD7**, TR 3:00-3:50, Neha Gupta

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FRONT OF ROOM – 314 Altgeld Hall

1. (3 points each) Suppose that f is an odd function which is integrable on the interval $[-5, 5]$. If $\int_0^2 f(x)\,dx = 4$ and $\int_2^3 f(x)\,dx = 10$, then evaluate the following quantities.

(a) $\int_0^5 f(x)\,dx + \int_5^3 f(x)\,dx = \int_0^3 f(x)dx = \int_0^2 f(x)dx + \int_2^3 f(x)\,dx$

$= 4 + 10 = 14$

(b) $\int_{-2}^2 f(x)\,dx = 0$ since f is odd.

(c) $\int_{-2}^2 f(|x|)\,dx = \int_{-2}^0 f(|x|)dx + \int_0^2 f(|x|)dx$

there may be more intuitive ways to see this.

$= \int_{-2}^0 f(-x)dx + \int_0^2 f(x)\,dx$

let $u = -x$
$du = -dx$

$= \int_2^0 -f(u)du + \int_0^2 f(x)dx = \int_0^2 f(u)du + \int_0^2 f(x)dx$

$= 4 + 4 = 8$

2. (10 points each) Evaluate the following definite integrals. Simplify each answer.

(a) $\int_2^{18} \frac{1}{2x}\,dx = \frac{1}{2}\int_2^{18} \frac{1}{x}\,dx$

$= \frac{1}{2}\left[\ln|x|\right]_2^{18}$

$= \frac{1}{2}\left[\ln 18 - \ln 2\right]$

$= \frac{1}{2}\ln\left(\frac{18}{2}\right)$

$= \frac{1}{2}\ln(9)$

$= \frac{1}{2}\ln(3^2)$

$= \frac{1}{2}\cdot 2\ln 3$

$= \ln 3$

(b) $\int_0^1 \frac{8}{1+x^2}\,dx$ $= 8\int_0^1 \frac{1}{1+x^2}\,dx$

$= 8\left[\tan^{-1}x\right]_0^1$

$= 8\left[\tan^{-1}(1) - \tan^{-1}(0)\right]$

$= 8\left[\frac{\pi}{4} - 0\right]$

$= 2\pi$

3. (10 points each) Evaluate the following indefinite integrals.

(a) $\int \frac{12x}{1+3x^2}\,dx$ $= \int \frac{2}{u}\,du = 2\int \frac{1}{u}\,du$

$u = 1+3x^2$

$du = 6x\,dx$

$2du = 12x\,dx$

$= 2\ln|u| + C$

$= 2\ln|1+3x^2| + C$

$= 2\ln(1+3x^2) + C$

(b) $\int \tan x \sec^5 x \, dx = \int \sec^4 x \sec x \tan x \, dx$

$u = \sec x$
$du = \sec x \tan x \, dx$

$= \int u^4 \, du$

$= \frac{1}{5} u^5 + C$

$= \frac{1}{5} \sec^5 x + C$

or $\int \tan x \sec^5 x \, dx = \int \frac{\sin x}{\cos^6 x} dx$

$u = \cos x$
$du = -\sin x \, dx$

$= \int \frac{-du}{u^6}$

$= \int -u^{-6} du$

$= \frac{-u^{-5}}{-5} + C = \frac{1}{5\cos^5 x} + C$

4. (5 points) Evaluate the following indefinite integral.

$\int 2x^5 (x^2+1)^{35} \, dx = \int x^4 (x^2+1)^{35} 2x \, dx$

$u = x^2 + 1$
$du = 2x \, dx$
$u = x^2 + 1 \Rightarrow$
$u - 1 = x^2$

$= \int (x^2)^2 (x^2+1)^{35} 2x \, dx$

$= \int (u-1)^2 u^{35} du$

$= \int (u^{37} - 2u^{36} + u^{35}) du$

$= \frac{1}{38} u^{38} - \frac{2}{37} u^{37} + \frac{1}{36} u^{36} + C$

$= \frac{1}{38}(x^2+1)^{38} - \frac{2}{37}(x^2+1)^{37} + \frac{1}{36}(x^2+1)^{36} + C$

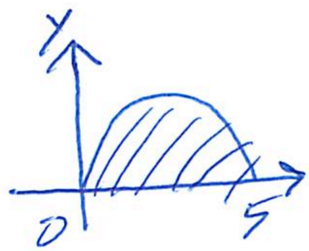
5. (5 points each) Let **R** be the finite region bounded by the graph of $f(x) = 5x - x^2$ and the x-axis on the interval $[0, 5]$. Set up, but do not evaluate, definite integrals which represent the given quantities. Use proper notation.

(a) The average value of f on the interval $[0, 5]$.

$$f_{ave} = \frac{1}{5-0}\int_0^5 (5x - x^2)\,dx$$
$$= \frac{1}{5}\int_0^5 (5x - x^2)\,dx$$

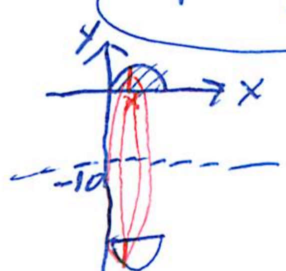
(b) The area of **R**.

$$\text{area} = \int_0^5 (5x - x^2)\,dx$$



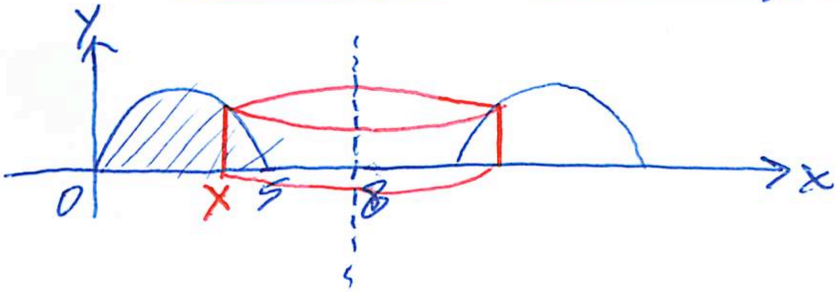
(c) The volume of the solid obtained when **R** is revolved around the horizontal line $y = -10$.

$$V = \int_0^5 \Big(\pi(\underbrace{5x - x^2 + 10}_{\text{big radius}})^2 - \pi(\underset{\uparrow\ \text{little radius}}{10})^2\Big)\,dx$$



(d) The volume of the solid obtained when **R** is revolved around the vertical line $x = 8$.

$$V = \int_0^5 2\pi\underset{\uparrow\ \text{rad}}{(8-x)}\underset{\uparrow\ \text{height}}{(5x-x^2)}\,dx$$



6. (5 points) Fill in the missing information for the following theorem.

Rolle's Theorem Let f be a function that satisfies the following three hypotheses:

(1) f is continuous on the closed interval $[a, b]$.

(2) f is differentiable on the open interval (a, b).

(3) $f(a) = f(b)$.

Then there is a number c in (a, b) such that $f'(c) = 0$.

7. (5 points) If Newton's Method is used to approximate a solution to the equation $f(x) = 0$, then it generates a sequence of approximations $x_1, x_2, x_3, x_4, \dots$. Which one of the following correctly shows how x_n can be used to determine the next approximation x_{n+1} ?

(a) $x_{n+1} = \dfrac{x_n + f'(x_n)}{f(x_n)}$

(b) $x_{n+1} = x_n + \dfrac{f'(x_n)}{f(x_n)}$

(c) $x_{n+1} = \dfrac{x_n + f(x_n)}{f'(x_n)}$

(d) $x_{n+1} = x_n + \dfrac{f(x_n)}{f'(x_n)}$

(e) $x_{n+1} = \dfrac{x_n - f'(x_n)}{f(x_n)}$

(f) $x_{n+1} = x_n - \dfrac{f'(x_n)}{f(x_n)}$

(g) $x_{n+1} = \dfrac{x_n - f(x_n)}{f'(x_n)}$

(h) $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

8. (5 points) Fill in the missing information to show that the given definite integral can be expressed as the limit of a Riemann sum. The only variables appearing in your limit should be n and k. You do not need to evaluate this limit.

$$\int_2^6 (x^5 + 8)^4 \, dx = \lim_{n\to\infty} \underbrace{\sum_{k=1}^{n} \left[\left(\left(2 + k \cdot \frac{4}{n}\right)^5 + 8 \right)^4 \cdot \frac{4}{n} \right]}_{\text{right Riemann sum}}$$

9. (6 points) Suppose that a polynomial f satisfies the following conditions.

- $f(1) = 8$
- $f'(1) = 2$
- $f''(1) = 3$
- $f'''(1) = 5$

Use a linear approximation to estimate the value of $f(0.8)$. Simplify and write your answer in decimal form.

point: $(1, 8)$

SLOPE: $f'(1) = 2$

tangent line: $y - 8 = 2(x-1)$

$y = 8 + 2(x-1)$

$f(x) \approx 8 + 2(x-1)$ for x near 1

$f(0.8) \approx 8 + 2(0.8-1)$

$f(0.8) \approx 8 - 0.4 = 7.6$

10. (5 points) Suppose that F and F' are each differentiable (and thus continuous) everywhere and that r and s are constants. Circle the choice below which most clearly states part 2 of the Fundamental Theorem of Calculus.

(a) $\int_r^s F'(t)\,dt = F'(r) - F'(s)$

(b) $\int_r^s F(t)\,dt = F'(r) - F'(s)$

(c) $\int_r^s F'(t)\,dt = F(r) - F(s)$

(d) $\int_r^s F(t)\,dt = F(r) - F(s)$

(e) $\int_r^s F'(t)\,dt = F'(s) - F'(r)$

(f) $\int_r^s F(t)\,dt = F'(s) - F'(r)$

(g) $\int_r^s F'(t)\,dt = F(s) - F(r)$

(h) $\int_r^s F(t)\,dt = F(s) - F(r)$

Students – do not write on this page!

1. (9 points) ______________

2a. (10 points) ______________

2b. (10 points) ______________

3a. (10 points) ______________

3b. (10 points) ______________

4. (5 points) ______________

5a. (5 points) ______________

5b. (5 points) ______________

5c. (5 points) ______________

5d. (5 points) ______________

6. (5 points) ______________

7. (5 points) ______________

8. (5 points) ______________

9. (6 points) ______________

10. (5 points) ______________

TOTAL (100 points) __________