AL1 (MWF 10:00-10:50) AL3 (MWF 1:00-1:50) AL2 (MWF 3:00-3:50)

MATH 220 **Test 2** **Fall 2018**

Name Solutions **NetID** _____.

UIN _____

Circle your TA discussion section.

- ▷ **AD1**, TR 11:00-12:50, Adriana Morales
- ▷ **AD2**, TR 9:00-10:50, Hannah Burson
- ▷ **AD3**, TR 1:00-2:50, Dana Neidinger
- ▷ **ADA**, TR 8:00-8:50, Gayana Jayasinghe
- ▷ **ADB**, TR 9:00-9:50, Felix Clemen
- ▷ **ADC**, TR 10:00-10:50, Lutian Zhao
- ▷ **ADD**, TR 11:00-11:50, Gidon Orelowitz
- ▷ **ADE**, TR 12:00-12:50, Josh Wen
- ▷ **ADF**, TR 1:00-1:50, Nachiketa Adhikari
- ▷ **ADG**, TR 2:00-2:50, Lutian Zhao
- ▷ **ADH**, TR 3:00-3:50, Stathis Chrontsios
- ▷ **ADI**, TR 4:00-4:50, Stathis Chrontsios
- ▷ **ADJ**, TR 9:00-9:50, Gayana Jayasinghe
- ▷ **ADK**, TR 10:00-10:50, Madina Bolat
- ▷ **ADL**, TR 11:00-11:50, Chris Loa
- ▷ **ADM**, TR 12:00-12:50, Heeyeon Kim
- ▷ **ADN**, TR 1:00-1:50, Josh Wen
- ▷ **ADO**, TR 2:00-2:50, Kesav Krishnan
- ▷ **ADQ**, TR 10:00-10:50, Felix Clemen
- ▷ **ADR**, TR 9:00-9:50, Madina Bolat
- ▷ **ADS**, TR 12:00-12:50, Chris Loa
- ▷ **ADT**, TR 2:00-2:50, Nachiketa Adhikari
- ▷ **ADU**, TR 3:00-3:50, Kesav Krishnan
- ▷ **ADZ**, TR 9:00-9:50, Gidon Orelowitz

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

	◇	◇	◇	◇
	Q1	Q2	Q3	Q4
	P1	P2	P3	P4
N1	N2	N3	N4	N5
M1	M2	M3	M4	M5
L1	L2	L3	L4	L5
K1	K2	K3	K4	K5
J1	J2	J3	J4	J5
H1	H2	H3	H4	H5
G1	G2	G3	G4	G5
F1	F2	F3	F4	F5
E1	E2	E3	E4	E5
D1	D2	D3	D4	D5
C1	C2	C3	C4	C5
B1	B2	B3	B4	B5
A1	◇	◇	◇	◇

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9		
P1	P2	P3	P4	P5	P6	P7	P8	P9		
N1	N2	N3	N4	N5	N6	N7	N8	N9	N10	
M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	
L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	
K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	
J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	
H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	
G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	
F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	
E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	
D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	
C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	
B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	

R1	R2	R3		
Q1	Q2	Q3	Q4	
P1	P2	P3	P4	P5
N1	N2	N3	N4	N5
M1	M2	M3	M4	M5
L1	L2	L3	L4	L5
K1	K2	K3	K4	K5
J1	J2	J3	J4	J5
H1	H2	H3	H4	H5
G1	G2	G3	G4	G5
F1	F2	F3	F4	F5
E1	E2	E3	E4	E5
D1	D2	D3	D4	D5
C1	C2	C3	C4	C5
B1	B2	B3	B4	B5
◇	◇	◇	◇	A5

FRONT OF ROOM – 114 David Kinley Hall

1. (10 points) Find $w'(x)$ given that $w(x) = 5x^9 e^{12x}$

$$w'(x) = \frac{d}{dx}(5x^9) \cdot e^{12x} + 5x^9 \cdot \frac{d}{dx}(e^{12x})$$

$$w'(x) = 45x^8 e^{12x} + 5x^9 \cdot 12e^{12x}$$

$$w'(x) = 45x^8 e^{12x} + 60x^9 e^{12x}$$

2. (10 points) Find $g'(x)$ given that $g(x) = \dfrac{x^8 + 3\cos(x)}{x^4 + 5\sin(x)}$

$$g'(x) = \frac{\frac{d}{dx}(x^8+3\cos(x)) \cdot (x^4+5\sin(x)) - (x^8+3\cos(x)) \cdot \frac{d}{dx}(x^4+5\sin(x))}{(x^4+5\sin(x))^2}$$

$$g'(x) = \frac{(8x^7 - 3\sin(x))(x^4+5\sin(x)) - (x^8+3\cos(x))(4x^3+5\cos(x))}{(x^4+5\sin(x))^2}$$

3. (10 points) Find $f'(x)$ given that $f(x) = \cot(\ln(\sec(5x)))$

$$f'(x) = -\csc^2(\ln(\sec(5x))) \cdot \frac{d}{dx}(\ln(\sec(5x)))$$

$$f'(x) = -\csc^2(\ln(\sec(5x))) \cdot \frac{1}{\sec(5x)} \cdot \frac{d}{dx}(\sec(5x))$$

$$f'(x) = -\csc^2(\ln(\sec(5x))) \cdot \frac{1}{\sec(5x)} \cdot \sec(5x)\tan(5x) \cdot \frac{d}{dx}(5x)$$

$$f'(x) = -\csc^2(\ln(\sec(5x))) \cdot \frac{1}{\sec(5x)} \cdot \sec(5x)\tan(5x) \cdot 5$$

4. (10 points) Find $\frac{dy}{dx}$ and write your answer in terms of x given the function $y = (8x)^{(9/x)}$

$$y = (8x)^{9/x} = e^{\ln((8x)^{9/x})} = e^{\frac{9}{x}\ln(8x)}$$

$$y = e^{\frac{9\ln(8x)}{x}}$$

$$\frac{dy}{dx} = e^{\frac{9\ln(8x)}{x}} \cdot \frac{d}{dx}\left(\frac{9\ln(8x)}{x}\right)$$

$$\frac{dy}{dx} = e^{\frac{9\ln(8x)}{x}} \left(\frac{(9 \cdot \frac{1}{8x} \cdot 8)(x) - (9\ln(8x))(1)}{(x)^2}\right)$$

$$\frac{dy}{dx} = e^{\frac{9\ln(8x)}{x}} \left(\frac{9 - 9\ln(8x)}{x^2}\right)$$

$$\frac{dy}{dx} = (8x)^{9/x} \left(\frac{9 - 9\ln(8x)}{x^2}\right)$$

5. (10 points) Find the slope of the line tangent to the curve $x^4y^2 = 75x - 2y$ at the point $(2, 3)$.

$$\frac{d}{dx}(x^4y^2) = \frac{d}{dx}(75x - 2y)$$

$$\frac{d}{dx}(x^4)\cdot y^2 + x^4 \cdot \frac{d}{dx}(y^2) = 75 - 2\frac{dy}{dx}$$

$$4x^3y^2 + x^4 \cdot 2y\frac{dy}{dx} = 75 - 2\frac{dy}{dx}$$

$$2x^4y\frac{dy}{dx} + 2\frac{dy}{dx} = 75 - 4x^3y^2$$

$$\frac{dy}{dx}(2x^4y + 2) = 75 - 4x^3y^2$$

$$\frac{dy}{dx} = \frac{75 - 4x^3y^2}{2x^4y + 2}$$

$$\left.\frac{dy}{dx}\right|_{(x,y)=(2,3)} = \frac{75 - 4(2)^3(3)^2}{2(2)^4(3) + 2} = \frac{-213}{98}$$

6. (10 points) Evaluate the following limit. Simplify your answer.

$$\lim_{x\to 0} \frac{e^{5x} - 5x - 1}{1 - \cos(9x)} \quad (\to 0 \text{ over } \to 0)$$

$$\overset{H}{=} \lim_{x\to 0} \frac{5e^{5x} - 5}{9\sin(9x)} \quad (\to 0 \text{ over } \to 0)$$

$$\overset{H}{=} \lim_{x\to 0} \frac{25e^{5x}}{81\cos(9x)}$$

$$= \frac{25}{81}$$

7. (10 points) Determine the x-coordinate for the absolute minimum value of the following function.

$$f(x) = 2\ln\left(64x^2+1\right) - 320\arctan\left(8x\right)$$

$$f'(x) = 2\cdot\frac{1}{64x^2+1}\cdot 128x - 320\cdot\frac{1}{(8x)^2+1}\cdot 8$$

$$= \frac{256x}{64x^2+1} - \frac{2560}{64x^2+1}$$

$$= \frac{256x - 2560}{64x^2+1}$$

$$= \frac{256(x-10)}{64x^2+1}$$

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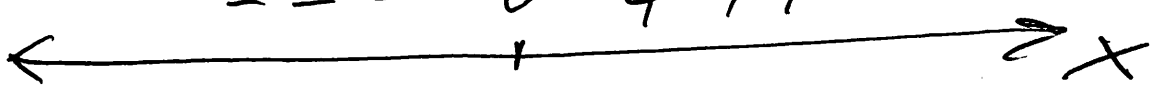
values of $f'(x)$

$---\ 0\ +++$



f dec. 10 f incr.

abs. min at $x = 10$

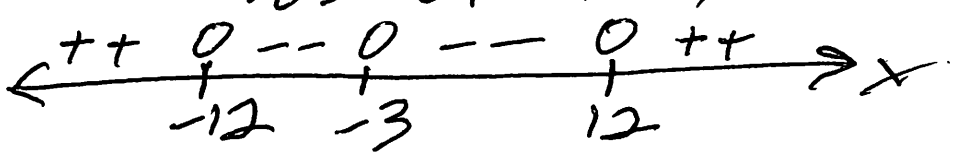
is $f(10) = 2\ln(6401) - 320\arctan(80)$

8. (10 points) A function $f(x)$ is differentiable everywhere and has the following second derivative.

$$f''(x) = \frac{(2x^2 - 288)(x+3)^{42}(x^2+25)}{20e^{16-x}} = \frac{2(x-12)(x+12)(x+3)^{42}(x^2+25)}{20e^{16-x}}$$

Find the intervals of concavity for $f(x)$ and state each x-value at which the graph of $f(x)$ has an inflection point.

values of $f''(x)$



f concave up
$(-\infty, -12)$
$(12, \infty)$

f concave down
$(-12, -3)$
$(-3, 12)$
or $(-12, 12)$

inflection points at these x-values
$x = -12$
$x = 12$

9. (10 points) The curve $y = f(x)$ has the property that the slope of the curve is always equal to its y-coordinate multiplied by $1/4$. If the curve goes through the point $(\ln(81), 36)$, then find a formula for $f(x)$. Simplify your answer.

$$\frac{dy}{dx} = \frac{1}{4}y \Rightarrow y = Ce^{\frac{1}{4}x}$$

$$36 = Ce^{\frac{1}{4}(\ln(81))}$$

$$C = \frac{36}{e^{\frac{1}{4}\ln(81)}}$$

$$= \frac{36}{e^{\ln(81)^{1/4}}}$$

$$= \frac{36}{e^{\ln(3)}}$$

$$= \frac{36}{3}$$

$$= 12$$

$$y = 12e^{\frac{1}{4}x}$$

10. (10 points) A spherical balloon is being inflated so that its diameter is increasing at a constant rate of 6 cm/min. How quickly is the volume of the balloon increasing when the diameter is 50 cm ?

D = diameter ($D = 2r$)

Given: $\frac{dD}{dt} = 6 \text{ cm/min}$

want: $\left.\frac{dV}{dt}\right|_{D=50 \text{ cm}}$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi \left(\frac{D}{2}\right)^3$

$V = \frac{\pi}{6} D^3$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{6} D^3\right)$

$\frac{dV}{dt} = \frac{\pi}{2} D^2 \frac{dD}{dt}$

$\frac{dV}{dt} = \frac{\pi}{2}(50)^2 \cdot (6)$

$\frac{dV}{dt} = 7500\pi \text{ cm}^3/\text{min}$

Students – do not write on this page!

1. (10 points) _____
2. (10 points) _____
3. (10 points) _____
4. (10 points) _____
5. (10 points) _____
6. (10 points) _____
7. (10 points) _____
8. (10 points) _____
9. (10 points) _____
10. (10 points) _____

TOTAL (100 points) _____