MATH 220 Test 1 Fall 2019

Name Solutions NetID _____

UIN _____

Circle your TA discussion section.

- ▷ **AD1**, TR 11:00-12:50, Mina Nahvi
- ▷ **AD2**, TR 9:00-10:50, Adriana Morales
- ▷ **AD3**, TR 1:00-2:50, Vincent Villalobos
- ▷ **AD@**, TR 9:00-9:50, Phuong "Sophie" Le
- ▷ **ADA**, TR 8:00-8:50, Scott Harman
- ▷ **ADB**, TR 9:00-9:50, Lutian Zhao
- ▷ **ADC**, TR 10:00-10:50, Lutian Zhao
- ▷ **ADD**, TR 11:00-11:50, Dara Zirlin
- ▷ **ADE**, TR 12:00-12:50, David Altizio
- ▷ **ADF**, TR 1:00-1:50, Saaber Pourmotabbed
- ▷ **ADG**, TR 2:00-2:50, John "Connor" Grady
- ▷ **ADH**, TR 3:00-3:50, Sarah Simpson
- ▷ **ADI**, TR 4:00-4:50, Ryan McConnell
- ▷ **ADJ**, TR 9:00-9:50, Robert "Bob" Krueger
- ▷ **ADK**, TR 10:00-10:50, Sarah Simpson
- ▷ **ADL**, TR 11:00-11:50, Rocco Davino
- ▷ **ADM**, TR 12:00-12:50, Dara Zirlin
- ▷ **ADN**, TR 1:00-1:50, John "Connor" Grady
- ▷ **ADO**, TR 2:00-2:50, Shuyu "Sonya" Xiao
- ▷ **ADQ**, TR 10:00-10:50, Saaber Pourmotabbed
- ▷ **ADR**, TR 9:00-9:50, Scott Harman
- ▷ **ADS**, TR 12:00-12:50, Rocco Davino
- ▷ **ADT**, TR 2:00-2:50, Ryan McConnell
- ▷ **ADU**, TR 3:00-3:50, Shuyu "Sonya" Xiao
- ▷ **ADW**, TR 8:00-8:50, Robert "Bob" Krueger

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

310	311	312	R	313	314	315	316	317	318	—	—	319	320	321	322	323	R	324	325	326
291	292	293	Q	294	295	296	297	298	299	300	301	302	303	304	305	306	Q	307	308	309
272	273	274	P	275	276	277	278	279	280	281	282	283	284	285	286	287	P	288	289	290
253	254	255	O	256	257	258	259	260	261	262	263	264	265	266	267	268	O	269	270	271
234	235	236	N	237	238	322	240	241	242	243	244	245	246	247	248	249	N	250	251	252
216	217	218	M	219	220	221	222	223	224	225	226	227	228	229	230		M	231	232	233
199	200	201	L	202	203	204	205	206	207	208	209	210	211	212	213		L	214	215	216
181	182	183	K	184	185	186	187	188	189	190	191	192	193	194	195		K	196	197	198
163	164	165	J	166	167	168	169	170	171	172	173	174	175	176	177		J	178	179	180
145	146	147	I	148	149	150	151	152	153	154	155	156	157	158	159		I	160	161	162
127	128	129	H	130	131	132	133	134	135	136	137	138	139	140	141		H	142	143	144
109	110	111	G	112	113	114	115	116	117	118	119	120	121	122	123		G	124	125	126
91	92	93	F	94	95	96	97	98	99	100	101	102	103	104	105		F	106	107	108
73	74	75	E	76	77	78	79	80	81	82	83	84	85	86	87		E	88	89	90
55	56	57	D	58	59	60	61	62	63	64	65	66	67	68	69		D	70	71	72
38	39	40	C	41	42	43	44	45	46	47	48	49	50	51			C	52	53	54
21	22	23	B	24	25	26	27	28	29	30	31	32	33	34			B	35	36	37
5	6	7	A	8	9	10	11	12	13	14	15	16	17				A	18	19	20
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FRONT OF ROOM – 141 Wohlers Hall

1. (10 points) Let $f(x) = 5x - 21x^2$.

Use the definition of a derivative as a limit to prove that $f'(x) = 5 - 42x$.

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{5(x+h) - 21(x+h)^2 - (5x - 21x^2)}{h}$$

$$= \lim_{h \to 0} \frac{5x + 5h - 21(x^2 + 2xh + h^2) - 5x + 21x^2}{h}$$

$$= \lim_{h \to 0} \frac{5x + 5h - 21x^2 - 42xh - 21h^2 - 5x + 21x^2}{h}$$

$$= \lim_{h \to 0} \frac{5h - 42xh - 21h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(5 - 42x - 21h)}{h}$$

$$= \lim_{h \to 0} (5 - 42x - 21h)$$

$$= 5 - 42x$$

2. (10 points) Determine all values of x which satisfy the equation below.

$$3\ln(5) + e^{2\ln(4-3x)} = \ln\left(125e^{241-24x}\right)$$

For domain, $4-3x>0 \Rightarrow x<4/3$

$3\ln(5)+e^{\ln((4-3x)^2)} = \ln(125)+\ln(e^{241-24x})$

$3\ln(5) + (4-3x)^2 = \ln(5^3) + 241-24x$

$3\ln(5) + 16-24x+9x^2 = 3\ln(5)+241-24x$

$16-24x+9x^2 = 241-24x$

$9x^2 = 241-16 = 225$

$x^2 = \frac{225}{9} = 25$

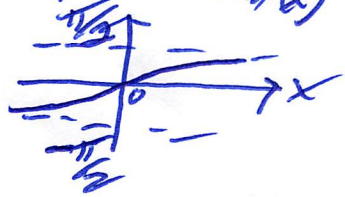
$x = \pm 5$

-5 is only solution since 5 is not in domain

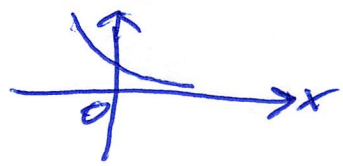
3. (10 points) Find an equation for each horizontal asymptote on the graph of the function $f(x)$.

$$f(x) = \begin{cases} 20\arctan(x) + 52\pi & \text{if } x < 0 \\ 80e^{-2x} - 5 & \text{if } x \geq 0 \end{cases}$$

$y = \arctan(x)$



$y = e^{-2x}$



$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (20\arctan(x)+52\pi)$

$= 20\left(\frac{-\pi}{2}\right)+52\pi$

$= 42\pi$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (80e^{-2x}-5) = 80\cdot 0 - 5 = -5$

hor. asy.) $y = 42\pi$ and $y = -5$

4. (10 points) The function $f(x) = 4e^{8x} - 2x + 1$ has derivative $f'(x) = 32e^{8x} - 2$.
Determine a formula for the line which is tangent to the graph of $f(x)$ at its y-intercept.

At y-intercept, $x = 0$

Point: $(0, f(0)) = (0, 5)$

slope: $f'(0) = 30$

$f(0) = 4e^{8\cdot 0} - 2\cdot 0 + 1 = 5$

$f'(0) = 32e^{8\cdot 0} - 2 = 30$

$y = 30x + 5$

5. (10 points) The function $R(x) = \dfrac{9e^{3x}}{5 + 2e^{3x}}$ is one-to-one. Find a formula for its inverse $R^{-1}(x)$.

$y = \dfrac{9e^{3x}}{5+2e^{3x}}$

$x = \dfrac{9e^{3y}}{5+2e^{3y}}$

$x(5 + 2e^{3y}) = 9e^{3y}$

$5x + 2xe^{3y} = 9e^{3y}$

$2xe^{3y} - 9e^{3y} = -5x$

$e^{3y}(2x - 9) = -5x$

$e^{3y} = \dfrac{-5x}{2x-9}$

$\ln(e^{3y}) = \ln\left(\dfrac{-5x}{2x-9}\right)$

$3y = \ln\left(\dfrac{-5x}{2x-9}\right)$

$y = \dfrac{1}{3}\ln\left(\dfrac{-5x}{2x-9}\right)$

$R^{-1}(x) = \dfrac{1}{3}\ln\left(\dfrac{-5x}{2x-9}\right)$

6. (10 points) Use interval notation to state the domain of the given function.

$$f(x) = \frac{\cos(x^2 - 121)}{x - \sqrt{42 - x}}$$

no restriction on domain for numerator

From $\sqrt{42-x}$, $42-x \geq 0 \Rightarrow x \leq 42$

From denominator, $x - \sqrt{42-x} = 0 \Rightarrow$

$x = \sqrt{42-x} \Rightarrow$

$x^2 = 42 - x \Rightarrow$

$x^2 + x - 42 = 0 \Rightarrow$

$(x+7)(x-6) = 0 \Rightarrow$

$x = -7$ or 6

but -7 is an extraneous solution (since $-7 - \sqrt{42-(-7)} = -14 \neq 0$)

domain

$x \leq 42$ and $x \neq 6$

$(-\infty, 6) \cup (6, 42]$

7. (8 points each) Evaluate the following limits and write your answers in simplified form. For infinite limits, you must clearly show whether the limit is ∞ or $-\infty$. We will learn l'Hospital's Rule and other shortcuts for obtaining limits later. For now you are not allowed to use these approaches.

(a) $\lim_{x \to \infty} \frac{8 + 9x^{1/6}}{2x^{1/6} + 3}$ ($\to \infty$ / $\to \infty$)

$= \lim_{x \to \infty} \frac{8 + 9x^{1/6}}{2x^{1/6} + 3} \cdot \frac{1/x^{1/6}}{1/x^{1/6}}$

$= \lim_{x \to \infty} \frac{8/x^{1/6} + 9}{2 + 3/x^{1/6}}$

$= \frac{0 + 9}{2 + 0}$

$= \frac{9}{2}$

(b) $\lim_{x\to\infty} \frac{8\sin(x)+20}{x^{42}}$

$-1 \le \sin(x) \le 1$

$-8 \le 8\sin(x) \le 8$

$12 \le 8\sin(x)+20 \le 28$

$\frac{12}{x^{42}} \le \frac{8\sin(x)+20}{x^{42}} \le \frac{28}{x^{42}}$

$\lim_{x\to\infty} \frac{12}{x^{42}} = 0, \quad \lim_{x\to\infty} \frac{28}{x^{42}} = 0$

by squeeze theorem

$\lim_{x\to\infty} \frac{8\sin(x)+20}{x^{42}} = 0$

(c) $\lim_{x\to 9/5} \frac{25x^2-81}{5x-9}$ $\to 0$

$= \lim_{x\to 9/5} \frac{(5x-9)(5x+9)}{5x-9}$

$= \lim_{x\to 9/5} (5x+9)$

$= 5\cdot\frac{9}{5} + 9$

$= 18$

(d) $\lim_{x\to 196^-} \dfrac{\sqrt{x}+7}{\sqrt{x}-7}$ $= \dfrac{\sqrt{196}+7}{\sqrt{196}-7} = \dfrac{14+7}{14-7} = \dfrac{21}{7} = 3$

(e) $\lim_{x\to 8^+} \dfrac{9e^{-5x}}{\ln(8/x)}$ $\to 9e^{-40}$ (a positive number)

$\to 0^-$

$\lim_{x\to 8^+} \dfrac{9e^{-5x}}{\ln(8/x)} = -\infty$

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note: $x > 8 \Rightarrow \dfrac{8}{x} < 1$

$\Rightarrow \ln\left(\dfrac{8}{x}\right) < \ln(1)$

$\Rightarrow \ln\left(\dfrac{8}{x}\right) < 0$

Thus, as $x \to 8^+$, $\ln\left(\dfrac{8}{x}\right) \to 0^-$

Students – do not write on this page!

1. (10 points) _____

2. (10 points) _____

3. (10 points) _____

4. (10 points) _____

5. (10 points) _____

6. (10 points) _____

7a. (8 points) _____

7b. (8 points) _____

7c. (8 points) _____

7d. (8 points) _____

7e. (8 points) _____

TOTAL (100 points) _____