MATH 220 Test 2 Fall 2019

Name Solutions NetID _____

UIN _____

Circle your TA discussion section.

- ▷ **AD1**, TR 11:00-12:50, Mina Nahvi
- ▷ **AD2**, TR 9:00-10:50, Adriana Morales
- ▷ **AD3**, TR 1:00-2:50, Vincent Villalobos
- ▷ **AD@**, TR 9:00-9:50, Phuong "Sophie" Le
- ▷ **ADA**, TR 8:00-8:50, Scott Harman
- ▷ **ADB**, TR 9:00-9:50, Lutian Zhao
- ▷ **ADC**, TR 10:00-10:50, Lutian Zhao
- ▷ **ADD**, TR 11:00-11:50, Dara Zirlin
- ▷ **ADE**, TR 12:00-12:50, David Altizio
- ▷ **ADF**, TR 1:00-1:50, Saaber Pourmotabbed
- ▷ **ADG**, TR 2:00-2:50, John "Connor" Grady
- ▷ **ADH**, TR 3:00-3:50, Sarah Simpson
- ▷ **ADI**, TR 4:00-4:50, Ryan McConnell
- ▷ **ADJ**, TR 9:00-9:50, Robert "Bob" Krueger
- ▷ **ADK**, TR 10:00-10:50, Sarah Simpson
- ▷ **ADL**, TR 11:00-11:50, Rocco Davino
- ▷ **ADM**, TR 12:00-12:50, Dara Zirlin
- ▷ **ADN**, TR 1:00-1:50, John "Connor" Grady
- ▷ **ADO**, TR 2:00-2:50, Shuyu "Sonya" Xiao
- ▷ **ADQ**, TR 10:00-10:50, Saaber Pourmotabbed
- ▷ **ADR**, TR 9:00-9:50, Scott Harman
- ▷ **ADS**, TR 12:00-12:50, Rocco Davino
- ▷ **ADT**, TR 2:00-2:50, Ryan McConnell
- ▷ **ADU**, TR 3:00-3:50, Shuyu "Sonya" Xiao
- ▷ **ADW**, TR 8:00-8:50, Robert "Bob" Krueger

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

310	311	312	R	313	314	315	316	317	318	—	—	319	320	321	322	323	R	324	325	326
291	292	293	Q	294	295	296	297	298	299	300	301	302	303	304	305	306	Q	307	308	309
272	273	274	P	275	276	277	278	279	280	281	282	283	284	285	286	287	P	288	289	290
253	254	255	O	256	257	258	259	260	261	262	263	264	265	266	267	268	O	269	270	271
234	235	236	N	237	238	322	240	241	242	243	244	245	246	247	248	249	N	250	251	252
216	217	218	M	219	220	221	222	223	224	225	226	227	228	229	230		M	231	232	233
199	200	201	L	202	203	204	205	206	207	208	209	210	211	212	213		L	214	215	216
181	182	183	K	184	185	186	187	188	189	190	191	192	193	194	195		K	196	197	198
163	164	165	J	166	167	168	169	170	171	172	173	174	175	176	177		J	178	179	180
145	146	147	I	148	149	150	151	152	153	154	155	156	157	158	159		I	160	161	162
127	128	129	H	130	131	132	133	134	135	136	137	138	139	140	141		H	142	143	144
109	110	111	G	112	113	114	115	116	117	118	119	120	121	122	123		G	124	125	126
91	92	93	F	94	95	96	97	98	99	100	101	102	103	104	105		F	106	107	108
73	74	75	E	76	77	78	79	80	81	82	83	84	85	86	87		E	88	89	90
55	56	57	D	58	59	60	61	62	63	64	65	66	67	68	69		D	70	71	72
38	39	40	C	41	42	43	44	45	46	47	48	49	50	51			C	52	53	54
21	22	23	B	24	25	26	27	28	29	30	31	32	33	34			B	35	36	37
5	6	7	A	8	9	10	11	12	13	14	15	16	17				A	18	19	20
	1	2																	3	4

FRONT OF ROOM – 141 Wohlers Hall

1. (10 points) If $f(x) = x^9 \arctan(x) + x^5 \arcsin(x)$, then find $f'(x)$.

$$f'(x) = \frac{d}{dx}(x^9 \arctan(x)) + \frac{d}{dx}(x^5 \arcsin(x))$$

$$= \frac{d}{dx}(x^9) \cdot \arctan(x) + x^9 \cdot \frac{d}{dx}(\arctan(x))$$

$$+ \frac{d}{dx}(x^5) \cdot \arcsin(x) + x^5 \cdot \frac{d}{dx}(\arcsin(x))$$

$$= 9x^8 \arctan(x) + x^9 \cdot \frac{1}{1+x^2} + 5x^4 \arcsin(x) + x^5 \cdot \frac{1}{\sqrt{1-x^2}}$$

2. (10 points) If $w(t) = \cot\left(\frac{\ln(t)}{t^9 + 3t}\right)$, then find $w'(t)$.

$$w'(t) = -\csc^2\left(\frac{\ln(t)}{t^9+3t}\right) \cdot \frac{d}{dt}\left(\frac{\ln(t)}{t^9+3t}\right)$$

$$= -\csc^2\left(\frac{\ln(t)}{t^9+3t}\right) \cdot \frac{\frac{d}{dt}(\ln(t)) \cdot (t^9+3t) - \ln(t) \cdot \frac{d}{dt}(t^9+3t)}{(t^9+3t)^2}$$

$$= -\csc^2\left(\frac{\ln(t)}{t^9+3t}\right) \cdot \frac{\frac{1}{t}(t^9+3t) - \ln(t) \cdot (9t^8+3)}{(t^9+3t)^2}$$

3. (10 points) If $p(\theta) = \csc^8\left(\sqrt{e^{5\theta}+3}\right)$, then find $p'(\theta)$.

$$p'(\theta) = 8\csc^7\left(\sqrt{e^{5\theta}+3}\right)\cdot\frac{d}{d\theta}\left(\csc\left(\sqrt{e^{5\theta}+3}\right)\right)$$

$$= 8\csc^7\left(\sqrt{e^{5\theta}+3}\right)\left(-\csc\left(\sqrt{e^{5\theta}+3}\right)\cot\left(\sqrt{e^{5\theta}+3}\right)\right)\cdot\frac{d}{d\theta}\left(\sqrt{e^{5\theta}+3}\right)$$

$$= 8\csc^7\left(\sqrt{e^{5\theta}+3}\right)\left(-\csc\left(\sqrt{e^{5\theta}+3}\right)\cot\left(\sqrt{e^{5\theta}+3}\right)\right)\cdot\frac{1}{2}\left(e^{5\theta}+3\right)^{-1/2}\cdot\frac{d}{d\theta}\left(e^{5\theta}+3\right)$$

$$= 8\csc^7\left(\sqrt{e^{5\theta}+3}\right)\left(-\csc\left(\sqrt{e^{5\theta}+3}\right)\cot\left(\sqrt{e^{5\theta}+3}\right)\right)\cdot\frac{1}{2}\left(e^{5\theta}+3\right)^{-1/2}\cdot e^{5\theta}\cdot 5$$

4. (10 points) Find the equation of the line tangent to the curve at its positive x-intercept.

$$x^5y + 25 = x^2 + y^{42}$$

at x-int, $y=0$

$$x^5\cdot 0 + 25 = x^2 + 0^{42}$$

$$25 = x^2$$

$$x = \pm 5$$

$x = 5$ for positive x-int

$$\frac{d}{dx}(x^5y + 25) = \frac{d}{dx}(x^2 + y^{42})$$

$$5x^4y + x^5\frac{dy}{dx} = 2x + 42y^{41}\frac{dy}{dx}$$

at point $(5,0)$, we have

$$5\cdot 5^4\cdot 0 + 5^5\frac{dy}{dx} = 2\cdot 5 + 42\cdot 0^{41}\cdot\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2\cdot 5}{5^5} = \frac{2}{5^4} = \frac{2}{625}$$

tangent line: $y - 0 = \frac{2}{625}(x-5)$

$$y = \frac{2}{625}x - \frac{2}{125}$$

5. (10 points) The slope of the tangent line at each point on the graph of $y = f(x)$ is equal to seven times the value of its y-coordinate. Given that $f(6) = 5$, determine a formula for $f(x)$.

$\frac{dy}{dx} = 7y \Rightarrow y = Ce^{7x}$

$f(6) = 5 \Rightarrow 5 = Ce^{7\cdot 6} \Rightarrow C = \frac{5}{e^{42}}$

$y = \frac{5}{e^{42}} \cdot e^{7x}$

$f(x) = \frac{5}{e^{42}} \cdot e^{7x} = 5e^{7x-42}$

6. (10 points) List each interval upon which the graph of $f(x)$ is increasing and each interval upon which the graph of $f(x)$ is decreasing.

$$f(x) = \frac{1 + 13x - x^2}{e^{13x}}$$

$f'(x) = \frac{\frac{d}{dx}(1+13x-x^2)\cdot e^{13x} - (1+13x-x^2)\cdot \frac{d}{dx}(e^{13x})}{(e^{13x})^2}$

$= \frac{(13-2x)e^{13x} - (1+13x-x^2)\cdot 13e^{13x}}{(e^{13x})^2}$

$= \frac{e^{13x}(13-2x-13(1+13x-x^2))}{(e^{13x})^2}$

$= \frac{13x^2 - 171x}{e^{13x}}$

$= \frac{x(13x-171)}{e^{13x}}$

values of $f'(x)$

$+ \; 0 \; - \; 0 \; +$ → x

$0 \qquad \frac{171}{13}$

f incr $(-\infty, 0)$

f decr $(0, 171/13)$

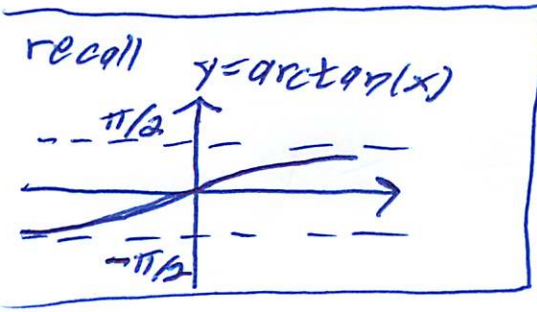
f incr $(\frac{171}{13}, \infty)$

7. (10 points) Evaluate the following limit. You must justify and simplify your answer.

$$\lim_{x\to\infty} \frac{10\arctan(3x) - 5\pi}{6\arctan(2x) - 3\pi}$$ $\to 0$ over $\to 0$

$\overset{H}{=} \lim_{x\to\infty} \frac{10 \cdot \frac{1}{1+(3x)^2}\cdot 3}{6 \cdot \frac{1}{1+(2x)^2}\cdot 2}$

recall $y = \arctan(x)$



$= \lim_{x\to\infty} \frac{30}{1+9x^2} \cdot \frac{1+4x^2}{12}$

$= \lim_{x\to\infty} \frac{30+120x^2}{12+108x^2}$ $\to \infty$ over $\to \infty$

$\overset{H}{=} \lim_{x\to\infty} \frac{240x}{216x} = \lim_{x\to\infty} \left(\frac{240}{216}\right) = \frac{240}{216}$

$= \frac{10}{9}$

8. (10 points) A polynomial $f(x)$ has the following second derivative.

$$f''(x) = -5x^{22}(x-2)^{35}\left((x+20)^2 - 484\right)$$

$= -5x^{22}(x-2)^{35}(x^2+40x+400-484)$

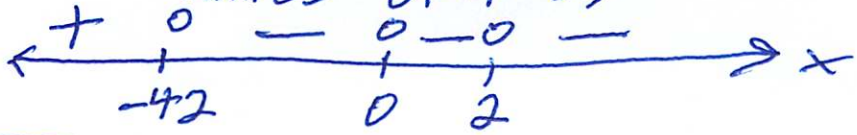
$= -5x^{22}(x-2)^{35}(x^2+40x-84)$

$= -5x^{22}(x-2)^{35}(x-2)(x+42)$

$= -5x^{22}(x-2)^{36}(x+42)$

(a) Determine the largest open interval upon which the graph of f(x) is concave up.

values of $f''(x)$



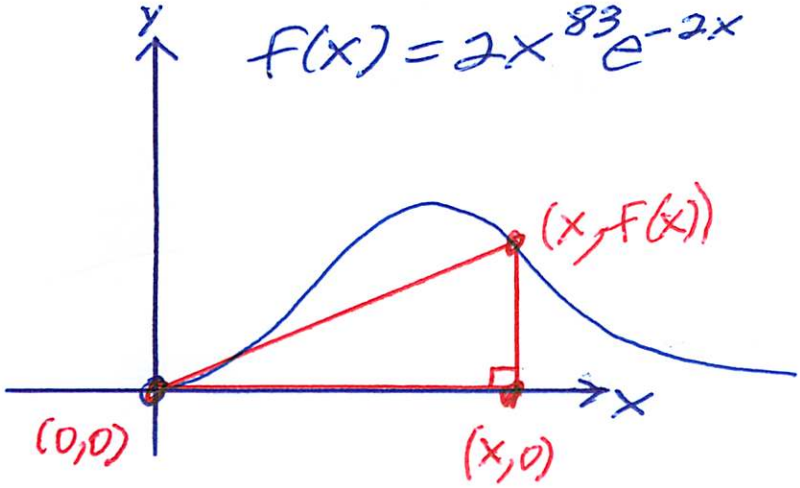
f is concave up on $(-\infty, -42)$

(b) Find each x-value at which the graph of $f(x)$ has an inflection point.

at $x = -42$

since f switches from concave up to concave down at that point

9. (10 points) For each $x > 0$, a triangle is formed with vertices $(0,0)$, $(x,0)$ and $(x, f(x))$ where $f(x) = 2x^{83}e^{-2x}$. What is the value of x which results in the triangle of largest area?



$f(x) = 2x^{83}e^{-2x}$

area $= \frac{1}{2}$(base)(height)

$A = \frac{1}{2} x \cdot f(x)$

$A = \frac{1}{2} x \cdot 2x^{83}e^{-2x}$

$A = x^{84}e^{-2x}$

maximize $A = x^{84}e^{-2x}$ for $x > 0$

$A' = 84x^{83}e^{-2x} + x^{84}(-2e^{-2x})$

$= 2x^{83}e^{-2x}(42 - x)$

values of A' for $x > 0$

(A incr on $(0, 42)$; A decr on $(42, \infty)$)

abs. max. area when $x = 42$

10. (10 points) The volume of a sphere is increasing at a rate of $42\pi\ cm^3/sec$. How quickly is the radius increasing when the diameter is 25 cm ?

note: diameter $= 25 \Rightarrow$ radius $= \frac{25}{2}$

Given: $\frac{dV}{dt} = 42\pi\ cm^3/sec$

want: $\left.\frac{dr}{dt}\right|_{r=\frac{25}{2}}$

$V = \frac{4}{3}\pi r^3$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\downarrow$

$42\pi = 4\pi \left(\frac{25}{2}\right)^2 \frac{dr}{dt}$

$42\pi = 625\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{42\pi}{625\pi} = \frac{42}{625}\ cm/sec$

Students – do not write on this page!

1. (10 points) _____
2. (10 points) _____
3. (10 points) _____
4. (10 points) _____
5. (10 points) _____
6. (10 points) _____
7. (10 points) _____
8. (10 points) _____
9. (10 points) _____
10. (10 points) _____

TOTAL (100 points) _____