AL1 (MWF 10:00-10:50) AL3 (MWF 1:00-1:50) AL2 (MWF 3:00-3:50)

MATH 220 **Test 3** **Fall 2019**

Name Solutions **NetID** ____________________

UIN ____________________

Circle your TA discussion section.

- ▷ **AD1**, TR 11:00-12:50, Mina Nahvi
- ▷ **AD2**, TR 9:00-10:50, Adriana Morales
- ▷ **AD3**, TR 1:00-2:50, Vincent Villalobos
- ▷ **AD@**, TR 9:00-9:50, Phuong "Sophie" Le
- ▷ **ADA**, TR 8:00-8:50, Scott Harman
- ▷ **ADB**, TR 9:00-9:50, Lutian Zhao
- ▷ **ADC**, TR 10:00-10:50, Lutian Zhao
- ▷ **ADD**, TR 11:00-11:50, Dara Zirlin
- ▷ **ADE**, TR 12:00-12:50, David Altizio
- ▷ **ADF**, TR 1:00-1:50, Saaber Pourmotabbed
- ▷ **ADG**, TR 2:00-2:50, John "Connor" Grady
- ▷ **ADH**, TR 3:00-3:50, Sarah Simpson
- ▷ **ADI**, TR 4:00-4:50, Ryan McConnell
- ▷ **ADJ**, TR 9:00-9:50, Robert "Bob" Krueger
- ▷ **ADK**, TR 10:00-10:50, Sarah Simpson
- ▷ **ADL**, TR 11:00-11:50, Rocco Davino
- ▷ **ADM**, TR 12:00-12:50, Dara Zirlin
- ▷ **ADN**, TR 1:00-1:50, John "Connor" Grady
- ▷ **ADO**, TR 2:00-2:50, Shuyu "Sonya" Xiao
- ▷ **ADQ**, TR 10:00-10:50, Saaber Pourmotabbed
- ▷ **ADR**, TR 9:00-9:50, Scott Harman
- ▷ **ADS**, TR 12:00-12:50, Rocco Davino
- ▷ **ADT**, TR 2:00-2:50, Ryan McConnell
- ▷ **ADU**, TR 3:00-3:50, Shuyu "Sonya" Xiao
- ▷ **ADW**, TR 8:00-8:50, Robert "Bob" Krueger

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say *START*.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

310	311	312	R	313	314	315	316	317	318	—	—	319	320	321	322	323	R	324	325	326
291	292	293	Q	294	295	296	297	298	299	300	301	302	303	304	305	306	Q	307	308	309
272	273	274	P	275	276	277	278	279	280	281	282	283	284	285	286	287	P	288	289	290
253	254	255	O	256	257	258	259	260	261	262	263	264	265	266	267	268	O	269	270	271
234	235	236	N	237	238	322	240	241	242	243	244	245	246	247	248	249	N	250	251	252
216	217	218	M	219	220	221	222	223	224	225	226	227	228	229	230		M	231	232	233
199	200	201	L	202	203	204	205	206	207	208	209	210	211	212	213		L	214	215	216
181	182	183	K	184	185	186	187	188	189	190	191	192	193	194	195		K	196	197	198
163	164	165	J	166	167	168	169	170	171	172	173	174	175	176	177		J	178	179	180
145	146	147	I	148	149	150	151	152	153	154	155	156	157	158	159		I	160	161	162
127	128	129	H	130	131	132	133	134	135	136	137	138	139	140	141		H	142	143	144
109	110	111	G	112	113	114	115	116	117	118	119	120	121	122	123		G	124	125	126
91	92	93	F	94	95	96	97	98	99	100	101	102	103	104	105		F	106	107	108
73	74	75	E	76	77	78	79	80	81	82	83	84	85	86	87		E	88	89	90
55	56	57	D	58	59	60	61	62	63	64	65	66	67	68	69		D	70	71	72
38	39	40	C	41	42	43	44	45	46	47	48	49	50	51			C	52	53	54
21	22	23	B	24	25	26	27	28	29	30	31	32	33	34			B	35	36	37
5	6	7	A	8	9	10	11	12	13	14	15	16	17				A	18	19	20
	1	2																3	4	

FRONT OF ROOM – 141 Wohlers Hall

1. (10 points) Evaluate the indefinite integral.

$$\int \left(x^6 + 9\cos(x) + 5\sin(x) + 3\csc(x)\cot(x) + 8\sec(x)\tan(x) + 9\sec^2(x) + 4\csc^2(x) + 2\right)\, dx$$

$$= \frac{1}{7}x^7 + 9\sin(x) - 5\cos(x) - 3\csc(x) + 8\sec(x) + 9\tan(x) - 4\cot(x) + 2x + C$$

2. (10 points) Evaluate the indefinite integral.

$$\int \frac{240x^5}{x^{12}+25}\, dx = \int \frac{40}{(x^6)^2+25}\, 6x^5 dx$$

$$\boxed{\begin{aligned} u &= x^6 \\ du &= 6x^5 dx \end{aligned}}$$

$$= \int \frac{40}{u^2+25}\, du$$

$$= \int \frac{40}{25\left(\left(\frac{u^2}{25}\right)+1\right)}\, du$$

$$= \int \frac{40}{25\left(\left(\frac{u}{5}\right)^2+1\right)}\, du$$

$$\boxed{\begin{aligned} w &= \frac{u}{5} \\ dw &= \frac{1}{5}du \\ 5dw &= du \end{aligned}}$$

$$= \int \frac{40}{25(w^2+1)}\, 5dw$$

$$= 8\int \frac{1}{w^2+1}\, dw$$

$$= 8\arctan(w) + C$$

$$= 8\arctan\left(\frac{u}{5}\right) + C$$

$$= 8\arctan\left(\frac{x^6}{5}\right) + C$$

3. (10 points) Evaluate the indefinite integral.

$$\int 81x\,(9x+4)^{40}\,dx$$

$$\boxed{\begin{array}{l} u = 9x+4 \Rightarrow x = \frac{u-4}{9} \\ du = 9dx \\ \frac{1}{9}du = dx \end{array}}$$

$$= \int 81\left(\frac{u-4}{9}\right) u^{40} \frac{1}{9}\,du$$

$$= \int \left(u^{41} - 4u^{40}\right) du$$

$$= \frac{1}{42}u^{42} - \frac{4}{41}u^{41} + C$$

$$= \frac{1}{42}(9x+4)^{42} - \frac{4}{41}(9x+4)^{41} + C$$

4. (10 points) Evaluate the indefinite integral.

version 1: $\int \tan^6(x)\sec^4(x)\,dx$

$$\boxed{\begin{array}{l} u = \tan(x) \\ du = \sec^2(x)dx \end{array}}$$

$\int \tan^6(x)\sec^4(x)\,dx =$

$\int \tan^6(x)\sec^2(x)\sec^2(x)\,dx =$

$\int \tan^6(x)(\tan^2(x)+1)\sec^2(x)\,dx =$

$\int u^6(u^2+1)\,du =$

$\int (u^8+u^6)\,du = \frac{1}{9}u^9 + \frac{1}{7}u^7 + C$

$= \frac{1}{9}\tan^9(x) + \frac{1}{7}\tan^7(x) + C$

version 2: $\int \sec^5(x)\tan^3(x)\,dx$

$$\boxed{\begin{array}{l} u = \sec(x) \\ du = \sec(x)\tan(x)dx \end{array}}$$

$\int \sec^5(x)\tan^3(x)\,dx =$

$\int \sec^4(x)\tan^2(x)\sec(x)\tan(x)\,dx =$

$\int \sec^4(x)(\sec^2(x)-1)\sec(x)\tan(x)\,dx =$

$\int u^4(u^2-1)\,du =$

$\int (u^6-u^4)\,du = \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$

$= \frac{1}{7}\sec^7(x) - \frac{1}{5}\sec^5(x) + C$

5. (10 points) Find the average value of the function $f(x) = \dfrac{32x}{\sqrt{2x^2+49}}$ on the interval $[0, 4]$. Simplify your answer.

$$f_{ave} = \frac{1}{4-0}\int_0^4 \frac{32x}{\sqrt{2x^2+49}}\,dx$$

$$= \frac{1}{4}\int_0^4 \frac{8}{\sqrt{2x^2+49}}\cdot 4x\,dx$$

$$\boxed{\begin{array}{l} u = 2x^2+49 \\ du = 4x\,dx \\ \hline x=0 \Rightarrow u = 49 \\ x=4 \Rightarrow u = 81 \end{array}}$$

$$= \frac{1}{4}\int_{49}^{81} \frac{8}{\sqrt{u}}\,du$$

$$= 2\int_{49}^{81} u^{-1/2}\,du$$

$$= 2\cdot 2u^{1/2}\Big|_{49}^{81}$$

$$= 4\sqrt{81} - 4\sqrt{49} = 4\cdot 9 - 4\cdot 7 = 8$$

6. (10 points) Suppose that $f(x)$ is a polynomial which satisfies the following conditions.

- $\displaystyle\int_2^9 f(x)\,dx = 30$
- $\displaystyle\int_4^9 f(x)\,dx = 34$

$$\int_2^4 f(x)\,dx = \int_2^9 f(x)\,dx + \int_9^4 f(x)\,dx$$

$$= \int_2^9 f(x)\,dx - \int_4^9 f(x)\,dx$$

$$= 30 - 34 = -4$$

Evaluate the following quantities.

(a) $\displaystyle\int_2^4 (8f(x)+5)\,dx$ $= 8\int_2^4 f(x)\,dx + \int_2^4 5\,dx$

$$= 8(-4) + 5x\Big|_2^4$$

$$= -32 + 5\cdot 4 - 5\cdot 2 = -22$$

(b) $\displaystyle\int_3^4 34x f(x^2-7)\,dx$ $= \int_3^4 17 f(x^2-7)\cdot 2x\,dx$

$$\boxed{\begin{array}{l} u = x^2-7 \\ du = 2x\,dx \\ \hline x=3 \Rightarrow u = 2 \\ x=4 \Rightarrow u = 9 \end{array}}$$

$$= \int_2^9 17 f(u)\,du$$

$$= 17\int_2^9 f(u)\,du = 17\cdot 30 = 510$$

7. (10 points) Given the function below, use a linear approximation to estimate $f(5.02)$. Simplify and write your answer in decimal form.

$f(x) = \frac{1}{15}\ln\left(x^3 - 124\right) + 6x + 12$

$f(5) = \frac{1}{15}\ln(1) + 6\cdot 5 + 12 = 42$

$f'(x) = \frac{1}{15}\cdot\frac{1}{x^3-124}\cdot 3x^2 + 6$

$f'(5) = \frac{1}{15}\cdot\frac{1}{1}\cdot 3\cdot 5^2 + 6 = 11$

point: $(5, f(5)) = (5, 42)$

slope: $f'(5) = 11$

tangent line at $x=5$: $y - 42 = 11(x-5)$

$y = 42 + 11(x-5)$

$f(x) \approx 42 + 11(x-5)$ near $x=5$

$f(5.02) \approx 42 + 11(5.02 - 5)$

$f(5.02) \approx 42.22$

8. (10 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit. Simplify your answer.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{60k}{n^2} + \frac{12}{n+5}\right) = \lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{60k}{n^2} + \sum_{k=1}^{n}\frac{12}{n+5}\right)$$

$$= \lim_{n\to\infty}\left(\frac{60}{n^2}\sum_{k=1}^{n}k + \frac{12}{n+5}\sum_{k=1}^{n}1\right)$$

$$= \lim_{n\to\infty}\left(\frac{60}{n^2}\cdot\frac{n(n+1)}{2} + \frac{12}{n+5}\cdot n\right)$$

$$= \lim_{n\to\infty}\left(\frac{60n^2+60n}{2n^2} + \frac{12n}{n+5}\right)$$

$$= \lim_{n\to\infty}\left(30 + \frac{30}{n} + \frac{12}{1+5/n}\right)$$

$$= 30 + 0 + 12$$

$$= 42$$

9. (10 points) Let $g(x) = 8x + \int_x^9 e^t(t-25)\,dt$. Determine the x-value for each inflection point of $g(x)$.

$$g(x) = 8x + \int_x^9 e^t(t-25)\,dt$$

$$g(x) = 8x - \int_9^x e^t(t-25)\,dt$$

$$g'(x) = 8 - e^x(x-25) \quad \text{by F.T.C. (part 1)}$$

$$g''(x) = 0 - \left(\frac{d}{dx}(e^x)(x-25) + e^x \cdot \frac{d}{dx}(x-25)\right)$$

$$g''(x) = -\left(e^x(x-25) + e^x \cdot 1\right)$$

$$g''(x) = -e^x(x-25+1)$$

$$g''(x) = -e^x(x-24)$$

values of $g''(x)$

$+ \quad 0 \quad -$

$\longleftarrow \overset{|}{24} \longrightarrow x$

$g''(x) > 0 \Rightarrow g$ is concave up on $(-\infty, 24)$

$g''(x) < 0 \Rightarrow g$ is concave down on $(24, \infty)$

g has an inflection point at $x = 24$

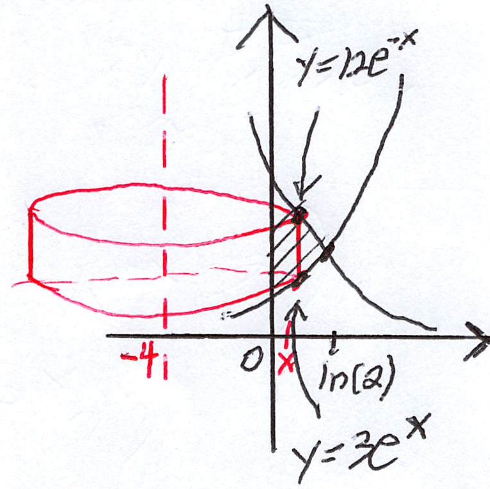
10. (10 points) The graphs of $v(x) = 3e^x$ and $w(x) = 12e^{-x}$ intersect at the point $(x, y) = (\ln(2), 6)$. Let $\mathbf{R}$ be the finite region bounded by $v(x)$, $w(x)$ and the y-axis. By integrating with respect to x, set up, but do not evaluate, definite integrals which represent the given quantities.

(a) The volume of the solid obtained when $\mathbf{R}$ is revolved around the vertical line $x = -4$.

$$\text{vol} = \int_{x_{min}}^{x_{max}} \begin{pmatrix} \text{surface} \\ \text{area} \end{pmatrix} dx$$

$$= \int_0^{\ln(2)} 2\pi r h \, dx$$

$$= \int_0^{\ln(2)} 2\pi (x-(-4))(12e^{-x} - 3e^x) dx$$

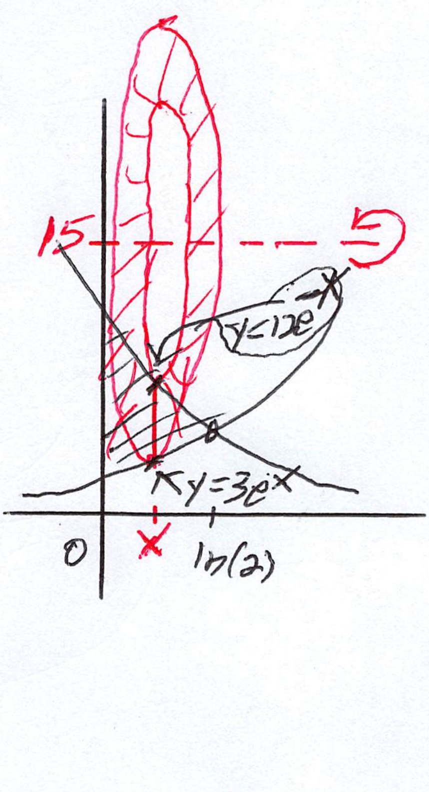


(b) The volume of the solid obtained when $\mathbf{R}$ is revolved around the horizontal line $y = 15$.

$$\text{vol} = \int_{x_{min}}^{x_{max}} \begin{pmatrix} \text{cross-sectional} \\ \text{area} \end{pmatrix} dx$$

$$= \int_0^{\ln(2)} (\pi r_{out}^2 - \pi r_{in}^2) dx$$

$$= \int_0^{\ln(2)} (\pi (15-3e^x)^2 - \pi (15-12e^{-x})^2) dx$$



Students – do not write on this page!

1. (10 points) ________________
2. (10 points) ________________
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8. (10 points) ________________
9. (10 points) ________________
10. (10 points) ________________

TOTAL (100 points) ____________