AL1 (MWF 9:00-9:50) AL2 (MWF 10:00-10:50)

MATH 220 **Test 1** **Spring 2019**

Name Solutions

NetID ______________

UIN ______________

Circle your TA discussion section.

- **AD1**, TR 9:00-10:50, Ran Ji
- **AD2**, TR 1:00-2:50, Cassie Christenson
- **AD3**, TR 11:00-12:50, Dana Neidinger
- **ADA**, TR 8:00-8:50, Eion Blanchard
- **ADB**, TR 9:00-9:50, Eion Blanchard
- **ADC**, TR 10:00-10:50, Yuxuan "Yuki" Zhang
- **ADD**, TR 11:00-11:50, Stathis Chrontsios
- **ADE**, TR 12:00-12:50, Kesav Krishnan
- **ADF**, TR 1:00-1:50, Souktik Roy
- **ADG**, TR 2:00-2:50, Gidon Orelowitz
- **ADH**, TR 3:00-3:50, Mina Nahvi
- **ADJ**, TR 9:00-9:50, Yuxuan "Yuki" Zhang
- **ADK**, TR 10:00-10:50, Souktik Roy
- **ADL**, TR 11:00-11:50, Gidon Orelowitz
- **ADM**, TR 12:00-12:50, Vincent Villalobos
- **ADN**, TR 1:00-1:50, Kesav Krishnan
- **ADO**, TR 2:00-2:50, Stathis Chrontsios
- **ADQ**, TR 4:00-4:50, Mina Nahvi
- **ADR**, TR 10:00-10:50, Vincent Villalobos

- Sit in your assigned seat (circled below).
- Do not open the test or write formulas upon it until I say *START*.
- Remove smartwatches and turn off all electronic devices.
- Put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

310	311	312	R	313	314	315	316	317	318	—	—	319	320	321	322	323	R	324	325	326
291	292	293	Q	294	295	296	297	298	299	300	301	302	303	304	305	306	Q	307	308	309
272	273	274	P	275	276	277	278	279	280	281	282	283	284	285	286	287	P	288	289	290
253	254	255	O	256	257	258	259	260	261	262	263	264	265	266	267	268	O	269	270	271
234	235	236	N	237	238	322	240	241	242	243	244	245	246	247	248	249	N	250	251	252
216	217	218	M	219	220	221	222	223	224	225	226	227	228	229	230		M	231	232	233
199	200	201	L	202	203	204	205	206	207	208	209	210	211	212	213		L	214	215	216
181	182	183	K	184	185	186	187	188	189	190	191	192	193	194	195		K	196	197	198
163	164	165	J	166	167	168	169	170	171	172	173	174	175	176	177		J	178	179	180
145	146	147	I	148	149	150	151	152	153	154	155	156	157	158	159		I	160	161	162
127	128	129	H	130	131	132	133	134	135	136	137	138	139	140	141		H	142	143	144
109	110	111	G	112	113	114	115	116	117	118	119	120	121	122	123		G	124	125	126
91	92	93	F	94	95	96	97	98	99	100	101	102	103	104	105		F	106	107	108
73	74	75	E	76	77	78	79	80	81	82	83	84	85	86	87		E	88	89	90
55	56	57	D	58	59	60	61	62	63	64	65	66	67	68	69		D	70	71	72
38	39	40	C	41	42	43	44	45	46	47	48	49	50	51			C	52	53	54
21	22	23	B	24	25	26	27	28	29	30	31	32	33	34			B	35	36	37
5	6	7	A	8	9	10	11	12	13	14	15	16	17				A	18	19	20
	1	2																3	4	

FRONT OF ROOM – 141 Wohlers Hall

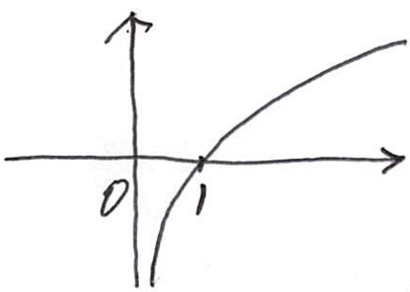
1. (8 points each) Evaluate the following limits and write your answers in simplified form. For infinite limits, you must clearly show whether the limit is ∞ or $-\infty$. We will learn l'Hospital's Rule and other shortcuts for obtaining limits later. For now ~~you are not~~ allowed to use these approaches.

(a) $\displaystyle\lim_{x\to 6^+} \frac{\cos(5\pi/x)}{\ln(7-x)}$ $\begin{matrix} \to -\sqrt{3}/2 \\ \to 0^- \end{matrix}$ $= +\infty$

as $x \to 6^+$, $\cos\left(\frac{5\pi}{x}\right) \to \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

as $x \to 6^+$, $7 - x \to 1^-$

Thus, $\ln(7-x) \to 0^-$ (see graph)



$y = \ln x$

(see 8c Spring 2014)

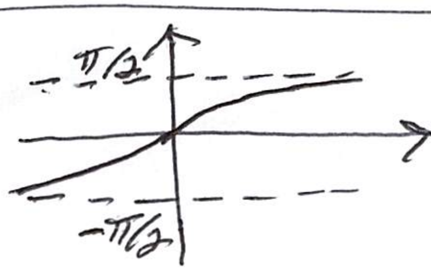
(b) $\displaystyle\lim_{x\to -\infty} \frac{10\arctan(8x) + 19\pi}{18\arctan(4x) + 16\pi} = \frac{10\left(-\frac{\pi}{2}\right) + 19\pi}{18\left(-\frac{\pi}{2}\right) + 16\pi} = \frac{14\pi}{7\pi} = 2$

as $x \to -\infty$, $8x \to -\infty$

thus, $\arctan(8x) \to -\frac{\pi}{2}$

as $x \to -\infty$, $4x \to -\infty$

thus, $\arctan(4x) \to -\frac{\pi}{2}$



$y = \arctan(x)$

(c) $\lim_{x\to\infty} \frac{6e^{2x}+4}{5e^x-2e^{2x}} = \lim_{x\to\infty} \frac{6e^{2x}+4}{5e^x-2e^{2x}} \cdot \frac{1/e^{2x}}{1/e^{2x}}$

$= \lim_{x\to\infty} \frac{6 - 4/e^{2x}}{5/e^x - 2}$

$= \frac{6-0}{0-2}$

$= -3$

2. (6 points) Simplify the following quantity.

$$3e^{5\ln(2)} + \ln\left(9e^{10}\right) - \ln\left(9e^2\right)$$

$= 3e^{\ln(2^5)} + \ln\left(\frac{9e^{10}}{9e^2}\right)$

$= 3\cdot 2^5 + \ln(e^8)$

$= 3\cdot 32 + 8$

$= 104$

3. (10 points) Suppose that $w(x)$ is odd, one-to-one, and its graph goes through the point $(-3, 1/8)$.

(a) Determine another point which must be on the graph of $w(x)$.

$w(-x) = -w(x)$ since w is odd

$w(3) = w(-(-3)) = -w(-3) = -1/8$

Thus $(3, -1/8)$ is a point on $w(x)$

(b) Determine a point which must be on the graph of $w^{-1}(x)$.

$(-3, 1/8)$ on $w(x) \Rightarrow (1/8, -3)$ on $w^{-1}(x)$

$(3, -1/8)$ on $w(x) \Rightarrow (-1/8, 3)$ on $w^{-1}(x)$

4. (10 points) For a given acute angle θ, it is known that $\sec(\theta) = 5$. Evaluate the following quantities.

(a) $\cos(2\theta)$ $= \cos^2(\theta) - \sin^2(\theta)$

$= \cos^2(\theta) - (1 - \cos^2(\theta))$

$= 2\cos^2(\theta) - 1$

$= 2\left(\frac{1}{5}\right)^2 - 1$

$= \frac{2}{25} - 1$

$= -23/25$

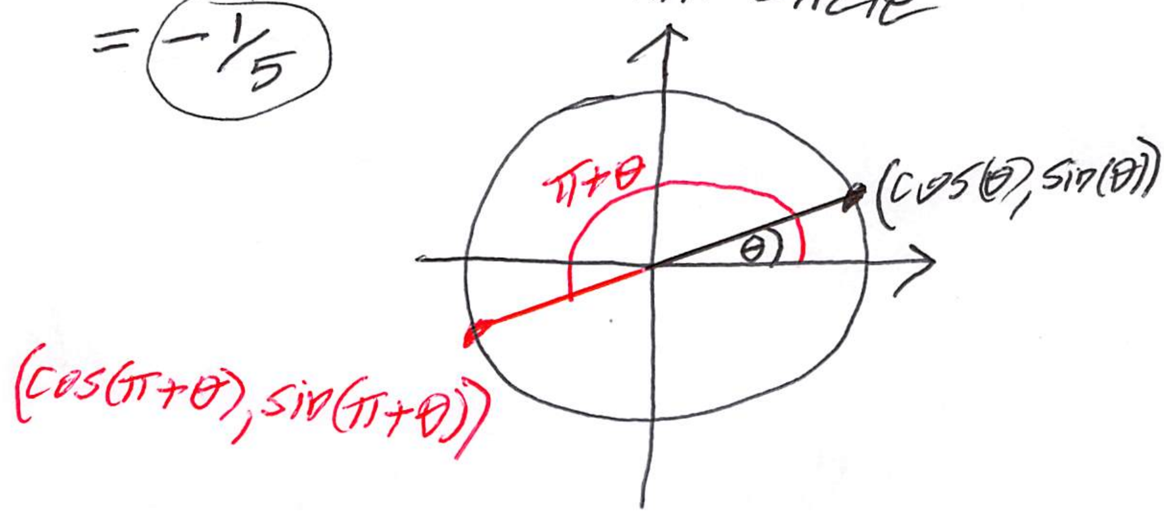
since $\sec(\theta) = \frac{1}{\cos(\theta)}$

$\sec(\theta) = 5 \Rightarrow$

$\cos(\theta) = 1/5$

(b) $\cos(\pi + \theta)$ $= -\cos(\theta)$ from unit circle

$= -1/5$



5. (10 points) Let $f(x) = 8x^2 + 9x$.

Use the definition of a derivative as a limit to prove that $f'(x) = 16x + 9$.

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{8(x+h)^2 + 9(x+h) - (8x^2 + 9x)}{h}$$

$$= \lim_{h\to 0} \frac{8(x^2 + 2xh + h^2) + 9x + 9h - 8x^2 - 9x}{h}$$

$$= \lim_{h\to 0} \frac{8x^2 + 16xh + 8h^2 + 9x + 9h - 8x^2 - 9x}{h}$$

$$= \lim_{h\to 0} \frac{16xh + 8h^2 + 9h}{h}$$

$$= \lim_{h\to 0} \frac{h(16x + 8h + 9)}{h}$$

$$= \lim_{h\to 0} (16x + 8h + 9)$$

$$= 16x + 9$$

6. (10 points) The function $f(x) = 5e^{4x} - 50$ has derivative $f'(x) = 20e^{4x}$. Determine the equation for the line which is tangent to the graph of $f(x)$ at its x-intercept.

To find x-intercept, set $f(x)=0$

$$5e^{4x}-50=0$$
$$5e^{4x}=50$$
$$e^{4x}=10$$
$$\ln(e^{4x})=\ln(10)$$
$$4x=\ln(10)$$
$$x=\tfrac{1}{4}\ln(10)$$

point: $(\tfrac{1}{4}\ln(10), 0)$

slope: $f'(\tfrac{1}{4}\ln(10)) = 20e^{4\cdot\frac{1}{4}\ln(10)} = 20e^{\ln(10)} = 20\cdot 10 = 200$

equation: $y-0=200(x-\tfrac{1}{4}\ln(10))$

$$y=200x-50\ln(10)$$

7. (10 points) The graphs of $f(x) = \ln(2) + 3\ln(-x)$ and $g(x) = \ln(-32x)$ intersect. Determine the x-value for each point of intersection. Simplify your answer.

$$\ln(2)+3\ln(-x)=\ln(-32x)$$
$$\ln(2)+\ln((-x)^3)=\ln(-32x)$$
$$\ln(2)+\ln(-x^3)=\ln(-32x)$$
$$\ln(-2x^3)=\ln(-32x)$$
$$-2x^3=-32x \quad \text{(since ln is one-to-one)}$$
$$x^3=16x$$
$$x^3-16x=0$$
$$x(x^2-16)=0$$
$$x(x+4)(x-4)=0$$

$\cancel{x=0}$, $x=-4$, or $\cancel{x=4}$ are possible solutions

however, only -4 is in the domain of $f(x)$ and $g(x)$

8. (10 points) A bacterial culture starts with 400 bacteria and doubles in size every 2 hours.

(a) Find a formula for the number of bacteria after t hours.

t	y
0	400
2	800
4	1600
6	3200
⋮	⋮

$y = C \cdot a^t$

$400 = C \cdot a^0 \Rightarrow C = 400$

$y = 400 \cdot a^t$

$800 = 400 \cdot a^2 \Rightarrow a^2 = 2 \Rightarrow a = \sqrt{2}$

☆ $y = 400(\sqrt{2})^t$

(b) At what time is the population equal to 2400 ?

$2400 = 400(\sqrt{2})^t$

$6 = (\sqrt{2})^t$

$\ln(6) = \ln((\sqrt{2})^t)$

$\Rightarrow \ln(6) = t\ln(\sqrt{2})$

$t = \dfrac{\ln(6)}{\ln(\sqrt{2})}$ hours

9. (10 points) Use interval notation to state the domain of the given function.

$$\frac{\ln(55-x)}{\sqrt{60-x}-\sqrt{x-24}}$$

from $\ln(55-x)$, $55-x>0 \Rightarrow x<55$

from $\sqrt{60-x}$, $60-x\geq 0 \Rightarrow x\leq 60$

from $\sqrt{x-24}$, $x-24\geq 0 \Rightarrow x\geq 24$

denominator = 0 when

$\sqrt{60-x} = \sqrt{x-24}$

$60-x = x-24$

$84 = 2x$

$x = 42$

Thus $x \neq 42$

domain of function is

$[24,42)\cup(42,55)$

Students – do not write on this page!

1a. (8 points) ____________________

1b. (8 points) ____________________

1c. (8 points) ____________________

2. (6 points) ____________________

3. (10 points) ____________________

4. (10 points) ____________________

5. (10 points) ____________________

6. (10 points) ____________________

7. (10 points) ____________________

8. (10 points) ____________________

9. (10 points) ____________________

TOTAL (100 points) ________________