AL1 (MWF 9:00-9:50) AL2 (MWF 10:00-10:50)

MATH 220 **Test 2** **Spring 2019**

Name Solutions

NetID ____________________

UIN ____________________

Circle your TA discussion section.

- ▷ **AD1**, TR 9:00-10:50, Ran Ji
- ▷ **AD2**, TR 1:00-2:50, Cassie Christenson
- ▷ **AD3**, TR 11:00-12:50, Dana Neidinger
- ▷ **ADA**, TR 8:00-8:50, Eion Blanchard
- ▷ **ADB**, TR 9:00-9:50, Eion Blanchard
- ▷ **ADC**, TR 10:00-10:50, Yuxuan "Yuki" Zhang
- ▷ **ADD**, TR 11:00-11:50, Stathis Chrontsios
- ▷ **ADE**, TR 12:00-12:50, Kesav Krishnan
- ▷ **ADF**, TR 1:00-1:50, Souktik Roy
- ▷ **ADG**, TR 2:00-2:50, Gidon Orelowitz
- ▷ **ADH**, TR 3:00-3:50, Mina Nahvi
- ▷ **ADJ**, TR 9:00-9:50, Yuxuan "Yuki" Zhang
- ▷ **ADK**, TR 10:00-10:50, Souktik Roy
- ▷ **ADL**, TR 11:00-11:50, Gidon Orelowitz
- ▷ **ADM**, TR 12:00-12:50, Vincent Villalobos
- ▷ **ADN**, TR 1:00-1:50, Kesav Krishnan
- ▷ **ADO**, TR 2:00-2:50, Stathis Chrontsios
- ▷ **ADQ**, TR 4:00-4:50, Mina Nahvi
- ▷ **ADR**, TR 10:00-10:50, Vincent Villalobos

- Sit in your assigned seat (circled below).
- Do not open the test or write formulas upon it until I say *START*.
- Remove smartwatches and turn off all electronic devices.
- Put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

310	311	312	R	313	314	315	316	317	318	—	—	319	320	321	322	323	R	324	325	326
291	292	293	Q	294	295	296	297	298	299	300	301	302	303	304	305	306	Q	307	308	309
272	273	274	P	275	276	277	278	279	280	281	282	283	284	285	286	287	P	288	289	290
253	254	255	O	256	257	258	259	260	261	262	263	264	265	266	267	268	O	269	270	271
234	235	236	N	237	238	322	240	241	242	243	244	245	246	247	248	249	N	250	251	252
216	217	218	M	219	220	221	222	223	224	225	226	227	228	229	230		M	231	232	233
199	200	201	L	202	203	204	205	206	207	208	209	210	211	212	213		L	214	215	216
181	182	183	K	184	185	186	187	188	189	190	191	192	193	194	195		K	196	197	198
163	164	165	J	166	167	168	169	170	171	172	173	174	175	176	177		J	178	179	180
145	146	147	I	148	149	150	151	152	153	154	155	156	157	158	159		I	160	161	162
127	128	129	H	130	131	132	133	134	135	136	137	138	139	140	141		H	142	143	144
109	110	111	G	112	113	114	115	116	117	118	119	120	121	122	123		G	124	125	126
91	92	93	F	94	95	96	97	98	99	100	101	102	103	104	105		F	106	107	108
73	74	75	E	76	77	78	79	80	81	82	83	84	85	86	87		E	88	89	90
55	56	57	D	58	59	60	61	62	63	64	65	66	67	68	69		D	70	71	72
38	39	40	C	41	42	43	44	45	46	47	48	49	50	51			C	52	53	54
21	22	23	B	24	25	26	27	28	29	30	31	32	33	34			B	35	36	37
5	6	7	A	8	9	10	11	12	13	14	15	16	17				A	18	19	20
	1	2																3	4	

FRONT OF ROOM – 141 Wohlers Hall

1. (10 points) Find $w'(x)$ given that $w(x) = 2x^3 + 8\sec(x) + 9\csc(x) + 3\tan(x) + 5\cot(x) + 6e^x + 4\ln(x)$

$$w'(x) = 6x^2 + 8\sec(x)\tan(x) - 9\csc(x)\cot(x) + 3\sec^2(x) - 5\csc^2(x) + 6e^x + \frac{4}{x}$$

2. (10 points) Find $g'(x)$ given that $g(x) = \left(\frac{x^9+8}{x^4+5}\right)^6$

$$g'(x) = 6\left(\frac{x^9+8}{x^4+5}\right)^5 \cdot \frac{d}{dx}\left(\frac{x^9+8}{x^4+5}\right)$$

$$g'(x) = 6\left(\frac{x^9+8}{x^4+5}\right)^5 \cdot \frac{\frac{d}{dx}(x^9+8)\cdot(x^4+5) - (x^9+8)\cdot\frac{d}{dx}(x^4+5)}{(x^4+5)^2}$$

$$g'(x) = 6\left(\frac{x^9+8}{x^4+5}\right)^5 \cdot \frac{9x^8\cdot(x^4+5) - (x^9+8)\cdot 4x^3}{(x^4+5)^2}$$

3. (10 points) Find $f'(x)$ given that $f(x) = \sqrt[4]{\arctan(x^9)}$ $= (\arctan(x^9))^{1/4}$

$$f'(x) = \frac{1}{4}(\arctan(x^9))^{-3/4} \cdot \frac{d}{dx}(\arctan(x^9))$$

$$f'(x) = \frac{1}{4}(\arctan(x^9))^{-3/4} \cdot \frac{1}{(x^9)^2+1} \cdot \frac{d}{dx}(x^9)$$

$$f'(x) = \frac{1}{4}(\arctan(x^9))^{-3/4} \cdot \frac{1}{x^{18}+1} \cdot 9x^8$$

4. (10 points) Find the equation of the line tangent to the following curve at its y-intercept.

$$14xy^4 = x^5 + 27y - 81$$

To find y-intercept, set $x=0$

$14(0)y^4 = 0^5 + 27y - 81$

$0 = 27y - 81$

$y = 3$

point: $(0,3)$

$\frac{d}{dx}(14xy^4) = \frac{d}{dx}(x^5 + 27y - 81)$

$\frac{d}{dx}(14x) \cdot y^4 + 14x \cdot \frac{d}{dx}(y^4) = 5x^4 + 27\frac{dy}{dx}$

$14y^4 + 14x \cdot 4y^3 \frac{dy}{dx} = 5x^4 + 27\frac{dy}{dx}$

plug in $(x,y) = (0,3)$

$14 \cdot 3^4 + 14(0) \cdot 4(3)^3 \frac{dy}{dx} = 5(0)^4 + 27\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{14 \cdot 3^4}{27}$

$= 42$

slope $= 42$

line

$y = 42x + 3$

5. (10 points) Suppose that A represents the number of grams of a radioactive substance at time t seconds. Given that $\frac{dA}{dt} = -0.125A$, how long does it take 12 grams of this substance to be reduced to 4 grams?

recall that $\frac{dy}{dx} = ky \Rightarrow y = Ce^{kx}$

so $\frac{dA}{dt} = -0.125A \Rightarrow A = Ce^{-0.125t}$

Plugging in $A = 12$ when $t = 0$ gives $12 = Ce^{-0.125(0)} = C$

Thus $A = 12e^{-0.125t}$

$4 = 12e^{-0.125t} \Rightarrow \frac{4}{12} = e^{-0.125t}$

$\ln\left(\frac{4}{12}\right) = \ln\left(e^{-0.125t}\right) = -0.125t$

$t = \frac{\ln(4/12)}{-0.125} = \frac{\ln(1/3)}{-1/8} = \frac{-\ln(3)}{-1/8} = 8\ln(3)$ seconds

6. (10 points) Determine the absolute minimum y-value on the graph of $y = 4e^{5x} - 60x + 30$.

$$y' = 4e^{5x} \cdot 5 - 60$$
$$y' = 20e^{5x} - 60$$
$$y' = 20(e^{5x} - 3)$$
$$y' = 0 \text{ when } e^{5x} - 3 = 0$$
$$e^{5x} = 3$$
$$\ln(e^{5x}) = \ln(3)$$
$$5x = \ln(3)$$
$$x = \tfrac{1}{5}\ln(3)$$

values of y'

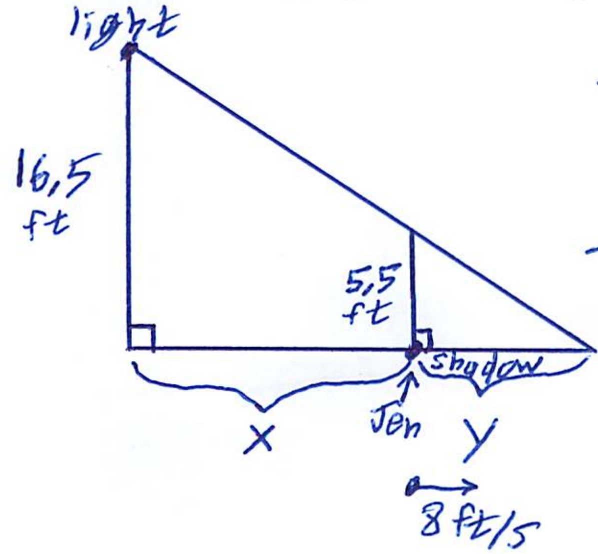
$- - - \quad 0 \quad + + +$

y decr. $\quad \frac{1}{5}\ln(3) \quad$ y incr. $\quad x$

abs. min when $x = \frac{1}{5}\ln(3)$ is

$$y = 4e^{5(\frac{1}{5}\ln(3))} - 60(\tfrac{1}{5}\ln(3)) + 30$$
$$= 4e^{\ln(3)} - 12\ln(3) + 30$$
$$= 4 \cdot 3 - 12\ln(3) + 30$$
$$= 42 - 12\ln(3)$$

7. (10 points) A street light is mounted at the top of a 16.5 ft tall pole. Jennifer is 5.5 ft tall. She walks away from the pole with a speed of 8 ft/s along a straight path. How quickly is the length of her shadow on the ground increasing when she is 15 ft from the pole?

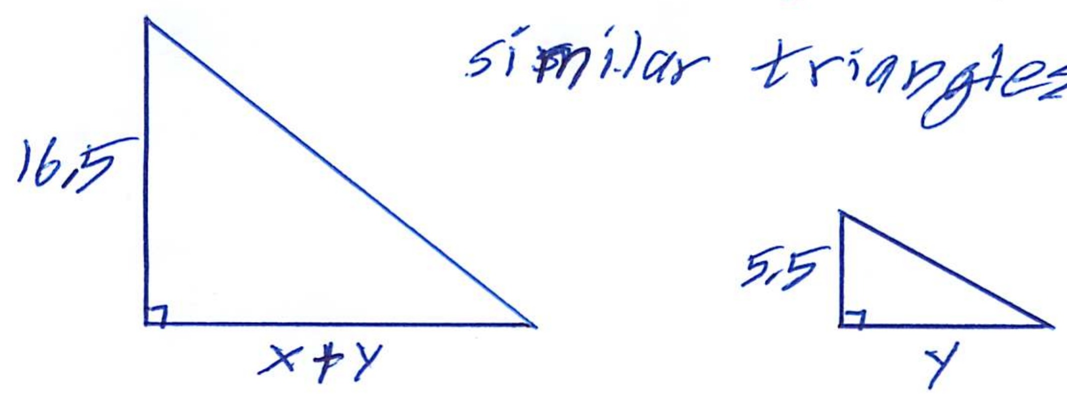


Given: $\frac{dx}{dt} = 8 \text{ ft/s}$

Want: $\left.\frac{dy}{dt}\right|_{x=15 ft}$

note in our solution below ~~that~~ her distance from the pole does not affect the answer

similar triangles



$$\frac{x+y}{16.5} = \frac{y}{5.5} \Rightarrow x+y = \frac{y}{5.5} \cdot 16.5 = 3y$$

$$x = 2y$$

$$\frac{d}{dt}(x) = \frac{d}{dt}(2y)$$

related rates: $\frac{dx}{dt} = 2\frac{dy}{dt}$

$$8 = 2\frac{dy}{dt}$$

$$\frac{dy}{dt} = 4$$

her shadow is increasing at 4 ft/s

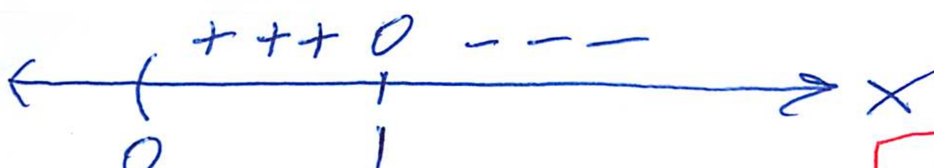
8. (10 points) For each $x > 0$, the function $f(x)$ is differentiable and has the following first derivative.

$$f'(x) = \frac{-8\ln(x)}{x^3}$$

(a) Where is the graph of $f(x)$ increasing and where is it decreasing? Use interval notation.

$f'(x) = 0 \Rightarrow \ln(x) = 0 \Rightarrow e^{\ln(x)} = e^0 \Rightarrow x = 1$

values of $f'(x)$



$f(x)$ is increasing on $(0, 1)$

$f(x)$ is decreasing on $(1, \infty)$

note
by definition of incr./decr. $[0, 1]$ and $[1, \infty)$ are also acceptable

(b) Where is the graph of $f(x)$ concave up and where is it concave down? Use interval notation.

$$f''(x) = \frac{\frac{d}{dx}(-8\ln(x)) \cdot x^3 - (-8\ln(x)) \cdot \frac{d}{dx}(x^3)}{(x^3)^2}$$

$$f''(x) = \frac{-8 \cdot \frac{1}{x} \cdot x^3 + 8\ln(x) \cdot 3x^2}{x^6}$$

$$f''(x) = \frac{-8x^2(1 - 3\ln(x))}{x^6}$$

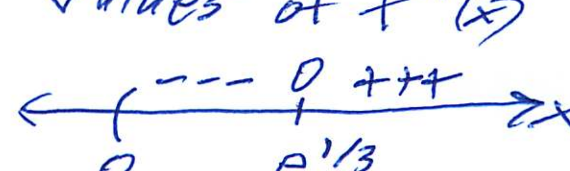
$$f''(x) = \frac{-8(1 - 3\ln(x))}{x^4}$$

$f''(x) = 0 \Rightarrow 1 - 3\ln(x) = 0$

$\Rightarrow \ln(x) = 1/3$

$\Rightarrow x = e^{1/3}$

values of $f''(x)$



$f(x)$ is concave down on $(0, e^{1/3})$

$f(x)$ is concave up on $(e^{1/3}, \infty)$

9. (20 points) Circle the correct limit.

(a) $\lim_{x\to 0} \frac{e^{12x}-1}{6x}$ $\genfrac{}{}{0pt}{}{\to 0}{\to 0}$ $\overset{H}{=} \lim_{x\to 0} \frac{12e^{12x}}{6} = \frac{12e^{12(0)}}{6} = \frac{12}{6} = 2$

(a) $-\infty$ (b) -2 (c) -1 (d) 0 (e) 1 (f) 2 (g) ∞

(b) $\lim_{x\to 7} \frac{e^{x-7}+1}{13-2x} = \frac{e^0+1}{13-2(7)} = \frac{2}{-1} = -2$

(a) $-\infty$ (b) -2 (c) -1 (d) 0 (e) 1 (f) 2 (g) ∞

(c) $\lim_{x\to\infty} \frac{2\ln x}{x^{42}}$ $\genfrac{}{}{0pt}{}{\to \infty}{\to \infty}$ $\overset{H}{=} \lim_{x\to\infty} \frac{2\cdot 1/x}{42x^{41}} = \lim_{x\to\infty} \frac{1}{21x^{42}} = 0$

(a) $-\infty$ (b) -2 (c) -1 (d) 0 (e) 1 (f) 2 (g) ∞

or using orders of growth to ∞ $\lim_{x\to\infty} \frac{2\ln(x) \to \infty \text{ slowly}}{x^{42} \to \infty \text{ quickly}} = 0$

(d) $\lim_{x\to\pi} \frac{2\cos x}{(\pi-x)^6}$ $\genfrac{}{}{0pt}{}{\to -2}{\to 0^+}$ $= -\infty$

(a) $-\infty$ (b) -2 (c) -1 (d) 0 (e) 1 (f) 2 (g) ∞

Students – do not write on this page!

1. (10 points) ______________________

2. (10 points) ______________________

3. (10 points) ______________________

4. (10 points) ______________________

5. (10 points) ______________________

6. (10 points) ______________________

7. (10 points) ______________________

8. (10 points) ______________________

9. (20 points) ______________________

TOTAL (100 points) ________________