AL1 (MWF 9:00-9:50) AL2 (MWF 10:00-10:50)

MATH 220 **Test 3** **Spring 2019**

Name Solutions **NetID** ______________

UIN ______________

Circle your TA discussion section.

- ▷ **AD1**, TR 9:00-10:50, Ran Ji
- ▷ **AD2**, TR 1:00-2:50, Cassie Christenson
- ▷ **AD3**, TR 11:00-12:50, Dana Neidinger
- ▷ **ADA**, TR 8:00-8:50, Eion Blanchard
- ▷ **ADB**, TR 9:00-9:50, Eion Blanchard
- ▷ **ADC**, TR 10:00-10:50, Yuxuan "Yuki" Zhang
- ▷ **ADD**, TR 11:00-11:50, Stathis Chrontsios
- ▷ **ADE**, TR 12:00-12:50, Kesav Krishnan
- ▷ **ADF**, TR 1:00-1:50, Souktik Roy
- ▷ **ADG**, TR 2:00-2:50, Gidon Orelowitz
- ▷ **ADH**, TR 3:00-3:50, Mina Nahvi
- ▷ **ADJ**, TR 9:00-9:50, Yuxuan "Yuki" Zhang
- ▷ **ADK**, TR 10:00-10:50, Souktik Roy
- ▷ **ADL**, TR 11:00-11:50, Gidon Orelowitz
- ▷ **ADM**, TR 12:00-12:50, Vincent Villalobos
- ▷ **ADN**, TR 1:00-1:50, Kesav Krishnan
- ▷ **ADO**, TR 2:00-2:50, Stathis Chrontsios
- ▷ **ADQ**, TR 4:00-4:50, Mina Nahvi
- ▷ **ADR**, TR 10:00-10:50, Vincent Villalobos

- Sit in your assigned seat (circled below).
- Do not open the test or write formulas upon it until I say *START*.
- Remove smartwatches and turn off all electronic devices.
- Put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say *STOP*.
- Quickly turn in your test to me or a TA and show your Student ID.

310	311	312	R	313	314	315	316	317	318	—	—	319	320	321	322	323	R	324	325	326
291	292	293	Q	294	295	296	297	298	299	300	301	302	303	304	305	306	Q	307	308	309
272	273	274	P	275	276	277	278	279	280	281	282	283	284	285	286	287	P	288	289	290
253	254	255	O	256	257	258	259	260	261	262	263	264	265	266	267	268	O	269	270	271
234	235	236	N	237	238	322	240	241	242	243	244	245	246	247	248	249	N	250	251	252
216	217	218	M	219	220	221	222	223	224	225	226	227	228	229	230		M	231	232	233
199	200	201	L	202	203	204	205	206	207	208	209	210	211	212	213		L	214	215	216
181	182	183	K	184	185	186	187	188	189	190	191	192	193	194	195		K	196	197	198
163	164	165	J	166	167	168	169	170	171	172	173	174	175	176	177		J	178	179	180
145	146	147	I	148	149	150	151	152	153	154	155	156	157	158	159		I	160	161	162
127	128	129	H	130	131	132	133	134	135	136	137	138	139	140	141		H	142	143	144
109	110	111	G	112	113	114	115	116	117	118	119	120	121	122	123		G	124	125	126
91	92	93	F	94	95	96	97	98	99	100	101	102	103	104	105		F	106	107	108
73	74	75	E	76	77	78	79	80	81	82	83	84	85	86	87		E	88	89	90
55	56	57	D	58	59	60	61	62	63	64	65	66	67	68	69		D	70	71	72
38	39	40	C	41	42	43	44	45	46	47	48	49	50	51			C	52	53	54
21	22	23	B	24	25	26	27	28	29	30	31	32	33	34			B	35	36	37
5	6	7	A	8	9	10	11	12	13	14	15	16	17				A	18	19	20
	1	2																3	4	

FRONT OF ROOM – 141 Wohlers Hall

1. (10 points) Evaluate the indefinite integral.

$$\int \left(91x^6 + 6\sin x + 5\sec^2 x + 9\csc^2 x + 8e^x + 4\right)\, dx \quad .$$

$= 91\cdot\frac{1}{7}x^7 + 6\cdot(-\cos(x)) + 5\cdot\tan(x) + 9\cdot(-\cot(x)) + 8\cdot e^x + 4x + C$

$= 13x^7 - 6\cos(x) + 5\tan(x) - 9\cot(x) + 8e^x + 4x + C$

2. (10 points) Evaluate the indefinite integral.

version 1: $\int \sin^{80}(x)\cos^3(x)\, dx$

$= \int \sin^{80}(x)\cos^2(x)\cos(x)dx$

$= \int \sin^{80}(x)(1-\sin^2(x))\cos(x)dx$

$= \int u^{80}(1-u^2)\,du$

$= \int (u^{80} - u^{82})\,du$

$= \frac{1}{81}u^{81} - \frac{1}{83}u^{83} + C$

$= \frac{1}{81}\sin^{81}(x) - \frac{1}{83}\sin^{83}(x) + C$

Substitution

$u = \sin(x)$

$du = \cos(x)dx$

version 2: $\int \cos^{60}(x)\sin^3(x)\, dx$

$= \int \cos^{60}(x)\sin^2(x)\sin(x)dx$

$= \int \cos^{60}(x)(1-\cos^2(x))\sin(x)dx$

$= \int u^{60}(1-u^2)(-du)$

$= \int (-u^{60} + u^{62})\,du$

$= -\frac{1}{61}u^{61} + \frac{1}{63}u^{63} + C$

$= -\frac{1}{61}\cos^{61}(x) + \frac{1}{63}\cos^{63}(x) + C$

Substitution

$u = \cos(x)$

$du = -\sin(x)dx$

$-du = \sin(x)dx$

3. (10 points) Evaluate the indefinite integral.

$$\int 225x\,(15x+4)^{28}\,dx = \int 225\left(\frac{u-4}{15}\right) u^{28} \frac{1}{15} du$$

$$\boxed{\begin{aligned} &u = 15x+4 \Rightarrow x = \frac{u-4}{15} \\ &du = 15dx \\ &\tfrac{1}{15} du = dx \end{aligned}}$$

$$= \int (u-4) u^{28} du$$

$$= \int (u^{29} - 4u^{28}) du$$

$$= \frac{1}{30} u^{30} - \frac{4}{29} u^{29} + C$$

$$= \frac{1}{30}(15x+4)^{30} - \frac{4}{29}(15x+4)^{29} + C$$

4. (10 points) Evaluate the indefinite integral.

$$\int \frac{8000x^{79}}{x^{160}+400}\,dx = \int \frac{8000}{(x^{80})^2+400} \cdot x^{79} dx$$

$$\boxed{\begin{aligned} &u = x^{80} \\ &du = 80x^{79} dx \\ &\tfrac{1}{80} du = x^{79} dx \end{aligned}}$$

$$= \int \frac{8000}{u^2+400} \cdot \frac{1}{80} du$$

$$\boxed{\begin{aligned} &w = \tfrac{u}{20} \\ &dw = \tfrac{1}{20} du \\ &20dw = du \end{aligned}}$$

$$= \int \frac{100}{400\left(\frac{u^2}{400}+1\right)} du$$

$$= \frac{1}{4} \int \frac{1}{\left(\frac{u}{20}\right)^2+1} du$$

$$= \frac{1}{4} \int \frac{1}{w^2+1} 20dw$$

$$= 5 \int \frac{1}{w^2+1} dw$$

$$= 5\arctan(w) + C$$

$$= 5\arctan\left(\frac{u}{20}\right) + C$$

$$= 5\arctan\left(\frac{x^{80}}{20}\right) + C$$

5. (10 points) Find the average value of the function $f(x) = 40x^3e^{x^4}$ on the interval $[0,5]$. Simplify your answer.

$$f_{ave} = \frac{1}{5-0}\int_0^5 40x^3e^{x^4}\,dx$$

$$\boxed{\begin{array}{l} u = x^4 \\ du = 4x^3dx \\ \frac{1}{4}du = x^3dx \end{array}} \quad \boxed{\begin{array}{l} x=0 \Rightarrow \\ u=0^4=0 \\ x=5 \Rightarrow \\ u=5^4=625 \end{array}}$$

$$= \frac{1}{5}\int_0^{625} 40e^u \cdot \frac{1}{4}du$$

$$= 2\int_0^{625} e^u du = 2e^u\Big]_0^{625} = 2e^{625} - 2e^0$$

$$= 2e^{625} - 2$$

6. (10 points)

version 1: Determine the x-coordinate for the absolute maximum value of the following function.

$$g(x) = \int_6^{80x-5x^2} \left(\frac{1}{\sin^{20}(t) + 400}\right) dt$$

$$g'(x) = \frac{1}{\sin^{20}(80x-5x^2)+400} \cdot \frac{d}{dx}(80x-5x^2) \quad \left(\begin{array}{l}\text{by F.T.C. (Part 1)} \\ \text{and chain rule}\end{array}\right)$$

$$= \frac{80-10x}{\sin^{20}(80x-5x^2)+400}$$

$$= \frac{10(8-x)}{\sin^{20}(80x-5x^2)+400}$$

values of $g'(x)$

$+ \quad 0 \quad -$ (at $x = 8$)

abs. max. when $x=8$

version 2: Determine the x-coordinate for the absolute minimum value of the following function.

$$g(x) = \int_9^{6x^2-132x} \left(\frac{1}{\cos^{40}(t) + 100}\right) dt$$

$$g'(x) = \frac{1}{\cos^{40}(6x^2-132x)+100} \cdot \frac{d}{dx}(6x^2-132x) \quad \left(\begin{array}{l}\text{by F.T.C. (Part 1)} \\ \text{and chain rule}\end{array}\right)$$

$$= \frac{12x-132}{\cos^{40}(6x^2-132x)+100}$$

$$= \frac{12(x-11)}{\cos^{40}(6x^2-132x)+100}$$

values of $g'(x)$

$- \quad 0 \quad +$ (at $x = 11$)

abs. min. when $x=11$

7. (10 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit. Simplify your answer.

$$\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{4 + 18kn^4}{n^6} \right) = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{4}{n^6} + \frac{18kn^4}{n^6} \right)$$

$$= \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{4}{n^6} + \sum_{k=1}^{n} \frac{18k}{n^2} \right)$$

$$= \lim_{n\to\infty} \left(\frac{4}{n^6} \cdot \sum_{k=1}^{n} 1 + \frac{18}{n^2} \cdot \sum_{k=1}^{n} k \right)$$

$$= \lim_{n\to\infty} \left(\frac{4}{n^6} \cdot n + \frac{18}{n^2} \cdot \frac{n(n+1)}{2} \right)$$

$$= \lim_{n\to\infty} \left(\frac{4}{n^5} + \frac{18n(n+1)}{2n^2} \right)$$

$$= 0 + \frac{18}{2} = 9$$

8. (10 points) Suppose that $f(x)$ is a polynomial which satisfies the following conditions.

- $\displaystyle\int_5^{129} f(x)\,dx = 30$
- $\displaystyle\int_{42}^{129} f(x)\,dx = 38$

$$\int_5^{129} f(x)\,dx = \int_5^{42} f(x)\,dx + \int_{42}^{129} f(x)\,dx$$

$$30 = \int_5^{42} f(x)\,dx + 38$$

$$\int_5^{42} f(x)\,dx = 30 - 38 = -8$$

Evaluate the following quantities.

(a) $\displaystyle\int_5^{42} (2f(x) + 10)\,dx = 2\int_5^{42} f(x)\,dx + \int_5^{42} 10\,dx$

$$= 2(-8) + \left[10x\right]_5^{42}$$

$$= -16 + [10 \cdot 42 - 10 \cdot 5] = 354$$

(b) $\displaystyle\int_1^{5} 6x^2 f\left(x^3 + 4\right)\,dx = \int_5^{129} 2f(u)\,du = 2\int_5^{129} f(u)\,du$

$$u = x^3 + 4$$
$$du = 3x^2\,dx$$
$$2du = 6x^2\,dx$$

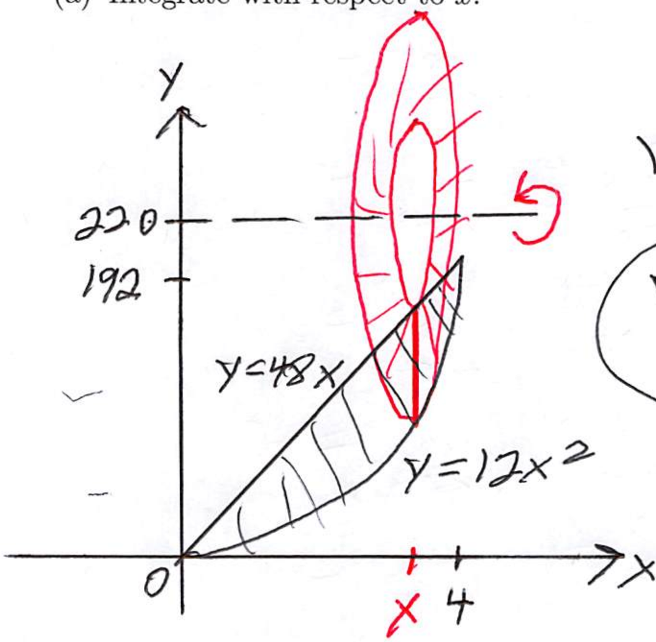
$x = 1 \Rightarrow$
$u = 1^3 + 4 = 5$
$x = 5 \Rightarrow$
$u = 5^3 + 4 = 129$

$$= 2 \cdot 30$$

$$= 60$$

9. (10 points) Let **R** be the finite region bounded by the graphs of $y = 12x^2$ and $y = 48x$. These curves intersect at the origin and at the point $(x, y) = (4, 192)$. Revolve **R** around the horizontal line $y = 220$ to form a solid. In the following manner, set up <u>but do not evaluate</u> definite integrals which represent the volume of the solid. Use proper notation.

(a) Integrate with respect to x.

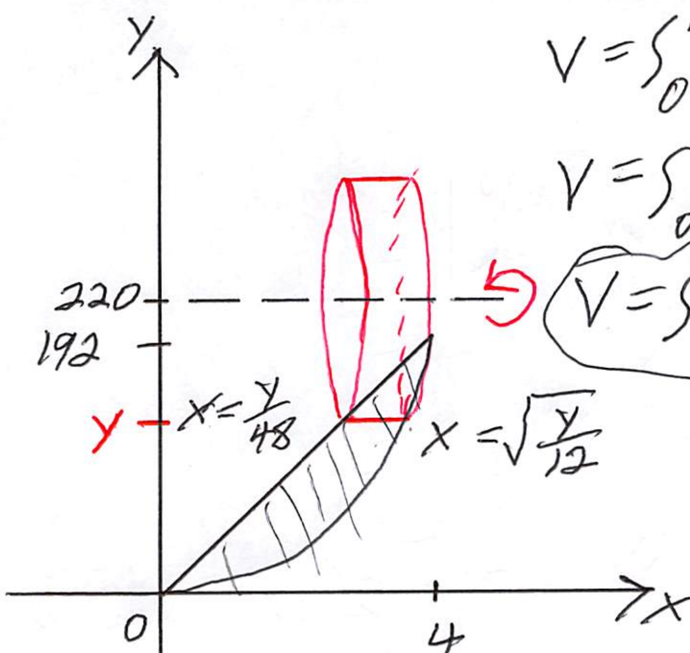


$$V = \int_0^4 \left(\text{cross-sectional area}\right) dx$$

$$V = \int_0^4 \left(\pi r_{out}^2 - \pi r_{in}^2\right) dx$$

$$V = \int_0^4 \left(\pi (220 - 12x^2)^2 - \pi (220 - 48x)^2\right) dx$$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)



$$V = \int_0^{192} \left(\text{surface area}\right) dy$$

$$V = \int_0^{192} 2\pi r h \, dy$$

$$V = \int_0^{192} 2\pi (220 - y)\left(\sqrt{\frac{y}{12}} - \frac{y}{48}\right) dy$$

note) $y = 48x \Rightarrow x = \frac{y}{48}$ and $y = 12x^2 \Rightarrow x = \pm\sqrt{\frac{y}{12}}$

here use $x = \sqrt{y/12}$

10. (10 points) Given the function below, use a linear approximation to estimate $g(1.03)$. Simplify and write your answer in decimal form.

$g(x) = 20\ln(x) - 8e^{x^2-1}$ $\Rightarrow g'(x) = 20 \cdot \frac{1}{x} - 8e^{x^2-1} \cdot 2x$

$$g'(x) = \frac{20}{x} - 16xe^{x^2-1}$$

Point: $(1, g(1)) = (1, 20\ln(1) - 8e^{1^2-1}) = (1, -8)$

slope: $g'(1) = \frac{20}{1} - 16(1)e^{1^2-1} = 4$

tangent line at $x=1$: $y-(-8) = 4(x-1)$

$$y = -8 + 4(x-1)$$

$g(x) \approx -8 + 4(x-1)$ for x near 1

$g(1.03) \approx -8 + 4(1.03-1)$

$g(1.03) \approx -8 + 0.12$

$g(1.03) \approx -7.88$

Students – do not write on this page!

1. (10 points) ____________________

2. (10 points) ____________________

3. (10 points) ____________________

4. (10 points) ____________________

5. (10 points) ____________________

6. (10 points) ____________________

7. (10 points) ____________________

8. (10 points) ____________________

9. (10 points) ____________________

10. (10 points) ____________________

TOTAL (100 points) _______________