Math 231 Exam 2

UIUC, October 25, 2010

Name ______________________________

Section (circle yours)

TTh 12-12:50 (Chayapa Darayon)	TTh 11-11:50 (Chayapa Darayon)	TTh 1-1:50 (Nate Orlow)	TTh 2-2:50 (Nate Orlow)	TTh 3-3:50 (Jian Liang)	TTh 4-4:50 (Jian Liang)
	WF 9-10:50 (Brian Ray)		WF 11-12:50 (Uma Ravat)		

Instructions. Please provide your answers in the space provided—using the back of sheets if necessary. No calculators, computers, friends or psychic reading are to be used while taking this test. If you are unclear about a question—please raise your hand and ask.

Question	Points	Score
1	15	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	15	
Total	90	

1. (15 points) The following are worth 3 points each and are short answer.

   (a) The series $\sum_{n=1}^{\infty} a_n$ has the property that the n-th partial sum for every n is given by

   $$s_n = \frac{n(n-1)}{2} = 1 + 2 + 3 + \cdots + n$$

   What is a_n for every n?

   (b) Show that for any number r,

   $$1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

   (c) Draw a diagram and give a brief explanation why $\sum_{n=2}^{9} \frac{1}{n}$ is less than $\int_1^9 \frac{1}{x}\,dx$.

   (d) The series $\sum_{n=1}^{\infty} a_n$ has the property that its n-th partial sum $s_n = a_1 + a_2 + \cdots + a_n$ for every n is $\frac{1}{n}$. Does the series converge and if so to what value?

   (e) Given an example of a series $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$ that diverges while the series $\sum_{n=1}^{\infty}(a_{2n-1} + a_{2n}) = (a_1 + a_2) + (a_3 + a_4) + (a_5 + a_6) + \cdots$ converges.

2. (10 points) Show that the following series diverges except for one value of c. Then compute the sum of the series for that value of c.

$$\sum_{n=1}^{\infty} \left(\frac{c}{2n} + \frac{1}{n+1} \right)$$

3. (10 points, 5 points each) Find the sum of the following series, if they exist, otherwise explain:

   (a) $5 + 2 + \frac{20}{25} + \frac{40}{125} + \dots.$

   (b) $\sum_{n=0}^{\infty} \frac{11^n}{(-8)^{n-1}}$.

4. (10 points, 5 points each)

(a) Does the *sequence* $\{\frac{3n^2+2}{4n^2-4}\}$ converge? If so, find the limit, if not, explain.

(b) Does the series $\sum_{n=4}^{\infty}(-1)^n \frac{3n^2+2}{4n^2-4}$ converge? If so, find the sum, if not, explain.

5. (10 points) Determine whether the series is **absolutely convergent**, **conditionally convergent** or **divergent**. Be sure to indicate your reasoning to receive full credit.

$$\sum_{n=0}^{\infty} \frac{n}{3n^4 + 4}$$

6. (10 points) Determine whether the series is **absolutely convergent**, **conditionally convergent** or **divergent**. Be sure to indicate your reasoning to receive full credit.

$$\sum_{n=0}^{\infty} (-1)^{n+1} \frac{n^2 2^n}{n!}$$

7. (10 points) Determine whether the series is **absolutely convergent**, **conditionally convergent** or **divergent**. Be sure to indicate your reasoning to receive full credit.

$$\sum_{n=3}^{\infty}(-1)^n \frac{1}{n\sqrt{\ln(n)}}$$

8. (15 points) Determine whether *one* of the following series is **absolutely convergent**, **conditionally convergent** or **divergent**. Be sure to indicate your reasoning to receive full credit and which attempt you want graded if you do work on both problems.

$$\sum_{n=0}^{\infty} \frac{3^n + (-2)^n}{6^n \sqrt[3]{5}} \qquad \text{or} \qquad \sum_{n=1}^{\infty} \frac{n!\, \tan^{-1}(n)}{(2n)!}$$