Math 231 Exam 3

UIUC, November 17, 2010

1 Short answer.

(a) The power series $\sum_{n=0}^{\infty} a_n x^n$ is known to diverge for $x = -2$ and $x = 5$. What is the maximum radius of convergence for this series?

(b) One can show that a power series representation for $x\sin(x^2)$ is given by

$$x\sin(x^2) = x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \frac{x^{15}}{7!} + \cdots$$

What is $(x\sin(x^2))^{(11)}(0)$ and why?

2 Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \sqrt{n} x^n$$

3 Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{n}$$

4 Find a series solution to the integral

$$\int_0^1 x^4 e^x \, dx$$

5

(a) How would the power series for $(1+4x)^{-3}$ be expressed using the binomial coefficient theorem?

(b) Find a power series for $\frac{1}{(1+4x)^3}$ using the fact that it is the second derivative of $\frac{1}{32(1+4x)}$.

(c) Using your results from (a) and (b), what is the value of $\binom{-3}{6}$?

6 Suppose you know that for some mysterious function f, $f(3) = 1, f'(3) = 2, f^{(2)}(3) = 3, f^{(3)}(3) = 4$ while $f^{(4)}(x) = \ln x$.

a) What is the the Taylor polynomial of degree 3 centered at 3 for f?

b) If you use the the degree 3 Taylor polynomial centered at 3 to approximate $f(e)$, use Taylor's Theorem to estimate your accuracy. Be sure to justify your answer.