

1. (8 points each) Evaluate the integral.

(a)  $\int x \cos(3x) dx$

(b)  $\int \sec^4(5x) dx$

(c)  $\int \frac{3 \cos^5 \alpha}{\sqrt{\sin \alpha}} d\alpha$

2. (8 points each) Evaluate the integral.

(a)  $\int \frac{dx}{(25 + x^2)^{\frac{3}{2}}}$

(b)  $\int \frac{x^2}{x^2 + 9}$

(c)  $\int \frac{x + a}{x^2 - x} dx$

3. (10 points) Determine whether the integral is convergent or divergent. If it is convergent, evaluate it.

(a)  $\int_1^{\infty} \frac{dx}{\sqrt[3]{x}}$

(b)  $\int_0^1 \frac{dx}{\sqrt[3]{x}}$

4. (8 points)

For any twice differentiable function  $f$  on  $[a, b]$ ,  $P_a^b(f)$  will approximate  $\int_a^b f(x) dx$  to an error no more than  $K_2 \frac{(b-a)^4}{32}$  when  $|f''(x)| \leq K_2$  for all  $x$  in  $[a, b]$ . You use  $P(f)$  to numerically approximate the integral  $\int_1^3 \sqrt{1+x^3} dx$  by subdividing the interval into 10 equal pieces and applying  $P(f)$  to each of the smaller intervals.

Using that the absolute value of the second derivative of  $\sqrt{1+x^3}$  is never more than 2 over  $[1, 3]$ , what is an upper bound for the error of your approximation (4 points) and why (6 points)?

5. (12 points, 8/4) Let  $a > 0$ . Evaluate the integrals.

(a)  $\int \sin(x)e^{-ax} dx$

(b)  $\int_0^\infty \sin(x)e^{-ax} dx$