

# Math 231/EL1 Exam 2

UIUC, March 14, 2013

Name \_\_\_\_\_

**WF Section** (circle yours)

9-9:50 (EDA)      10-10:50 (EDB)      11-11:50 (EDC)      12-12:50 (EDD)  
Nathan Fieldsteel    Nathan Fieldsteel    Matthew Mastroeni    Matthew Mastroeni

1-1:50 (EDE)      2-2:50 (ADF)      2-2:50 (ADG)      4-4:50 (ADH)  
Zhenghui Huo      Zhenghui Huo      Sarah Yeakel      Sarah Yeakel

**Instructions.** Please provide your answers in the space provided—using the back of sheets if necessary. No friends, calculators, electronics or psychic readings are to be used while taking this test. If you are unclear about a question—please raise your hand and ask.

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Question	Points	Score
1	30	
2	20	
3	5	
4	5	
5	3	
6	8	
7	8	
TA Correct	1	
Total	80	

1. (10 points each)

Determine if the series is **absolutely convergent**, **conditionally convergent** or **divergent**. Be sure to show your reasoning. No work, no credit.

(a)  $\sum_{n=5}^{\infty} \frac{1}{\sqrt{n^3 + 10n}}$

(b)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{6n + 3}$

(c)  $\sum_{n=2}^{\infty} (-1)^n \frac{n+2}{n}$

2. (10 points each)

Determine if the series is **absolutely convergent**, **conditionally convergent** or **divergent**. Be sure to show your reasoning. No work, no credit.

(a)  $\sum_{n=1}^{\infty} \frac{n^2}{6^n}$

(b)  $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

3. (5 points) Show that for any number  $r \neq 1$  and positive integer  $k$ ,

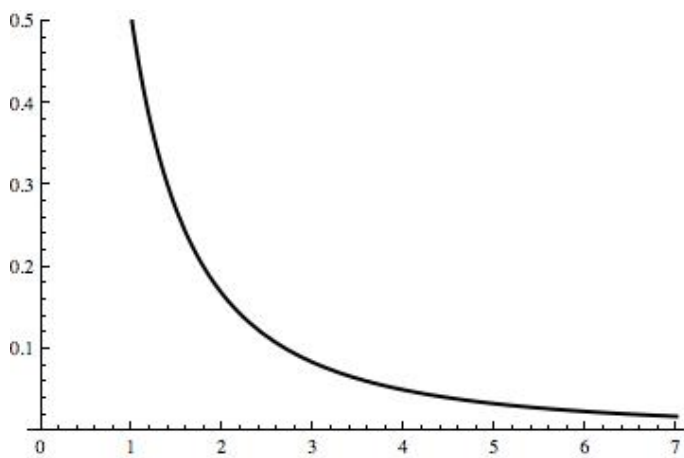
$$1 + r + r^2 + \cdots + r^k = \frac{1 - r^{k+1}}{1 - r}$$

4. (5 points) A series is defined by setting  $a_0 = 4$  and  $a_n = .5 a_{n-1}$  for all  $n > 0$ .

What is  $\sum_{n=0}^{\infty} a_n$ ?

5. (3 points) Draw on the diagram and give a brief explanation why

$$\sum_{n=1}^5 \frac{1}{n(n+1)} \geq \int_1^6 \frac{dx}{x(x+1)}$$

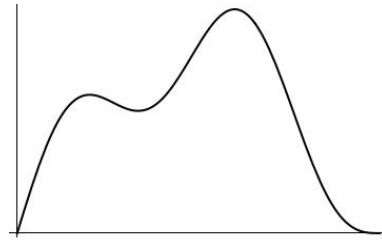


$$f(x) = \frac{1}{x(x+1)}$$

6. (8 points, 2/3/3)

This problem concerns the curve

$$y = 5 \sin x + \sin 5x, \quad 0 \leq x \leq \pi$$

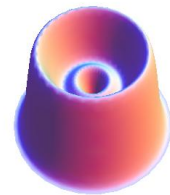


(a) Give an integral for the length of the curve. You do not need to evaluate the integral.

(b) Give an integral for the area of the surface obtained by rotating the curve about the  $x$ -axis. You do not need to evaluate the integral.



(c) Give an integral for the area of the surface obtained by rotating the curve about the  $y$ -axis. You do not need to evaluate the integral.



7. (8 points) Show that the following series diverges except for one value of  $c$ . Then compute the sum of the series for that value of  $c$ .

$$\sum_{n=2}^{\infty} \left( \frac{c}{n-1} + \frac{1}{n+1} \right)$$