Math 231 Exam III

UIUC, April 18, 2013

1. (*8 points*) Short answer.

   (a) Suppose that $c(x) = \sum_{n=0}^{\infty} c_n x^n$ converges for $x = -4$ but diverges for $x = 6$.

   i. $\sum_{n=0}^{\infty} 2^n c_n$ (absolutely converges/ conditionally converges/diverges).

   ii. $\sum_{n=0}^{\infty} (-8)^n c_n$ (absolutely converges/ conditionally converges/diverges).

   (b) Calculate the binomial coefficient $\binom{-3}{3} =$

   (c) Recall that $\ln 2 = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$. By the Alternating Series Estimation, how accurate is $1 - \frac{1}{2} + \cdots - \frac{1}{8} + \frac{1}{9} = \frac{1879}{2520}$ to the actual value of $\ln 2$?

2. (*10 points*) Find a series solution to the integral

$$\int_0^1 e^{-x^2}\, dx$$

3. (*12 points each*) Find the radius and interval of convergence for the power series. Be sure to indicate which points converge absolutely and which converge conditionally.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+2)^n}{2 \cdot 4 \cdot 6 \cdots (2n+2)}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(3x+2)^n}{(n+1)(n)}$

4. (*12 points*) Give the Taylor polynomial with degree 2 centered at 1 for $f(x) = \sqrt[5]{x}$. Then use Taylor's Inequality to estimate the accuracy of this approximation for x between 1 and 1.1.

5. (*12 points*) Let f be a function which has all derivatives and has the property that $f'' = f$. If $f(0) = 0$ and $f'(0) = 1$, what is the power series for f at 0?

6. (*12 points*) Determine a power series centered at 0 for $f(x) = \sin^{-1} x$ and use it to determine the 100-th derivative of $\sin^{-1} x$ at 0. You may find the following useful

$$\binom{-\frac{1}{2}}{n} = (-1)^n \frac{(1)(3)(5)\cdots(2n-1)}{2^n n!}$$