MATH231 SPRING 2022 MIDTERM 1 (50 POINTS)

- **No notes, calculators, or other aids are allowed.**

- Read instructions carefully and write your answers in the space provided.

- There are 24 questions in total. You have 45 minutes to answer them.

- To receive full credit, you must show **all** of your work. (That is, **you are supposed to justify the formulae which haven't been given in class or derived in home-work**.)

- Derivatives of inverse trigonometric functions

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}} \qquad (\arctan x)' = \frac{1}{1+x^2}$$

Q1–Q4. Evaluate the following integrals. You may use any method other than the hint.

1. (8 points) $\displaystyle\int \arctan x \, \mathrm{d}x$

2. (8 points) $\displaystyle\int \frac{1}{\sqrt{x^2 - 2x}} \, \mathrm{d}x$ (Hint: trig substitution)

3. (8 points) $\displaystyle\int \frac{5x}{(x-2)(x+3)}\,\mathrm{d}x$

4. (10 points) $\displaystyle\int 16\sin^2 x \cos^4 x\,\mathrm{d}x$ (Hint: both powers are even)

Q5–Q6. Improper integrals.

5. (8 points) Use the definition to show that $\displaystyle\int_e^\infty \frac{1}{x\sqrt{\ln x}}\,dx$ diverges.

6. (8 points) Use the comparison test to show $\displaystyle\int_\pi^\infty \frac{x\sin^2 x + 1}{x^4}\,dx$ converges.