MATH231 Spring 2022 Midterm 2 (50 points)

- **No notes, calculators, or other aids are allowed.**
- Read instructions carefully and write your answers in the space provided.
- There are 24 questions in total. You have 45 minutes to answer them.
- To receive full credit, you must show **all** of your work. (That is, **you are supposed to justify any formula which haven't been given in class or derived in homework**.)
- You may use a different method other than the one asked in the question. However, the maximum points you can get will be half of the total points.

Integrals $I = \int f(u)\,\mathrm{d}u$ containing square roots, where a is a positive number.

$f(u)$	I
$\sqrt{a^2-u^2}$	$\dfrac{u\sqrt{a^2-u^2}+a^2\arcsin(u/a)}{2}+C$
$\sqrt{u^2-a^2}$	$\dfrac{u\sqrt{u^2-a^2}-a^2\ln\left\lvert u+\sqrt{u^2-a^2}\right\rvert}{2}+C$
$\dfrac{1}{\sqrt{u^2-a^2}}$	$\ln\left\lvert u+\sqrt{u^2-a^2}\right\rvert+C$
$\sqrt{a^2+u^2}$	$\dfrac{u\sqrt{a^2+u^2}+a^2\ln\left\lvert u+\sqrt{a^2+u^2}\right\rvert}{2}+C$
$\dfrac{1}{\sqrt{a^2+u^2}}$	$\ln\left\lvert u+\sqrt{a^2+u^2}\right\rvert+C$

Q1–Q2. Arc length and the surface of revolution.

1. (10 points) Compute the arc length of the curve $y = \dfrac{x^2}{4} - \dfrac{\ln x}{2}$ for $1 \leq x \leq e$.

2. (10 points) Using surface of revolution integral to compute the surface area obtained by rotating the curve $y = \sqrt{r^2 - x^2}$ about the x-axis, where $r > 0$, constant and $-r \leq x \leq r$.

Q3–Q6. Series

3. (6 points) Compute $\sum_{n=0}^{\infty} \sqrt{5}^{1-2n} 2^{n+2}$. (Careful: n start from 0.)

4. (8 points) Use the divergence test to determine if $\sum_{n=1}^{\infty} \left(1 + \frac{1}{2n}\right)^n$ converges or not.

5. (10 points) Use the integral test to determine if $\sum_{n=1}^{\infty} ne^{-n^2}$ converges or not.

6. (6 points) Use the comparison test to determine if $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^6+n}}$ converges or not.