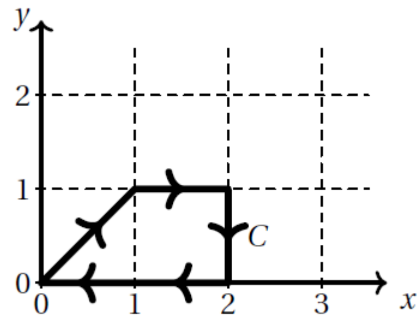
Question 1

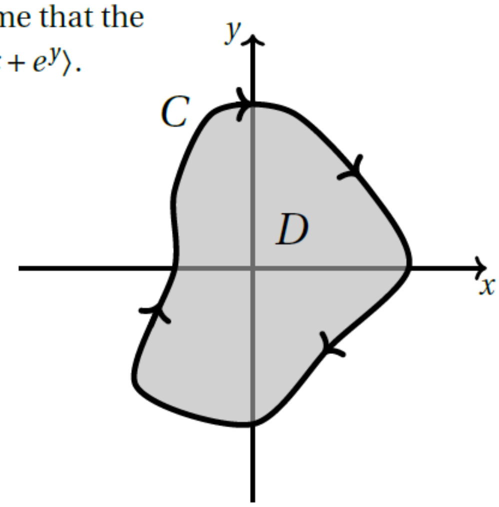
Let C be the oriented curve shown at right against a dashed grid of unit squares. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F} = \langle\, y^2 + \cos x,\ x + e^y \,\rangle$. **(6 points)**



$\int_C \mathbf{F}\cdot d\mathbf{r} =$

Question 2

Let C be the oriented curve shown at right, oriented **clockwise**. Assume that the region D bounded by C has area 6. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for $\mathbf{F} = \langle \sin x, 2x + e^y\rangle$.
(4 points)

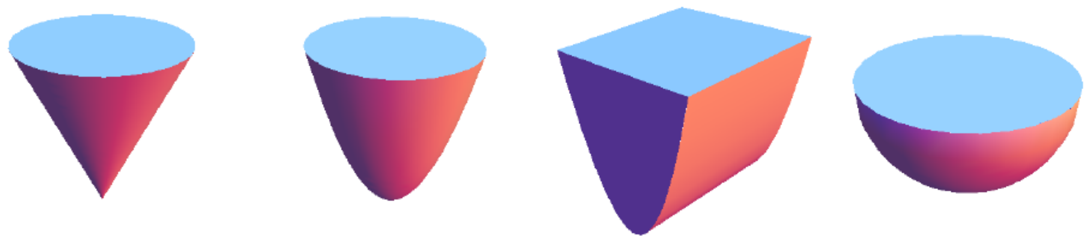


$\int_C \mathbf{F}\cdot d\mathbf{r} =$

Question 3

Consider the region R in $\mathbb{R}^3$ above the surface $x^2 + y^2 - z = 4$ and below the xy-plane. Also consider the vector field $\mathbf{F} = \langle 0, 0, z \rangle$.

(a) Circle the picture of R below. **(2 pts)**

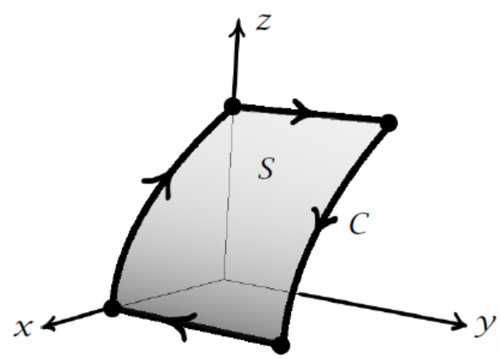


(b) Directly calculate the flux of $\mathbf{F}$ through the entire surface ∂R, with respect to the outward unit normals. **(10 pts)**

(c) Use the Divergence Theorem and your answer in (b) to compute the volume of R. **(3 pts)**

Question 4

Let C be the curve shown at right, which is the boundary of the portion of the surface $x + z^2 = 1$ in the positive octant where additionally $y \leq 1$.



(a) Label the four corners of C with their (x, y, z)-coordinates.
(1 pt)

(b) For $\mathbf{F} = \langle 0, xyz, xyz \rangle$, directly compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.
(6 pts)

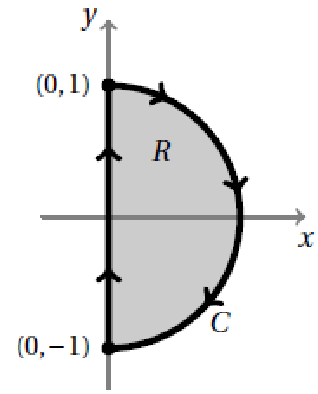
(c) Compute $\operatorname{curl} \mathbf{F}$. **(2 pts)**

(d) Use Stokes' Theorem to compute the flux of $\operatorname{curl} \mathbf{F}$ through the surface S where the normals point out from the origin. **(3 pts)**

(e) Give two distinct reasons why the vector field $\mathbf{F}$ is *not* conservative. **(2 pts)**

Question 5

Consider the vector field $\mathbf{F}(x, y) = \langle ye^x,\ e^x + x\rangle$. Let R be the half disk below, and let C be the boundary of R, oriented as shown.



(a) Use Green's Theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. **(4 points)**

$\int_C \mathbf{F} \cdot d\mathbf{r} =$

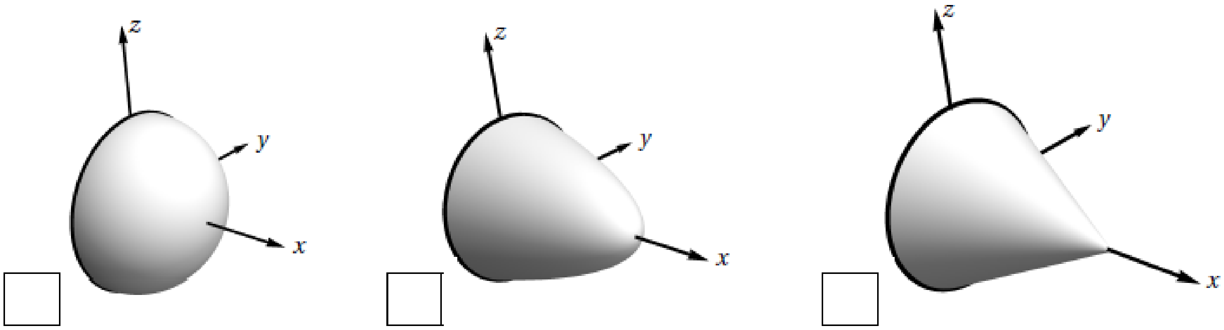
(b) Let C_0 be the round part of C, that is, just the semicircle from $(0, 1)$ to $(0, -1)$, not including the y-axis. Compute $\int_{C_0} \mathbf{F} \cdot d\mathbf{r}$. **(2 points)**

$\int_{C_0} \mathbf{F} \cdot d\mathbf{r} =$

Question 6

Let S be the surface in $\mathbb{R}^3$ parameterized by $\mathbf{r}(u,v) = \langle 2-2v^2,\ v\cos u,\ v\sin u\rangle$ for $0 \le u \le 2\pi$ and $0 \le v \le 1$.

(a) Mark the correct picture of S below. (**2 points**)



(b) For the vector field $\mathbf{F} = \langle 0,\ -z,\ y\rangle$, directly evaluate $\iint_S (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dA$ where $\mathbf{n}$ is unit normal vector field that points in the positive x-direction. (**5 points**)

$\iint_S (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dA =$

(c) Check your answer in (b) using Stokes' Theorem. (**3 points**)