Question 1

Consider the vector field $\mathbf{F}$ on $\mathbb{R}^2$ shown below right. For each part, circle the best answer. (**1 point each**)

(a) The line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ is:

negative	zero	positive

(b) At A, the vector $\operatorname{curl}\mathbf{F}$ is:

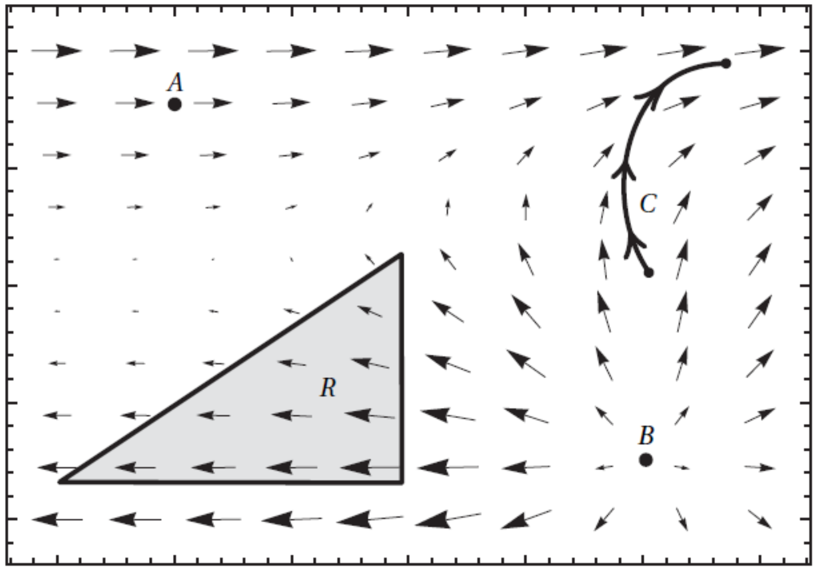
$\langle 1,0,0\rangle$	$\langle 0,0,-1\rangle$	$\langle 0,0,1\rangle$

(c) At B, the divergence $\operatorname{div}\mathbf{F}$ is:

negative	zero	positive

(d) If $\mathbf{F}=\langle P,Q\rangle$, then $\iint_R \frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\,dA$ is:

negative	zero	positive



(e) The vector field $\mathbf{F}$ is conservative:

True	False

Question 2

Consider the surfaces S and H show below right; the boundary of S is the unit circle in the xy-plane, and H has no boundary. For each part, circle the best answer.

(a) For $\mathbf{F}=\langle yz,\ xz+x,\ z\rangle$, the integral $\iint_H \mathbf{F}\cdot\mathbf{n}\,dA$ is:

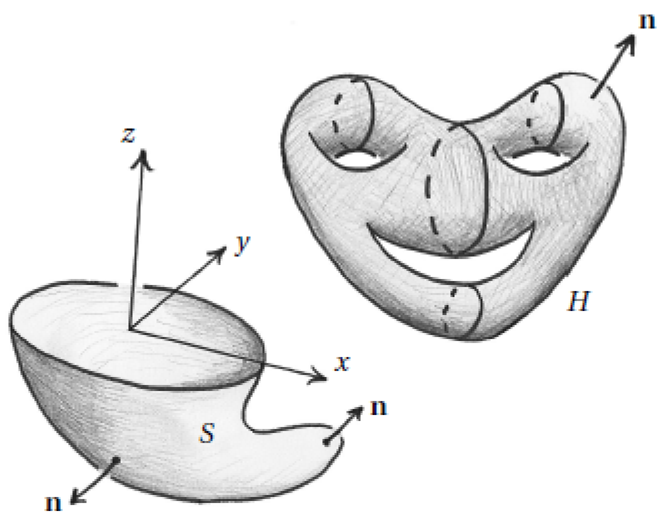
negative	zero	positive

(**1 point**)

(b) The flux of $\operatorname{curl}\mathbf{F}=\langle -x,y,1\rangle$ through H is:

negative	zero	positive

(**1 point**)



(c) The integral $\iint_S (\operatorname{curl}\mathbf{F})\cdot d\mathbf{S}$ is:

-2π	$-\pi$	0	π	2π

(**2 points**)

(d) For $\mathbf{G}=\langle y,\ z,\ 2\rangle$, the integral $\iint_S \mathbf{G}\cdot\mathbf{n}\,dA$ is:

-2π	$-\pi$	0	π	2π

(**2 points**)

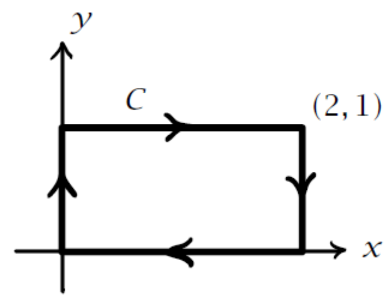
Question 3

Let $\mathbf{F}(x, y, z) = \left\langle \frac{x^3}{3},\ x^2\cos(z) + \frac{y^3}{3},\ \frac{z^3}{3} \right\rangle$ and let S denote the surface defined by $x^2 + y^2 + z^2 = 1$, equipped with the inward-pointing unit normal vector field $\mathbf{n}$. Compute $\iint_S \mathbf{F}\cdot\mathbf{n}\, dA$ by any valid method. **(6 points)**

$\iint_S \mathbf{F}\cdot\mathbf{n}\, dA =$

Question 4

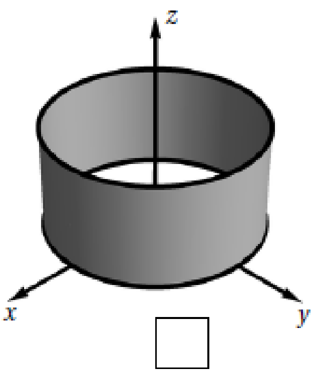
Let C be the oriented curve in $\mathbb{R}^2$ shown at right. For the vector field $F(x, y) = (x^3, x^2)$, use Green's theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$. **(6 points)**

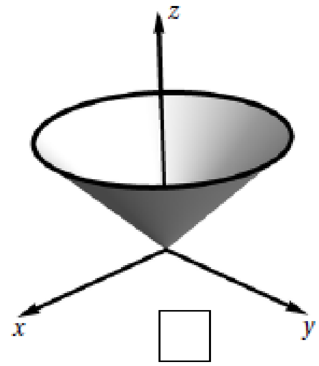


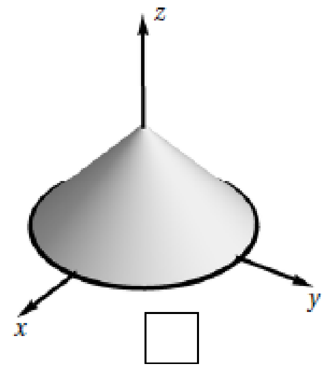
Question 5

Consider the surface S parameterized by $\mathbf{r}(u, v) = \langle \cos u,\ \sin u,\ v \rangle$ for $0 \le u \le 2\pi$ and $0 \le v \le 1$.

(a) Mark the picture of S below. **(2 points)**







(b) Consider the vector field $\mathbf{F} = \langle yz,\ -xz,\ 1 \rangle$ which has $\operatorname{curl}\mathbf{F} = \langle x,\ y,\ -2z \rangle$. Directly evaluate $\iint_S (\operatorname{curl}\mathbf{F}) \cdot \mathbf{n}\ dS$ via the given parameterization, where $\mathbf{n}$ is the outward normal vector field. **(4 points)**

$\iint_S (\operatorname{curl}\mathbf{F}) \cdot \mathbf{n}\ dS =$

(c) Check your answer in (b) using Stokes' Theorem. **(4 points)**

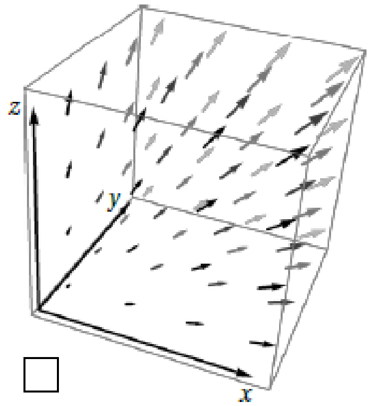
Question 6

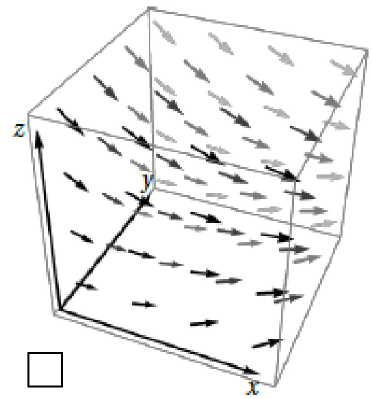
For each problem, circle the best answer. (**1 point each**)

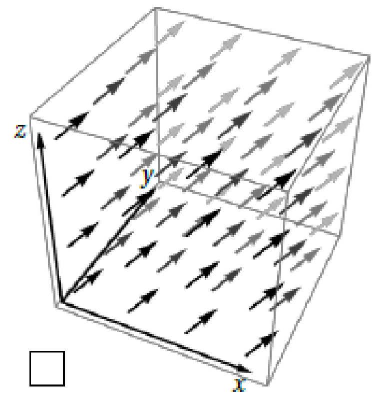
(a) Consider the vector field $\mathbf{F} = \langle 1,\ x,\ -z \rangle$. The vector field $\mathbf{F}$ is:

conservative	not conservative

(b) Mark the plot of $\mathbf{F}$ on the region where each of x, y, z is in $[0,1]$:



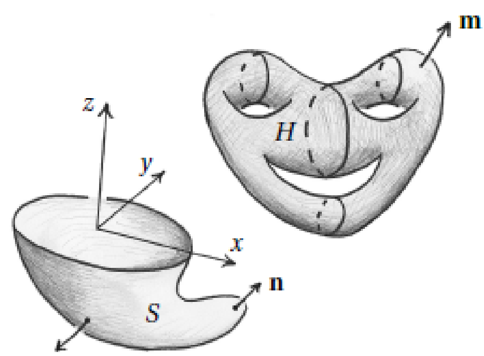




(c) For the leftmost vector field in part (b) is the divergence:

negative	zero	positive

Let S and H be the surfaces at right; the boundary of S is the unit circle in the xy-plane, and H has no boundary. Let $\mathbf{G} = \langle x,\ y,\ z \rangle$.



(d) The flux $\iint_H \mathbf{G} \cdot \mathbf{m}\, dS$ is:

negative	zero	positive

(e) The flux $\iint_S \mathbf{G} \cdot \mathbf{n}\, dS$ is:

negative	zero	positive

(f) The flux $\iint_S (\operatorname{curl} \mathbf{G}) \cdot \mathbf{n}\, dS$ is:

negative	zero	positive