Question 1

(a) Consider the vector field $\mathbf{F} = \langle\, yz,\ -xz,\ yx \,\rangle$ on $\mathbb{R}^3$. Compute the curl of $\mathbf{F}$. (**2 points**)

curl $\mathbf{F} = \langle \quad , \quad , \quad \rangle$

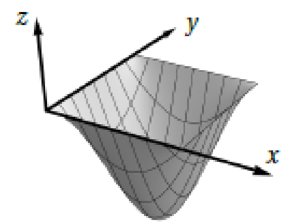
(b) Let S be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \leq 3$. Find the flux of curl $\mathbf{F}$ through S with respect to the outward pointing unit normal vector field. (**6 points**)

flux =

(c) Let D be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \geq 3$. Find the flux of curl $\mathbf{F}$ through D with respect to the outward pointing unit normal vector field. flux = (**1 point**)

Question 2

(a) Let S be the portion of the surface $z = -\sin(x)\sin(y)$ where $0 \le x \le \pi$ and $0 \le y \le \pi$ which is shown at right. Use a parameterization to find the flux of $\mathbf{F} = \langle 0, 0, 2z+1 \rangle$ through S with respect to the downward normals. **(5 points)**



flux =

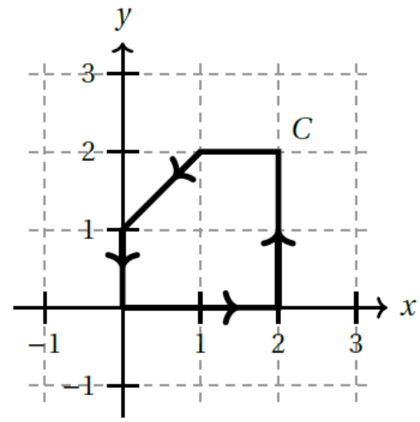
(b) Let E be the region below the xy-plane and above S. Use an integral theorem to compute the flux of $\mathbf{F}$ through ∂E with respect to the outward normals. **(4 points)**

flux =

(c) Your answers in (a) and (b) should differ. Explain what accounts for the difference. **(1 point)**

Question 3

Compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = \langle y + 2\cos(x),\ 3x + e^{y^2}\rangle$ and C is the oriented curve shown. **(5 points)**



$\int_C \mathbf{F}\cdot d\mathbf{r} =$ ☐

Question 4

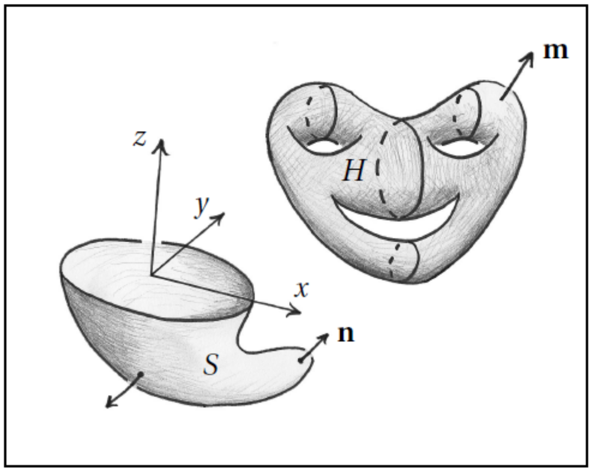
For this problem, $\mathbf{G} = \langle\, yz + 2x^2,\ 2xy,\ xy^2\,\rangle$ and S is the boundary of the cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$ oriented with the outward pointing normal vectors $\mathbf{n}$. Circle the best response for each of the following.

a) $\iint_S \mathbf{G}\cdot\mathbf{n}\, dS =$ | -5 -3 -1 0 1 3 5 | **(2 points)**

b) $\iint_S (\operatorname{curl}\mathbf{G})\cdot\mathbf{n}\, dS$ is | negative zero positive | **(1 point)**

Question 5

Let S and H be the surfaces at right; the boundary of S is the unit circle in the xy-plane, and H has no boundary. **1 pt each**



a) The integral $\iint_H x^2 y^2 + z^2 \, dS$ is:

negative	zero	positive

b) The vector field $\mathbf{F} = \langle y+z, -x, yz \rangle$ has $\operatorname{curl} \mathbf{F} = \langle z, 1, -2 \rangle$.

The flux $\iint_S (\operatorname{curl} \mathbf{F}) \cdot \mathbf{n} \, dS$ is:

-5π	-4π	-3π	-2π	$-\pi$	0	π	2π	3π	4π	5π

c) For $\mathbf{G} = \langle x, y, z \rangle$, the flux $\iint_S \mathbf{G} \cdot \mathbf{n} \, dS$ is:

negative	zero	positive

d) For $\mathbf{E} = \langle z, x, 2 \rangle$, the flux $\iint_S \mathbf{E} \cdot \mathbf{n} \, dS$ is:

-5π	-4π	-3π	-2π	$-\pi$	0	π	2π	3π	4π	5π

Question 6

Let $\mathbf{F} = (1 + x + yz)\mathbf{i} + 2y\mathbf{j} + (z + yx)\mathbf{k}$.

(a) Compute curl(F). (**1 point**)

curl(F) = $\langle \quad , \quad , \quad \rangle$.

(b) Let S be the portion of the cone $z = \sqrt{x^2 + y^2}$ with $z \leq 2$, with outward pointing normal vector. Compute the flux of curl(F) through S. (**5 points**)

flux =

(c) Let E be the sphere $x^2 + y^2 + z^2 = 1$ with outward pointing normal. Compute the flux of F through E. (**3 points**)

flux =

(d) Is F conservative? Yes No (**1 point**)