Math 241: Exam #1

Name:

NetID:

UIN:

- You do not need to show work on multiple choice questions. Otherwise, when space is provided, **show work which justifies your answer**.
- No calculators, notes, books, etc... are permitted.
- The exam lasts **60 minutes**.

Question 1 Consider the vector $\mathbf{n} = \langle -2, 2, 1 \rangle$ and points $P = (2, -2, -3)$ and $Q = (0, 1, 0)$. **(7 points)**

(a) Find the equation of the plane through Q with normal vector $\mathbf{n}$.

$\mathbf{n} =$ [ ]

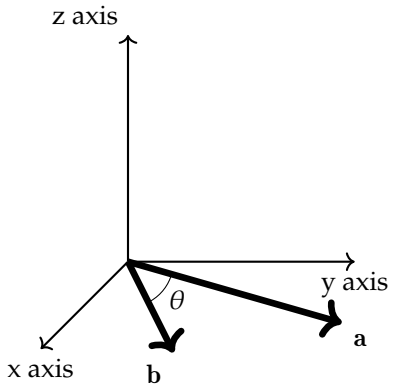
(b) Find the component of $\overrightarrow{QP}$ along $\mathbf{n}$.

$\text{comp}_{\mathbf{n}}(\overrightarrow{QP}) =$ [ ]

(c) Find the distance from the point P to the plane from part (a).

distance = [ ]

Question 2 The figure shows two vectors **a** and **b** in the xy-plane.
Their lengths are $|\mathbf{a}| = 7$ and $|\mathbf{b}| = 6$. The angle between them is $\theta = 30°$. **(5 points)**



(a) Find $|\mathbf{a} \times \mathbf{b}|$.

$|\mathbf{a} \times \mathbf{b}| =$ ☐

(b) The x component of $\mathbf{a} \times \mathbf{b}$ is

- ☐ negative
- ☐ zero
- ☐ positive

(c) The y component of $\mathbf{a} \times \mathbf{b}$ is

- ☐ negative
- ☐ zero
- ☐ positive

(d) The z component of $\mathbf{a} \times \mathbf{b}$ is

- ☐ negative
- ☐ zero
- ☐ positive

Question 3 Consider the function $f(x,y) = \sin^2(x)\left(\dfrac{y^2}{x^2+2y^2}\right)$ for $(x,y) \neq (0,0)$. Use the *Squeeze Theorem* to determine whether the limit below exists. **(5 points)**

(a) Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Circle your answer: **Yes** **No**

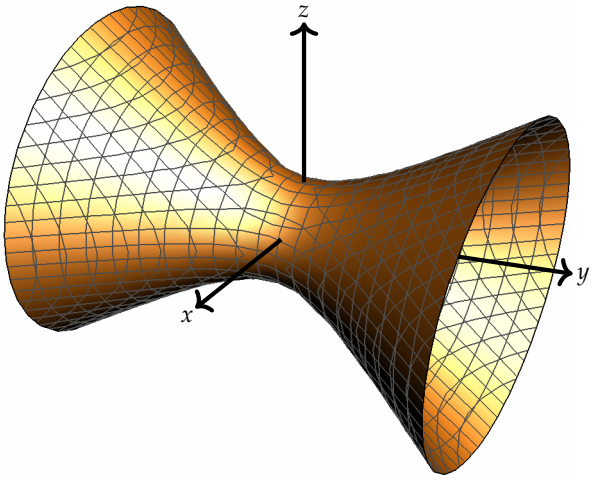
If the limit exists, determine its value (write DNE if it does not exist).

$\lim_{(x,y)\to(0,0)} f(x,y) =$ _____ .

(b) Give a complete justification for your answer using the *Squeeze Theorem*.

Question 4 Select the equation for the quadratic surface shown at right. **(2 points)**

- ☐ $x^2 + y^2 + z^2 = 1.$
- ☐ $x^2 + y^2 + z^2 = -1.$
- ☐ $x^2 - y^2 + z^2 = 1.$
- ☐ $x^2 - y^2 + z^2 = -1.$
- ☐ $x^2 + y^2 - z^2 = 1$
- ☐ $x^2 + y^2 - z^2 = -1.$



Question 5 $f(x, y)$ is a differentiable function. The tangent plane to the graph of f at the point $\big(1, 1, f(1, 1)\big)$ is given by $3x - y + z = 5$. Determine $f(1, 1)$, $\dfrac{\partial f}{\partial x}(1, 1)$, and $\dfrac{\partial f}{\partial y}(1, 1)$. **(6 points)**

$f(1, 1) =$ ☐

$\dfrac{\partial f}{\partial x}(1, 1) =$ ☐

$\dfrac{\partial f}{\partial y}(1, 1) =$ ☐

Question 6 (8 points)

(a) Let $f(x, y) = x^2y + y^3x + 1$

Compute $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

$\dfrac{\partial f}{\partial x} =$ ____

$\dfrac{\partial f}{\partial y} =$ ____

(b) Suppose x and y are differentiable functions of s and t and let $g(s, t) = f(x(s, t), y(s, t))$, where $f(x, y)$ is the function in part (a).

Use the table of values on the right, to calculate $\dfrac{\partial g}{\partial s}(0, 1)$.

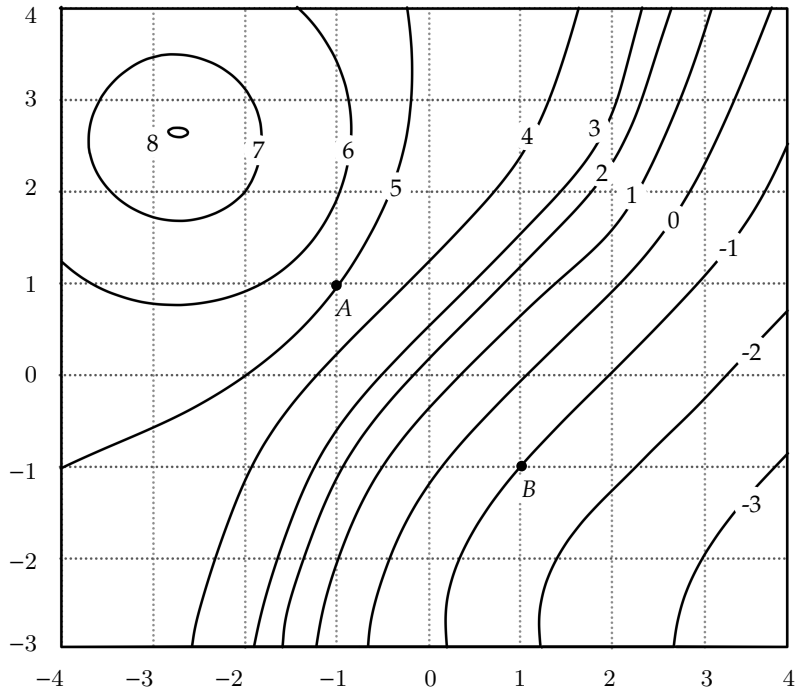
	x	y	$\dfrac{\partial x}{\partial s}$	$\dfrac{\partial y}{\partial s}$
$(0, 1)$	-1	-1	4	3
$(-1, -1)$	-3	-2	1	2

$\dfrac{\partial g}{\partial s}(0, 1) =$ ____

Question 7 The contour map of a differentiable function $f(x, y)$ is shown. Each level curve is labeled by the corresponding value of f. Choose the best answer for each question below. **(7 points)**

(a) At the point **B**, determine the sign of the following quantities

$f_y(\mathbf{B})$ is	☐	positive
	☐	negative
	☐	zero
$f_{xx}(\mathbf{B})$ is	☐	positive
	☐	negative
	☐	zero
$f_{yx}(\mathbf{B})$ is	☐	positive
	☐	negative
	☐	zero



(b) Let **u** be a unit vector with direction $\overrightarrow{\mathbf{BA}}$. Estimate $D_{\mathbf{u}}f(\mathbf{A})$, the directional derivative of f at **A** in the direction of **u**.

☐ 4

☐ 2

☐ 0

☐ -2

☐ -4