Math 241: Exam #1

Name:

NetID:

UIN:

- You do not need to show work on multiple choice questions. Otherwise, when space is provided, **show work which justifies your answer**.
- No calculators, notes, books, etc... are permitted.
- The exam lasts **60 minutes**.

Question 1 Consider the vector $\mathbf{n} = \langle -2, 2, 1 \rangle$ and points $P = (2, -2, -3)$ and $Q = (0, 1, 0)$. **(7 points)**

(a) Find the equation of the plane through Q with normal vector $\mathbf{n}$.

$$\boxed{-2x + 2(y-1) + z = 0}$$

(b) Find the component of $\overrightarrow{QP}$ along $\mathbf{n}$.

$$\text{Comp}_{\vec{n}} \vec{QP} = \frac{\vec{n} \cdot \vec{QP}}{|\vec{n}|} = \frac{\langle -2, 2, 1 \rangle \langle 2, -3, -3 \rangle}{|\vec{n}|} =$$

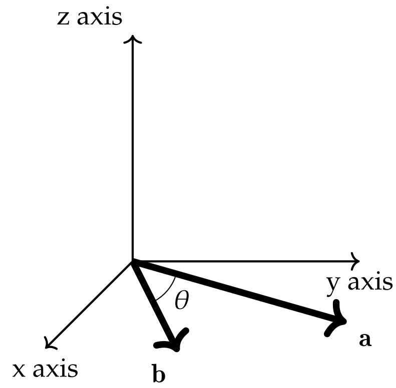
$$= \frac{-4 - 6 - 3}{\sqrt{4 + 4 + 1}} =$$

$\text{comp}_{\mathbf{n}}(\overrightarrow{QP}) =$ $\boxed{-13/3}$

(c) Find the distance from the point P to the plane from part (a).

distance = $\boxed{13/3}$

Question 2 The figure shows two vectors **a** and **b** in the xy-plane.
Their lengths are $|\mathbf{a}| = 7$ and $|\mathbf{b}| = 6$. The angle between them is $\theta = 30°$. **(5 points)**



(a) Find $|\mathbf{a} \times \mathbf{b}|$.

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta \Rightarrow |\vec{a} \times \vec{b}| = \frac{7 \cdot 6}{2}$

$|\mathbf{a} \times \mathbf{b}| = \boxed{21}$

(b) The x component of $\mathbf{a} \times \mathbf{b}$ is

- [ ] negative
- [x] zero
- [ ] positive

(c) The y component of $\mathbf{a} \times \mathbf{b}$ is

- [ ] negative
- [x] zero
- [ ] positive

(d) The z component of $\mathbf{a} \times \mathbf{b}$ is

- [x] negative
- [ ] zero
- [ ] positive

Question 3 Consider the function $f(x, y) = \sin^2(x)\left(\frac{y^2}{x^2 + 2y^2}\right)$ for $(x, y) \neq (0, 0)$. Use the *Squeeze Theorem* to determine whether the limit below exists. **(5 points)**

(a) Does the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Circle your answer: **Yes** **No**

If the limit exists, determine its value (write DNE if it does not exist).

$$\lim_{(x,y)\to(0,0)} f(x, y) = \boxed{0}.$$

(b) Give a complete justification for your answer using the *Squeeze Theorem*.

$$0 \leq \sin^2(x)\left[\frac{y^2}{x^2+2y^2}\right] \leq \sin^2(x)$$

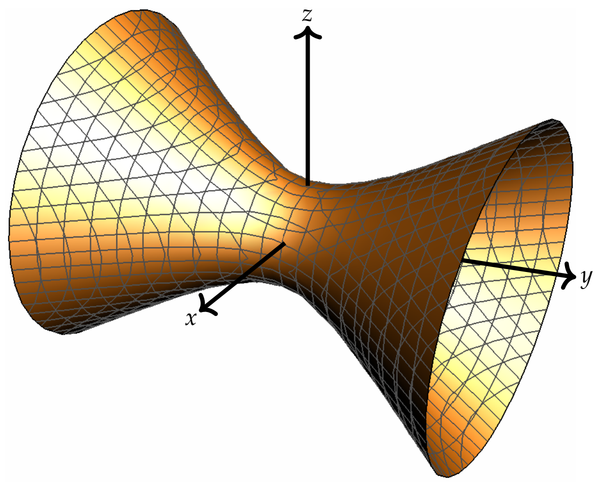
Since $\lim_{(x,y)\to(0,0)} \sin^2(x) = 0$

by squeeze theorem we have

$$\lim_{(x,y)\to(0,0)} f(x,y) = 0$$

Question 4 Select the equation for the quadratic surface shown at right. **(2 points)**

- [ ] $x^2 + y^2 + z^2 = 1.$
- [ ] $x^2 + y^2 + z^2 = -1.$
- [x] $x^2 - y^2 + z^2 = 1.$
- [ ] $x^2 - y^2 + z^2 = -1.$
- [ ] $x^2 + y^2 - z^2 = 1$
- [ ] $x^2 + y^2 - z^2 = -1.$



Question 5 $f(x, y)$ is a differentiable function. The tangent plane to the graph of f at the point $(1, 1, f(1, 1))$ is given by $3x - y + z = 5$. Determine $f(1, 1)$, $\frac{\partial f}{\partial x}(1, 1)$, and $\frac{\partial f}{\partial y}(1, 1)$. **(6 points)**

$f(1,1) = \boxed{3}$

$\frac{\partial f}{\partial x}(1,1) = \boxed{-3}$

$\frac{\partial f}{\partial y}(1,1) = \boxed{1}$

tangent plane is:

$f_x(1,1)(x-1) + f_y(1,1)(y-1) - (z - f(1,1)) = 0$

$\Rightarrow$

$-f_x(1,1)x - f_y(1,1)y + z = -f_x(1,1) - f_y(1,1) + f(1,1)$

Comparing coefficents
we find

$f_x(1,1) = -3$

$f_y(1,1) = 1$

plug in $x=1$, $y=1$ to get $f(1,1) = 5-3+1 = 3$

Question 6 (8 points)

(a) Let $f(x, y) = x^2y + y^3x + 1$

Compute $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

$\dfrac{\partial f}{\partial x} =$ $\boxed{2xy + y^3}$

$\dfrac{\partial f}{\partial y} =$ $\boxed{x^2 + 3y^2x}$

(b) Suppose x and y are differentiable functions of s and t and let $g(s,t) = f(x(s,t), y(s,t))$, where $f(x,y)$ is the function in part (a).

Use the table of values on the right, to calculate $\dfrac{\partial g}{\partial s}(0, 1)$.

	x	y	$\frac{\partial x}{\partial s}$	$\frac{\partial y}{\partial s}$
$(0,1)$	-1	-1	4	3
$(-1,-1)$	-3	-2	1	2

$x(0,1) = -1 \qquad f_x(-1,-1) = 2(-1)(-1) - 1 = 1$

$y(0,1) = -1 \qquad f_y(-1,-1) = (-1)^2 + 3(-1)^2(-1) = -2$

$$g_s(0,1) = f_x(-1,-1)\, x_s(0,1) + f_y(-1,-1)\, y_s(0,1)$$

$$= (1)(4) + (-2)(3)$$

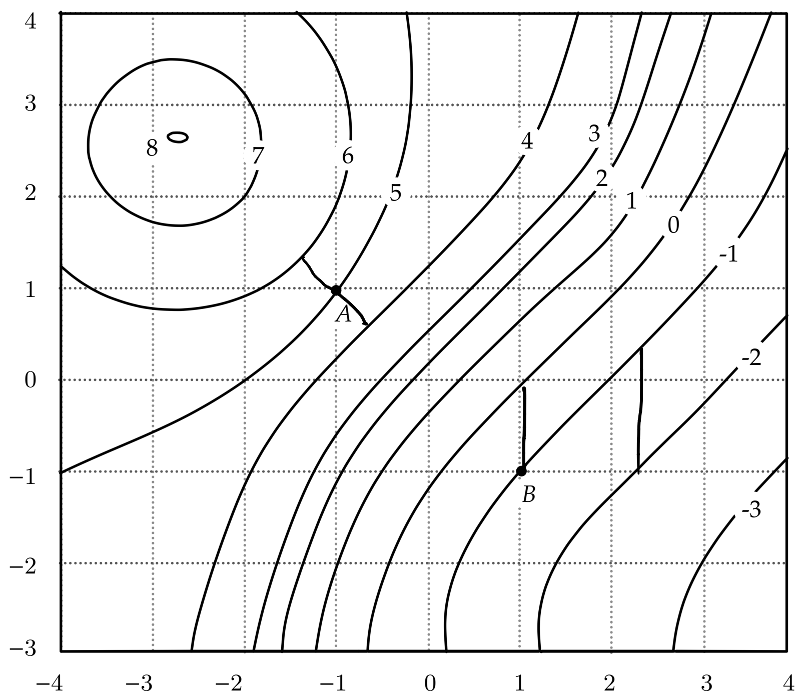
$$= -2$$

$\dfrac{\partial g}{\partial s}(0, 1) =$ $\boxed{-2}$

Question 7 The contour map of a differentiable function $f(x, y)$ is shown. Each level curve is labeled by the corresponding value of f. Choose the best answer for each question below. **(7 points)**

(a) At the point **B**, determine the sign of the following quantities

$f_y(\mathbf{B})$ is	☒	positive
	☐	negative
	☐	zero
$f_{xx}(\mathbf{B})$ is	☒	positive
	☐	negative
	☐	zero
$f_{yx}(\mathbf{B})$ is	☐	positive
	☒	negative
	☐	zero



(b) Let $\mathbf{u}$ be a unit vector with direction $\overrightarrow{\mathbf{BA}}$. Estimate $D_{\mathbf{u}}f(\mathbf{A})$, the directional derivative of f at $\mathbf{A}$ in the direction of $\mathbf{u}$.

☐ 4

☒ 2

☐ 0

☐ −2

☐ −4

$$D_u f(A) \approx \frac{6-4}{1}$$