Math 241: Exam #1

Name:

NetID:

UIN:

- You do not need to show work on multiple choice questions. Otherwise, when space is provided, **show work which justifies your answer**.
- No calculators, notes, books, etc... are permitted.
- The exam lasts **60 minutes**.

Question 1 Consider the plane $2x + y - 2z = 2$ and points $P = (2, -2, -3)$ and $Q = (1, 0, 0)$. **(7 points)**

(a) Find a normal vector **n** to this plane.

$$\mathbf{n} =$$

(b) Find the projection of the vector $\overrightarrow{PQ}$ onto **n**.

$$\text{proj}_{\mathbf{n}}(\overrightarrow{PQ}) =$$

(c) Find the distance from the point P to the plane.

$$\text{distance} =$$

Question 2 The figure shows a vector **a** in the direction **k** and a vector **b** in the xy-plane.
Their lengths are $|\mathbf{a}| = 4$ and $|\mathbf{b}| = 5$. **(5 points)**
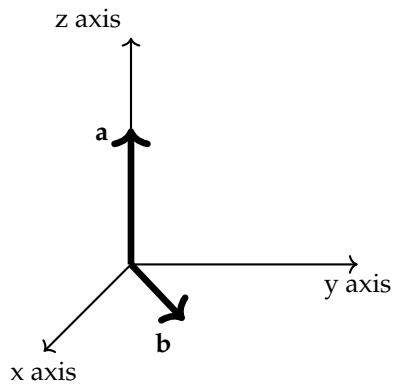
(a) Find $|\mathbf{a} \times \mathbf{b}|$.

$$|\mathbf{a} \times \mathbf{b}| = \boxed{}$$

(b) The x component of $\mathbf{a} \times \mathbf{b}$ is

☐ negative

☐ zero

☐ positive

(c) The y component of $\mathbf{a} \times \mathbf{b}$ is

☐ negative

☐ zero

☐ positive

(d) The z component of $\mathbf{a} \times \mathbf{b}$ is

☐ negative

☐ zero

☐ positive

Question 3 Consider the function $f(x,y) = x^2 \cos^2\left(\dfrac{1}{x^2+y^2}\right)$ for $(x,y) \neq (0,0)$. Use the *Squeeze Theorem* to determine whether the limit below exists. **(5 points)**

(a) Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Circle your answer: **Yes** **No**

If the limit exists, determine its value (write DNE if it does not exist).

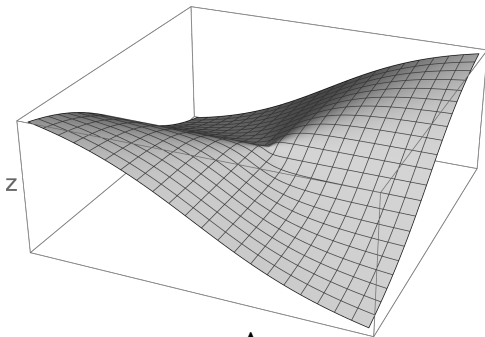$$\lim_{(x,y)\to(0,0)} f(x,y) = \boxed{}$$
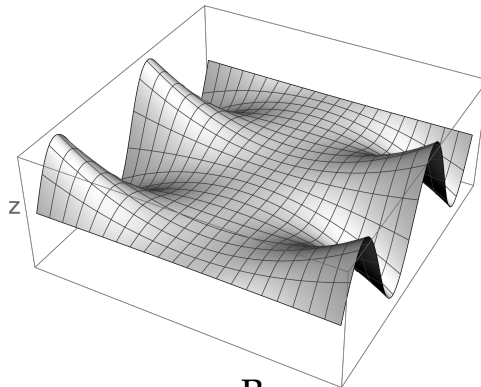
(b) Give a complete justification for you answer using the *Squeeze Theorem*.

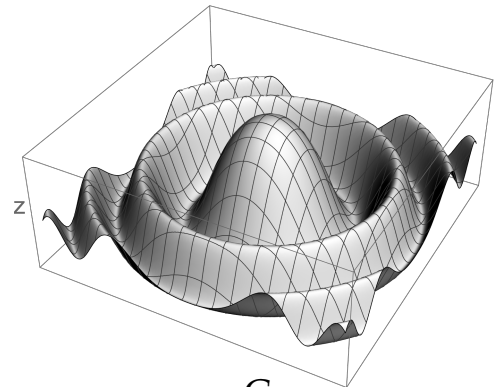**Question 4** Select the graph of $f(x, y) = x^2 \sin(y)$. Write your answer in the box: ⬚ .
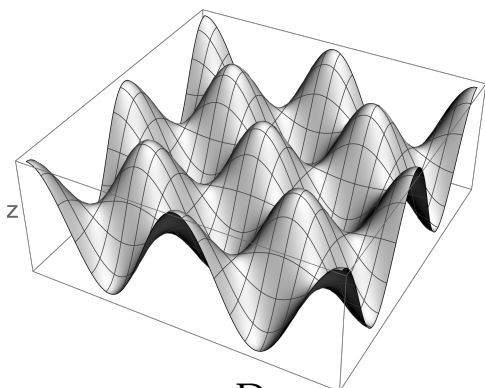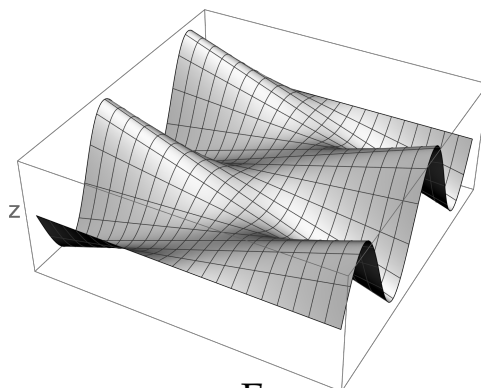
Only the z-axis is labeled.   **(2 points)**
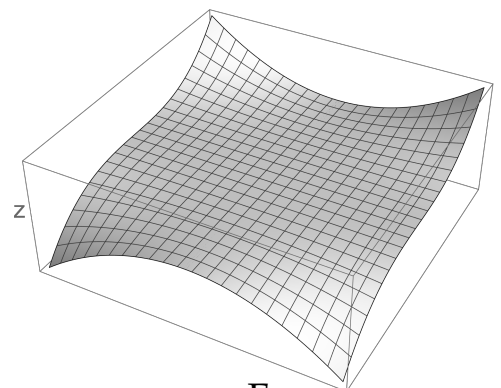
z

A

z

B

z

C

z

D

z

E

z

F

Question 5 $f(x, y)$ is a differentiable function. The tangent plane to the graph of f at the point $\big(1, 1, f(1, 1)\big)$ is given by $-3x + y + z = 4$. Determine $f(1, 1)$, $\dfrac{\partial f}{\partial x}(1, 1)$, and $\dfrac{\partial f}{\partial y}(1, 1)$.   **(6 points)**

$f(1, 1) =$

$\dfrac{\partial f}{\partial x}(1, 1) =$

$\dfrac{\partial f}{\partial y}(1, 1) =$

Question 6 (8 points)

(a) Let $x(s,t) = t^2 + 3st$ and $y(s,t) = 2t^2s + s^2 - t$.

Compute $\dfrac{\partial x}{\partial s}$ and $\dfrac{\partial y}{\partial s}$.

$$\frac{\partial x}{\partial s} = \boxed{3t}$$

$$\frac{\partial y}{\partial s} = \boxed{2t^2 + 2s}$$

(b) Suppose $f(x,y)$ is a differentiable function of x and y and let $g(s,t) = f(x(s,t), y(s,t))$, where $x(s,t)$ and $y(s,t)$ are the functions in part (a).

Use the table of values on the right, to calculate $\dfrac{\partial g}{\partial s}(0,1)$.

	g	f	$\dfrac{\partial f}{\partial x}$	$\dfrac{\partial f}{\partial y}$
$(0,1)$	-2	3	-4	-5
$(1,-1)$	-3	-2	1	2

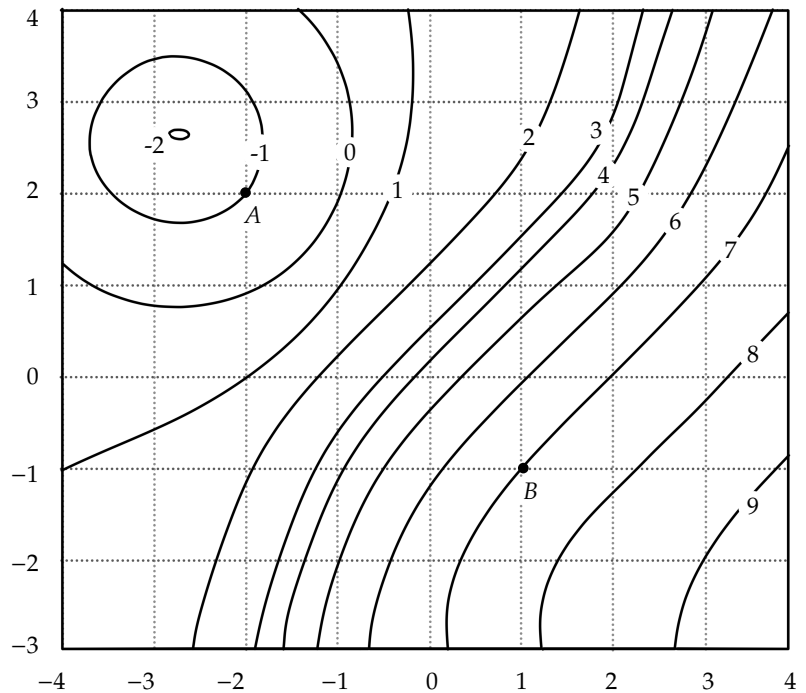$$\frac{\partial g}{\partial s}(0,1) = \boxed{7}$$

Question 7 The contour map of a differentiable function $f(x, y)$ is shown. Each level curve is labeled by the corresponding value of f. Choose the best answer for each question below.   **(7 points)**

(a) At the point **B**, determine the sign of the following quantities

$f_x(\mathbf{B})$ is ☐ positive

☐ negative

☐ zero

$f_{yy}(\mathbf{B})$ is ☐ positive

☐ negative

☐ zero

$f_{xy}(\mathbf{B})$ is ☐ positive

☐ negative

☐ zero

(b) Let **u** be a unit vector with direction $\overrightarrow{\mathbf{BA}}$. Estimate $D_{\mathbf{u}}f(\mathbf{A})$, the directional derivative of f at **A** in the direction of **u**.

☐ 4

☐ 1

☐ 0

☐ −1

☐ −4