Question 1 Consider the plane $2x + y - 2z = 2$ and points $P = (2, -2, -3)$ and $Q = (1, 0, 0)$. **(7 points)**

(a) Find a normal vector $\mathbf{n}$ to this plane.

$\mathbf{n} = \boxed{\langle 2, 1, -2 \rangle}$

(b) Find the projection of the vector $\overrightarrow{PQ}$ onto $\mathbf{n}$.

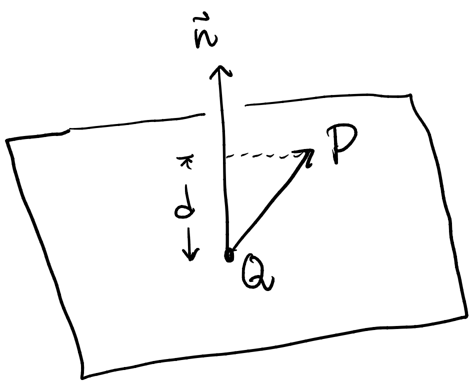
$$\overrightarrow{PQ} = \langle 1-2, 0-2, 0-{-3} \rangle$$
$$= \langle -1, 2, 3 \rangle$$

$$\text{proj}_{\vec{n}}(\overrightarrow{PQ}) = \frac{\overrightarrow{PQ} \cdot \vec{n}}{\vec{n} \cdot \vec{n}} = \frac{\langle -1, 2, 3 \rangle \cdot \langle 2, 1, -2 \rangle}{\langle 2, 1, -2 \rangle \cdot \langle 2, 1, -2 \rangle} \vec{n}$$

$$= \frac{-2+2+6}{4+1+4} \vec{n} = \frac{-6}{9} \vec{n}$$

$\text{proj}_{\mathbf{n}}(\overrightarrow{PQ}) = \boxed{-\frac{2}{3} \langle 2, 1, -2 \rangle}$

(c) Find the distance from the point P to the plane.

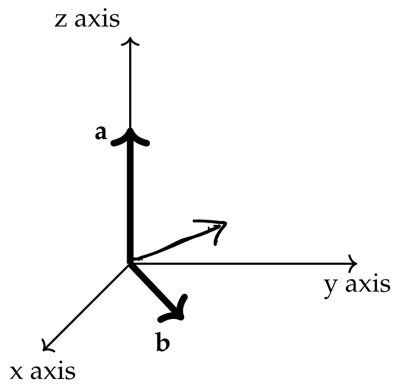


$$d = \left| \text{proj}_{\vec{n}}(\overrightarrow{QP}) \right| = \left| \text{proj}_{\vec{n}}(\overrightarrow{PQ}) \right|$$

$$= \left| -\frac{2}{3} \langle 2, 1, -2 \rangle \right| = \frac{2}{3} \sqrt{4+1+4} = 2$$

distance $= \boxed{2}$

Question 2 The figure shows a vector **a** in the direction **k** and a vector **b** in the xy-plane. Their lengths are $|\mathbf{a}| = 4$ and $|\mathbf{b}| = 5$. **(5 points)**



(a) Find $|\mathbf{a} \times \mathbf{b}|$.

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin(\pi/2)$$

$|\mathbf{a} \times \mathbf{b}| =$ $\boxed{20}$

(b) The x component of $\mathbf{a} \times \mathbf{b}$ is

- [x] negative
- [ ] zero
- [ ] positive

(c) The y component of $\mathbf{a} \times \mathbf{b}$ is

- [ ] negative
- [ ] zero
- [x] positive

(d) The z component of $\mathbf{a} \times \mathbf{b}$ is

- [ ] negative
- [x] zero
- [ ] positive

Question 3 Consider the function $f(x, y) = x^2 \cos^2\left(\frac{1}{x^2 + y^2}\right)$ for $(x, y) \neq (0, 0)$. Use the *Squeeze Theorem* to determine whether the limit below exists. **(5 points)**

(a) Does the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Circle your answer: **Yes** **No**

If the limit exists, determine its value (write DNE if it does not exist).

$$\lim_{(x,y)\to(0,0)} f(x, y) = \boxed{0}\ .$$

(b) Give a complete justification for you answer using the *Squeeze Theorem*.

$$0 \leq x^2 \cos^2\left(\frac{1}{x^2+y^2}\right) \leq x^2$$

Since $\lim_{(x,y)\to(0,0)} x^2 = 0$ by squeeze theorem

we have

$$\lim_{(x,y)\to(0,0)} f(x,y) = 0$$

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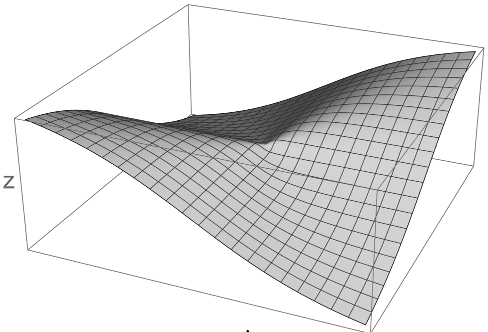
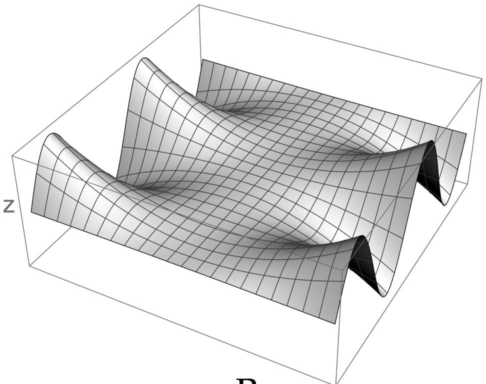
Question 4 Select the graph of $f(x,y) = x^2 \sin(y)$. Write your answer in the box: $\boxed{B}$.

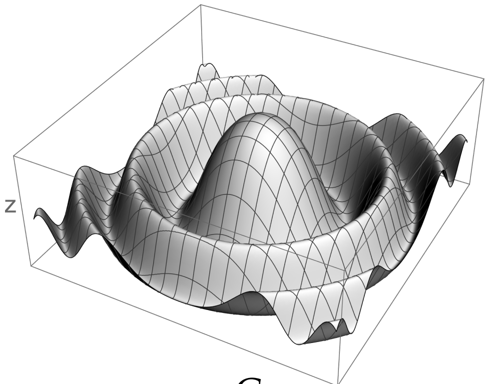
Only the z-axis is labeled. **(2 points)**



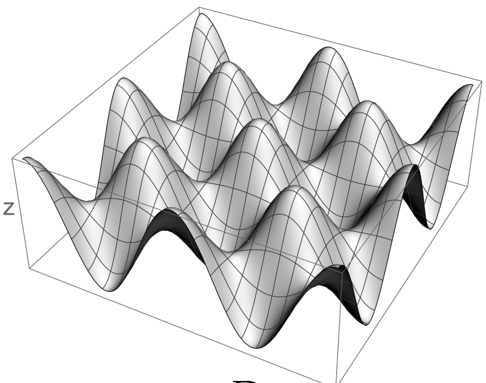
A



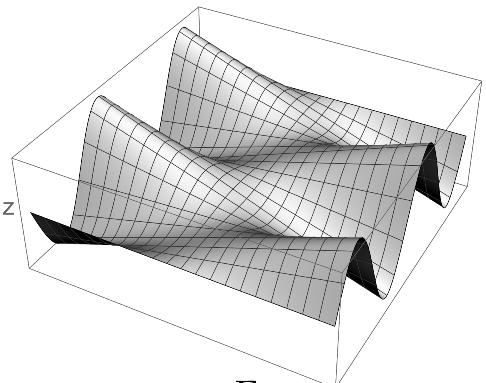
B



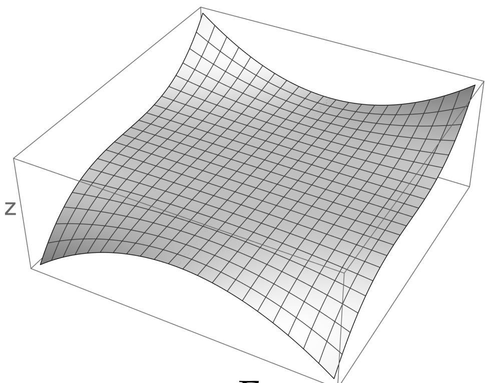
C



D



E



F

Question 5 $f(x,y)$ is a differentiable function. The tangent plane to the graph of f at the point $(1, 1, f(1,1))$ is given by $-3x + y + z = 4$. Determine $f(1,1)$, $\dfrac{\partial f}{\partial x}(1,1)$, and $\dfrac{\partial f}{\partial y}(1,1)$. **(6 points)**

tangent plane is

$$z = f_x(1,1)(x-1) + f_y(1,1)(y-1) + f(1,1)$$
$$= f_x(1,1)\,x + f_y(1,1)\,y + \left(f(1,1) - f_x(1,1) - f_y(1,1)\right)$$

comparing to

$z = 3x - y - 4$ conclude: $f_x(1,1) = 6$
$f_y(1,1) = 3$

plug in $(1,1)$ to find $f(1,1) = 6$

$f(1,1) = \boxed{6}$

$\dfrac{\partial f}{\partial x}(1,1) = \boxed{3}$

$\dfrac{\partial f}{\partial y}(1,1) = \boxed{-1}$

Question 6 **(8 points)**

(a) Let $x(s,t) = t^2 + 3st$ and $y(s,t) = 2t^2 s + s^2 - t$.

Compute $\frac{\partial x}{\partial s}$ and $\frac{\partial y}{\partial s}$.

$\frac{\partial x}{\partial s} = \boxed{3t}$

$\frac{\partial y}{\partial s} = \boxed{2t^2 + 2s}$

(b) Suppose $f(x,y)$ is a differentiable function of x and y and let $g(s,t) = f(x(s,t), y(s,t))$, where $x(s,t)$ and $y(s,t)$ are the functions in part (a).

Use the table of values on the right, to calculate $\frac{\partial g}{\partial s}(0,1)$.

	g	f	$\frac{\partial f}{\partial x}$	$\frac{\partial f}{\partial y}$
$(0,1)$	-2	3	-4	-5
$(1,-1)$	-3	-2	1	2

$x(0,1) = 1$
$y(0,1) = -1$

$g_s(0,1) = f_x(1,-1)\, x_s(0,1) + f_y(1,-1)\, y_s(0,1)$

$= (1)(3) + (2)(2)$

$\frac{\partial g}{\partial s}(0,1) = \boxed{7}$

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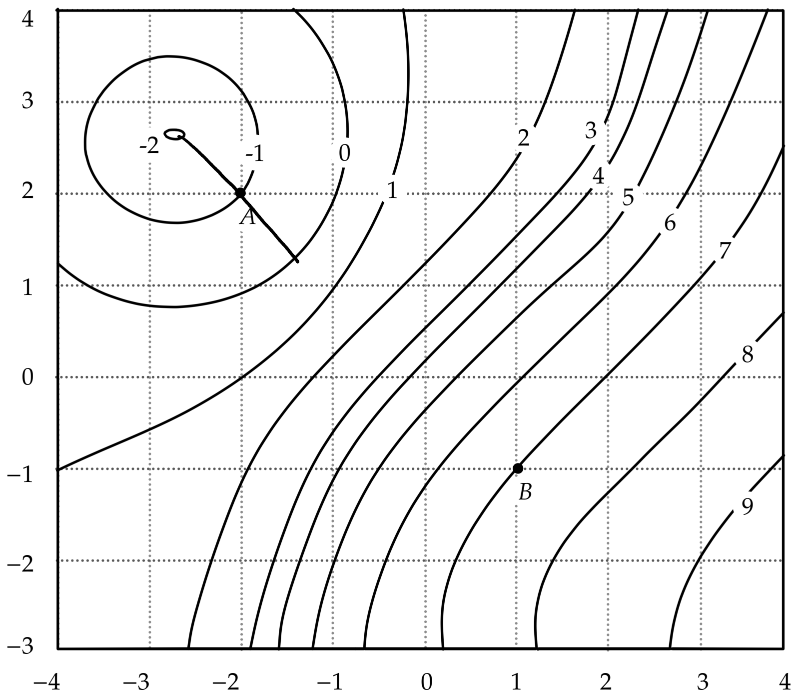
Question 7 The contour map of a differentiable function $f(x, y)$ is shown. Each level curve is labeled by the corresponding value of f. Choose the best answer for each question below. **(7 points)**

(a) At the point **B**, determine the sign of the following quantities

$f_x(\mathbf{B})$ is	☒	positive
	☐	negative
	☐	zero
$f_{yy}(\mathbf{B})$ is	☐	positive
	☒	negative
	☐	zero
$f_{xy}(\mathbf{B})$ is	☒	positive
	☐	negative
	☐	zero



(b) Let $\mathbf{u}$ be a unit vector with direction $\overrightarrow{\mathbf{BA}}$. Estimate $D_{\mathbf{u}}f(\mathbf{A})$, the directional derivative of f at $\mathbf{A}$ in the direction of $\mathbf{u}$.

- ☐ 4
- ☐ 1
- ☐ 0
- ☒ −1
- ☐ −4

$$D_{\vec{u}}f(A) \approx \frac{-2-0}{2} = -1$$

E8999

Do not write in box

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