Question 1 Consider the plane $x + 2y - 2z = 1$ and points $P = (2, -2, -3)$ and $Q = (1, 0, 0)$. **(7 points)**

(a) Find a normal vector $\mathbf{n}$ to this plane.

$\mathbf{n} =$ ______

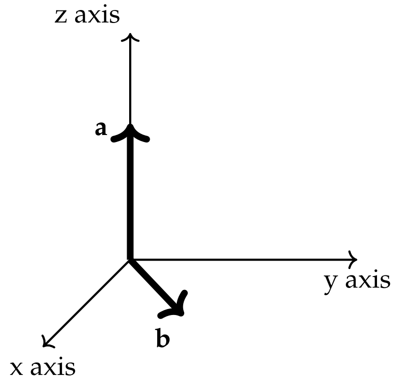
(b) Find the projection of the vector $\overrightarrow{PQ}$ onto $\mathbf{n}$.

$\text{proj}_{\mathbf{n}}(\overrightarrow{PQ}) =$ ______

(c) Find the distance from the point P to the plane.

distance = ______

Question 2 The figure shows a vector $\mathbf{a}$ in the direction $\mathbf{k}$ and a vector $\mathbf{b}$ in the xy-plane. Their lengths are $|\mathbf{a}| = 4$ and $|\mathbf{b}| = 5$. **(5 points)**



(a) Find $|\mathbf{a} \times \mathbf{b}|$.

$|\mathbf{a} \times \mathbf{b}| =$ ☐

(b) The x component of $\mathbf{a} \times \mathbf{b}$ is

☐ negative

☐ zero

☐ positive

(c) The y component of $\mathbf{a} \times \mathbf{b}$ is

☐ negative

☐ zero

☐ positive

(d) The z component of $\mathbf{a} \times \mathbf{b}$ is

☐ negative

☐ zero

☐ positive

Question 3 Consider the function $f(x, y) = x^2 \sin^2\left(\frac{1}{x^2 + y^2}\right)$ for $(x, y) \neq (0, 0)$. Use the *Squeeze Theorem* to determine whether the limit below exists. **(5 points)**

(a) Does the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Circle your answer: **Yes** **No**

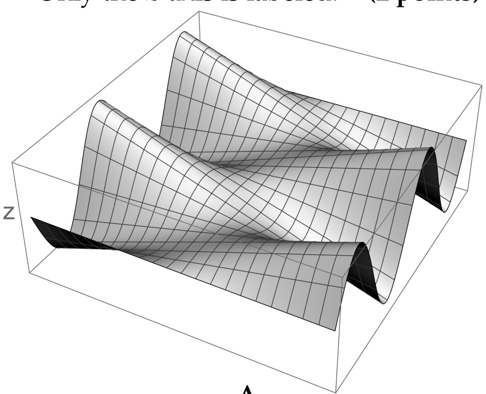
If the limit exists, determine its value (write DNE if it does not exist).

$$\lim_{(x,y)\to(0,0)} f(x, y) = \boxed{}\ .$$

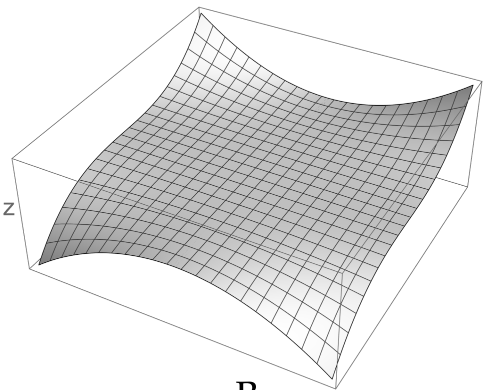
(b) Give a complete justification for your answer using the *Squeeze Theorem*.

Question 4 Select the graph of $f(x, y) = x^2 \sin(y)$. Write your answer in the box: $\boxed{}$.

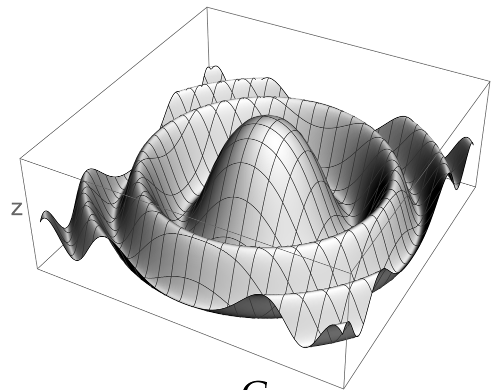
Only the z-axis is labeled. **(2 points)**



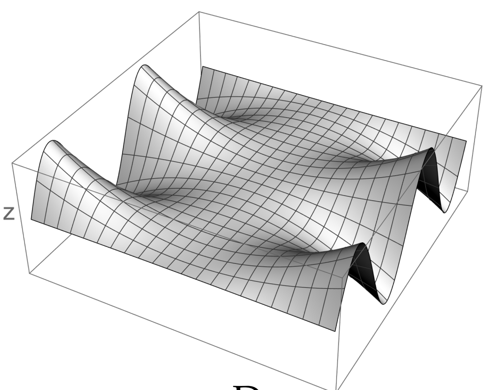
A



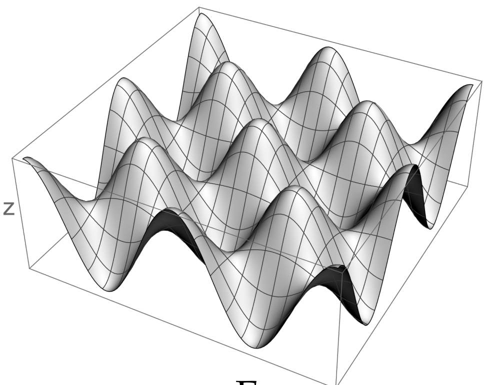
B



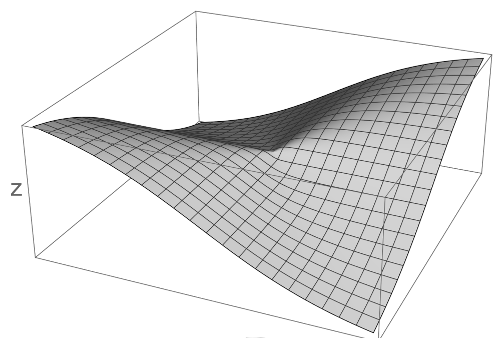
C



D



E



F

Question 5 $f(x, y)$ is a differentiable function. The tangent plane to the graph of f at the point $\big(1, 1, f(1, 1)\big)$ is given by $2x - y + z = 5$. Determine $f(1, 1)$, $\dfrac{\partial f}{\partial x}(1, 1)$, and $\dfrac{\partial f}{\partial y}(1, 1)$. **(6 points)**

$f(1, 1) =$

$\dfrac{\partial f}{\partial x}(1, 1) =$

$\dfrac{\partial f}{\partial y}(1, 1) =$

Question 6 (8 points)

(a) Let $x(s,t) = s^2t + t^3 - s$ and $y(s,t) = 2st - t^2$.

Compute $\frac{\partial x}{\partial s}$ and $\frac{\partial y}{\partial s}$.

$\frac{\partial x}{\partial s} =$ [ ]

$\frac{\partial y}{\partial s} =$ [ ]

(b) Suppose $f(x,y)$ is a differentiable function of x and y and let $g(s,t) = f(x(s,t), y(s,t))$, where $x(s,t)$ and $y(s,t)$ are the functions in part (a).

Use the table of values on the right, to calculate $\frac{\partial g}{\partial s}(0,1)$.

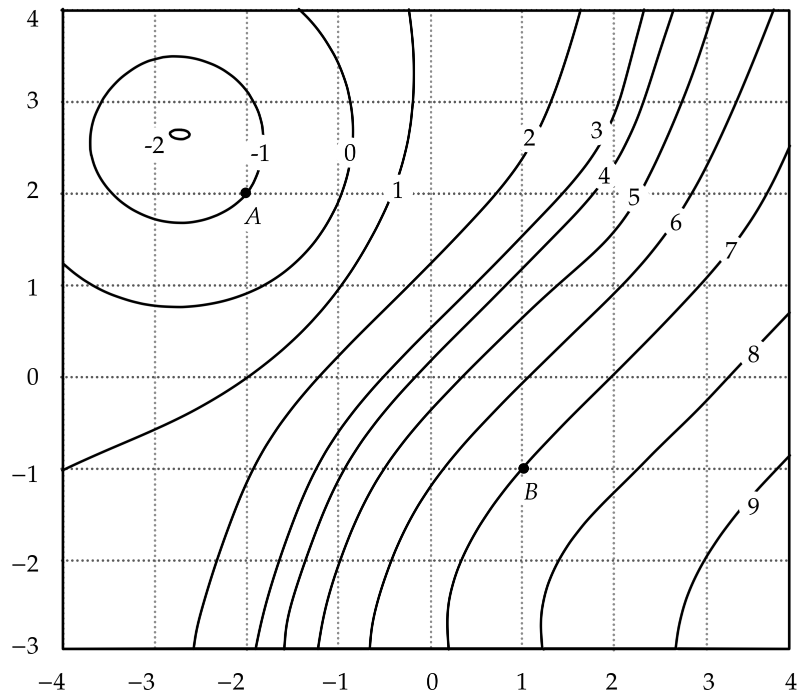
	g	f	$\frac{\partial f}{\partial x}$	$\frac{\partial f}{\partial y}$
$(0,1)$	-3	3	4	7
$(1,-1)$	-4	-3	1	5

$\frac{\partial g}{\partial s}(0,\ 1) =$ [ ]

Question 7 The contour map of a differentiable function $f(x, y)$ is shown. Each level curve is labeled by the corresponding value of f. Choose the best answer for each question below. **(7 points)**

(a) At the point **B**, determine the sign of the following quantities

$f_x(\mathbf{B})$ is	☐	positive
	☐	negative
	☐	zero
$f_{yy}(\mathbf{B})$ is	☐	positive
	☐	negative
	☐	zero
$f_{xy}(\mathbf{B})$ is	☐	positive
	☐	negative
	☐	zero



(b) Let $\mathbf{u}$ be a unit vector with direction $\overrightarrow{\mathbf{BA}}$. Estimate $D_{\mathbf{u}}f(\mathbf{A})$, the directional derivative of f at $\mathbf{A}$ in the direction of $\mathbf{u}$.

- ☐ 4
- ☐ 1
- ☐ 0
- ☐ −1
- ☐ −4