Question 1 Consider the plane $x + 2y - 2z = 1$ and points $P = (2, -2, -3)$ and $Q = (1, 0, 0)$. **(7 points)**

(a) Find a normal vector **n** to this plane.

$\mathbf{n} = \boxed{\langle 1, 2, -2 \rangle}$

(b) Find the projection of the vector $\overrightarrow{PQ}$ onto **n**.

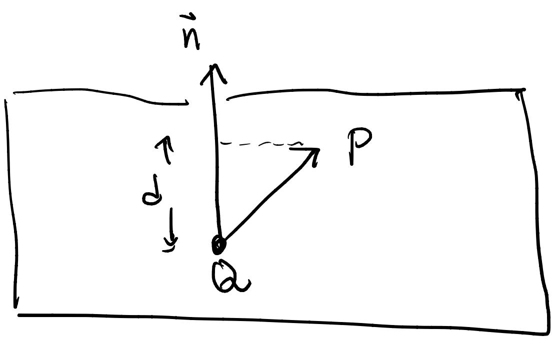
$\overrightarrow{PQ} = \langle 1-2,\ 0-{-2},\ 0-{-3} \rangle = \langle -1, 2, 3 \rangle$

$\text{proj}_{\vec{n}}(\overrightarrow{PQ}) = \dfrac{\overrightarrow{PQ} \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \vec{n} = \dfrac{\langle -1,2,3 \rangle \cdot \langle 1,2,-2 \rangle}{\langle 1,2,-2 \rangle \cdot \langle 1,2,-2 \rangle} \vec{n}$

$= \dfrac{-1+4-6}{1+4+4} = \dfrac{-3}{9} \langle 1, 2, -2 \rangle$

$\text{proj}_{\mathbf{n}}(\overrightarrow{PQ}) = \boxed{-\tfrac{1}{3} \langle 1, 2, -2 \rangle}$

(c) Find the distance from the point P to the plane.



$d = |\text{proj}_{\vec{n}}(\overrightarrow{QP})| = |\text{proj}_{\vec{n}}(\overrightarrow{PQ})|$

$= \left| \frac{-1}{3} \langle 1, 2, -2 \rangle \right|$

$= \frac{1}{3} \sqrt{1+4+4} = \sqrt{9}/3$

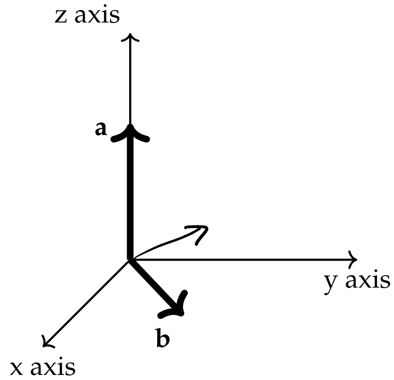
distance $= \boxed{1}$

E9000

Do not write in box

E9000

Question 2 The figure shows a vector $\mathbf{a}$ in the direction $\mathbf{k}$ and a vector $\mathbf{b}$ in the xy-plane. Their lengths are $|\mathbf{a}| = 4$ and $|\mathbf{b}| = 5$. **(5 points)**



(a) Find $|\mathbf{a} \times \mathbf{b}|$.

$|\vec{a}||\vec{b}| \sin(\pi/2)$

$|\mathbf{a} \times \mathbf{b}| =$ 20

(b) The x component of $\mathbf{a} \times \mathbf{b}$ is

- [x] negative
- [ ] zero
- [ ] positive

(c) The y component of $\mathbf{a} \times \mathbf{b}$ is

- [ ] negative
- [ ] zero
- [x] positive

(d) The z component of $\mathbf{a} \times \mathbf{b}$ is

- [ ] negative
- [x] zero
- [ ] positive

Question 3 Consider the function $f(x, y) = x^2 \sin^2\left(\frac{1}{x^2+y^2}\right)$ for $(x, y) \neq (0, 0)$. Use the *Squeeze Theorem* to determine whether the limit below exists. **(5 points)**

(a) Does the limit $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Circle your answer: **Yes** **No**

If the limit exists, determine its value (write DNE if it does not exist).

$\lim_{(x,y)\to(0,0)} f(x, y) =$ $\boxed{0}$.

(b) Give a complete justification for your answer using the *Squeeze Theorem*.

$$0 \leq x^2 \sin^2(1/x^2+y^2) \leq x^2$$

Since $\lim_{(x,y)\to(0,0)} x^2 = 0$ by squeeze theorem

we find that $\lim_{(x,y)\to(0,0)} f(x,y) = 0$

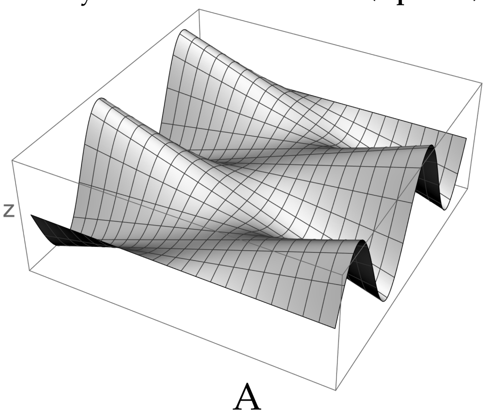
E9000

Do not write in box

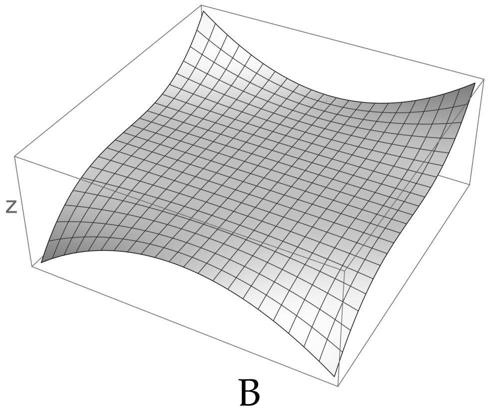
E9000

Question 4 Select the graph of $f(x, y) = x^2 \sin(y)$. Write your answer in the box: $\boxed{D}$.

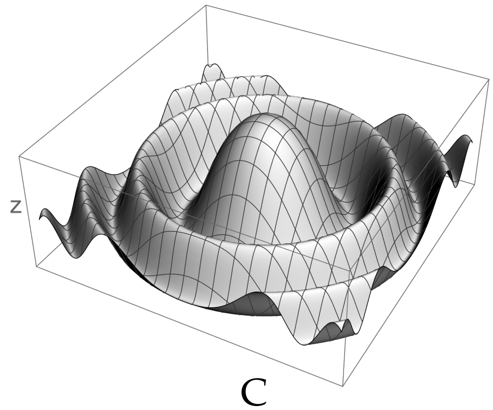
Only the z-axis is labeled. **(2 points)**



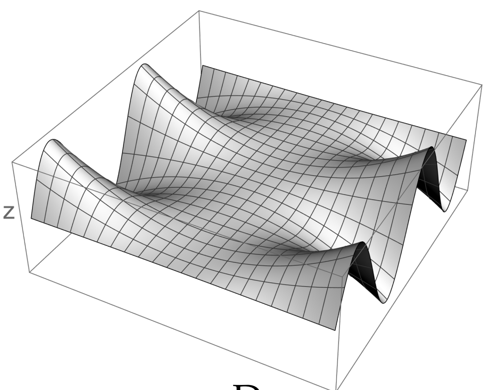
A



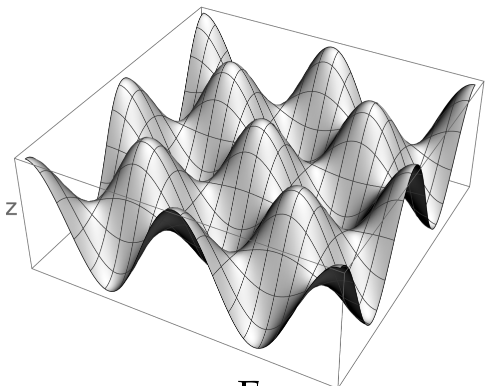
B



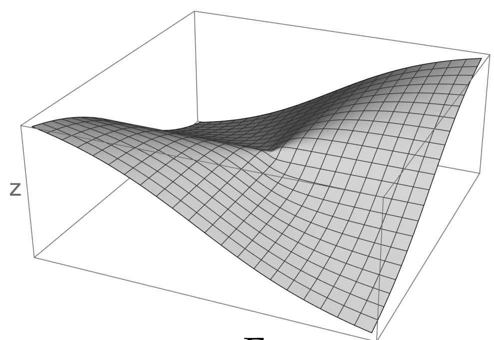
C



D



E



F

Question 5 $f(x, y)$ is a differentiable function. The tangent plane to the graph of f at the point $(1, 1, f(1, 1))$ is given by $2x - y + z = 5$. Determine $f(1, 1)$, $\dfrac{\partial f}{\partial x}(1, 1)$, and $\dfrac{\partial f}{\partial y}(1, 1)$. **(6 points)**

Tangent plane is:

$z = f_x(1,1)(x-1) + f_y(1,1)(y-1) + f(1,1)$

$= f_x(1,1)\,x + f_y(1,1)\,y + (f(1,1) - f_x(1,1) - f_y(1,1))$

compare to

$z = -2x + y + 5$

to find $\dfrac{\partial f}{\partial x}(1,1) = -2$, $\dfrac{\partial f}{\partial y}(1,1) = 1$

plug in $x=1$, $y=1$ to find $f(1,1)=4$

$f(1, 1) = \boxed{4}$

$\dfrac{\partial f}{\partial x}(1, 1) = \boxed{-2}$

$\dfrac{\partial f}{\partial y}(1, 1) = \boxed{1}$

Question 6 **(8 points)**

(a) Let $x(s,t) = s^2t + t^3 - s$ and $y(s,t) = 2st - t^2$.

Compute $\frac{\partial x}{\partial s}$ and $\frac{\partial y}{\partial s}$.

$\frac{\partial x}{\partial s} =$ $\boxed{2st - 1}$

$\frac{\partial y}{\partial s} =$ $\boxed{2t}$

(b) Suppose $f(x,y)$ is a differentiable function of x and y and let $g(s,t) = f(x(s,t), y(s,t))$, where $x(s,t)$ and $y(s,t)$ are the functions in part (a).

Use the table of values on the right, to calculate $\frac{\partial g}{\partial s}(0,1)$.

	g	f	$\frac{\partial f}{\partial x}$	$\frac{\partial f}{\partial y}$
$(0,1)$	-3	3	4	7
$(1,-1)$	-4	-3	1	5

$x(0,1) = 1$
$y(0,1) = -1$

$g_s(0,1) = f_x(1,-1)\, x_s(0,1) + f_y(1,-1)\, y_s(0,1)$

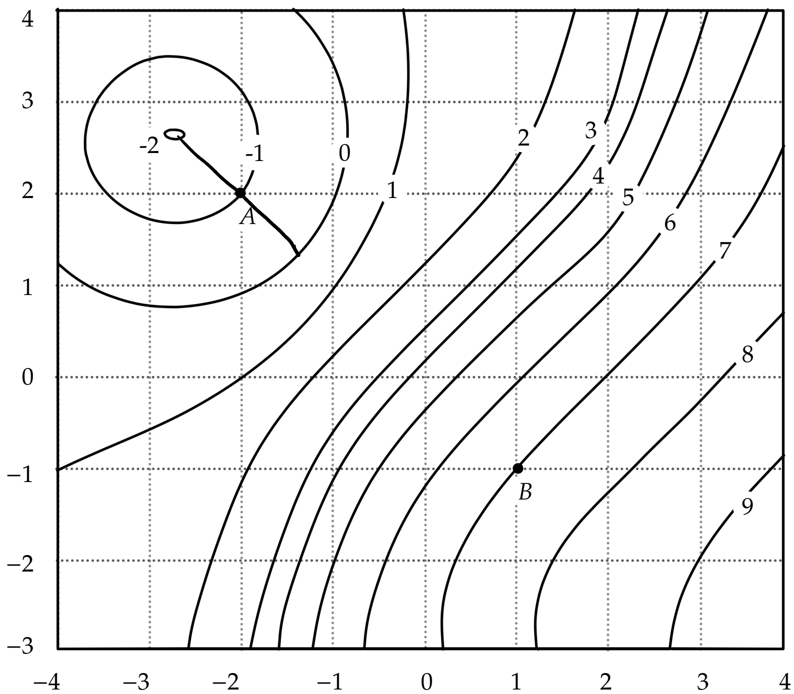
$= (1)(-1) + (5)(2) = 9$

$\frac{\partial g}{\partial s}(0,1) =$ $\boxed{9}$

Question 7 The contour map of a differentiable function $f(x, y)$ is shown. Each level curve is labeled by the corresponding value of f. Choose the best answer for each question below. **(7 points)**

(a) At the point **B**, determine the sign of the following quantities

$f_x(\mathbf{B})$ is	☒	positive
	☐	negative
	☐	zero
$f_{yy}(\mathbf{B})$ is	☐	positive
	☒	negative
	☐	zero
$f_{xy}(\mathbf{B})$ is	☒	positive
	☐	negative
	☐	zero



(b) Let $\mathbf{u}$ be a unit vector with direction $\overrightarrow{\mathbf{BA}}$. Estimate $D_{\mathbf{u}} f(\mathbf{A})$, the directional derivative of f at $\mathbf{A}$ in the direction of $\mathbf{u}$.

☐ 4

☐ 1

☐ 0

☒ −1

☐ −4

$$D_{\vec{u}} f(A) \approx \frac{-2-0}{2} = -1$$