Math 241: Exam #2

Name:

NetID:

- When space is provided, **show work which justifies your answer**. You do not need to show work on multiple choice questions unless otherwise specified.
- No calculators, notes, books, etc... are permitted.
- You do not need to numerically evaluate expressions such as $\sqrt{7}, 4/13, \cos(\pi/10)$, etc...
- The exam lasts **60 minutes**, has **6 pages** and consists of **7 questions**.

Question 1.

(9 points) The function f is differentiable and its second derivatives exist and are continuous. The table contains the values of f and its first and second-order partial derivatives at the points A, B, C, and D.

	f	f_x	f_y	f_{xx}	f_{xy}	f_{yy}
A(0,0)	0	0	1	0	2	1
B(1,1)	2	0	0	1	0	2
C(1,2)	1	0	0	1	1	0
D(2,2)	0	-1	0	0	0	0

(a) Which of the points $A(0,0)$, $B(1,1)$, $C(1,2)$, $D(2,2)$ are critical points? Mark all that apply.

☐ A ☐ B ☐ C ☐ D

(b) Use the second derivatives test to determine whether the critical points of f (that you found in part (a)) are local minima, local maxima, or saddle points. Write DNE if the corresponding type of critical point does not occur at the points A through D.
Show your work.

f has a local minimum at the point(s) ☐

f has a local maximum at the point(s) ☐

f has saddle point(s) at ☐

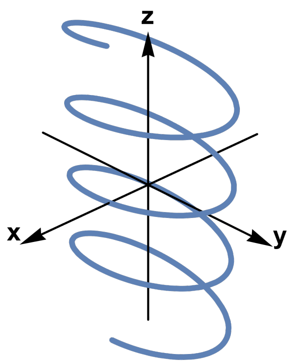
(c) Exactly one of the following statements is correct. Which?

○ the function f has an absolute maximum in the disk $\{(x, y) \mid x^2 + y^2 < 1\}$

○ the function f has an absolute maximum in the triangle $\{(x, y) \mid x \geq 0, y \geq 0, x + y \leq 5\}$

○ the function f has an absolute maximum in the whole plane $\mathbb{R}^2$

Question 2. **(2 points)** Which of the following is a parametrization of the depicted curve? Mark your answer.



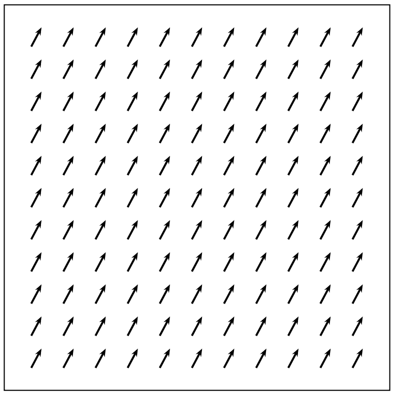
- ○ $\mathbf{r}(t) = \langle \cos t, 2\sin t, 3\cos t \rangle$, $\quad 0 \le t \le 2\pi$
- ○ $\mathbf{r}(t) = \langle 2\sin t, 2\cos t, \cos(3t) \rangle$, $\quad 0 \le t \le 2\pi$
- ○ $\mathbf{r}(t) = \langle \cos(4t), 2\sin(4t), t - \pi \rangle$, $\quad 0 \le t \le 2\pi$
- ○ $\mathbf{r}(t) = \langle t\cos(4t), t, t\sin(4t) \rangle$, $\quad 0 \le t \le 2\pi$

Question 3. **(4 points)** Consider the vector field $\mathbf{F}(x, y) = (6xy^3 + 9)\mathbf{i} + ax^2y^2\mathbf{j}$, where a is some real number. For what value(s) of a is $\mathbf{F}$ conservative?

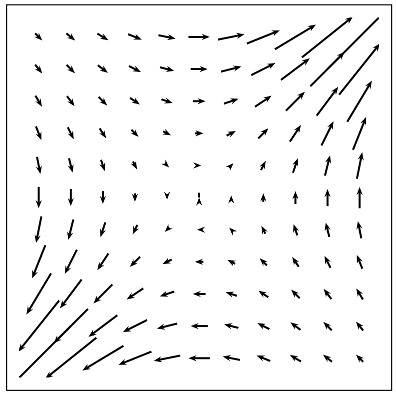
$a =$ [ ]

Question 4. **(4 points)** Exactly two of the following vector fields are *not* conservative. Which two?

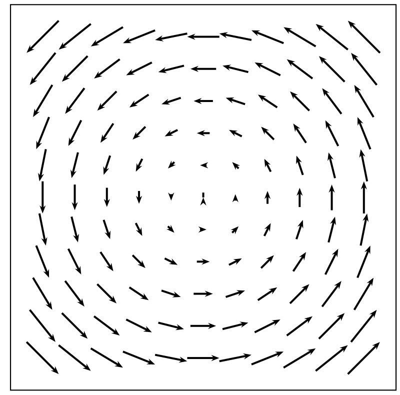
☐ A ☐ B ☐ C ☐ D ☐ E ☐ F



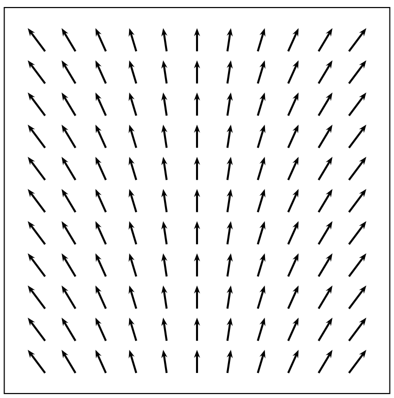
A



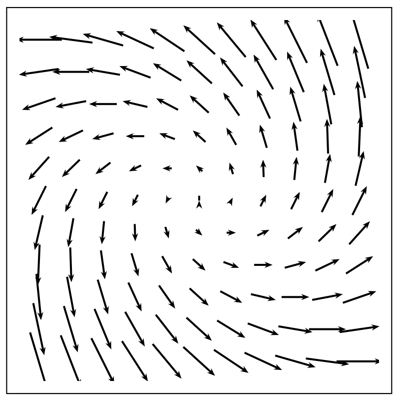
B



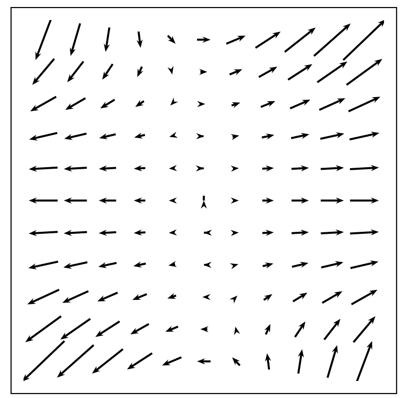
C



D



E

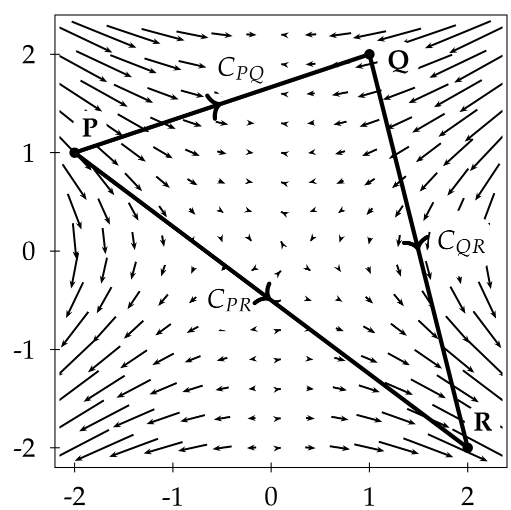


F

Question 5. **(5 points)** Let $g(x, y)$ be differentiable and consider the vector field $\mathbf{G} = \nabla g$. Suppose C_{PQ} is the line segment from $P(-2, 1)$ to $Q(1, 2)$, C_{QR} is the line segment from $Q(1, 2)$ to $R(2, -2)$, and C_{PR} the line segment from $P(-2, 1)$ to $R(2, -2)$. It is known that $\int_{C_{PQ}} \mathbf{G} \cdot d\mathbf{r} = 3$ and $\int_{C_{QR}} \mathbf{G} \cdot d\mathbf{r} = 0$.

Compute $\int_{C_{PR}} \mathbf{G} \cdot d\mathbf{r}$.

epeudeu



$\int_{C_{PR}} \mathbf{G} \cdot d\mathbf{r} =$ ☐

Question 6. **(8 points)** Set up and evaluate an integral to compute the area of a fence built over the parametrized curve $\mathbf{r}(t) = \langle \sin(3t), \cos(3t) \rangle$, with $t \in [0, \pi/6]$, where the height is described by the function $f(x, y) = x^2 y + 2$.

Area=

Question 7. **(8 points)** Use Lagrange multipliers to find the absolute minimum and absolute maximum of the function $f(x, y) = 2x - y + 2$ subject to the constraint $g(x, y) = x^2 + \frac{y^2}{2} = 2$.

minimum value of f =

at the point(s)

maximum value of f =

at the point(s)