Math 241: Exam #2

Name:

NetID:

- When space is provided, **show work which justifies your answer**. You do not need to show work on multiple choice questions unless otherwise specified.
- No calculators, notes, books, etc... are permitted.
- You do not need to numerically evaluate expressions such as $\sqrt{7}$, $4/13$, $\cos(\pi/10)$, etc...
- The exam lasts **60 minutes**, has **6 pages** and consists of **7 questions**.

Question 1. Consider the function $f(x, y) = 3x^2 - 3xy$. **(9 points)**

(a) Find one critical point P of f.

critical point P =

(b) Use the Second Derivatives test to determine whether the critical point P is

- ○ a local minimum of f,
- ○ a local maximum of f,
- ○ a saddle point of f , or
- ○ none of the above?

Show your work.

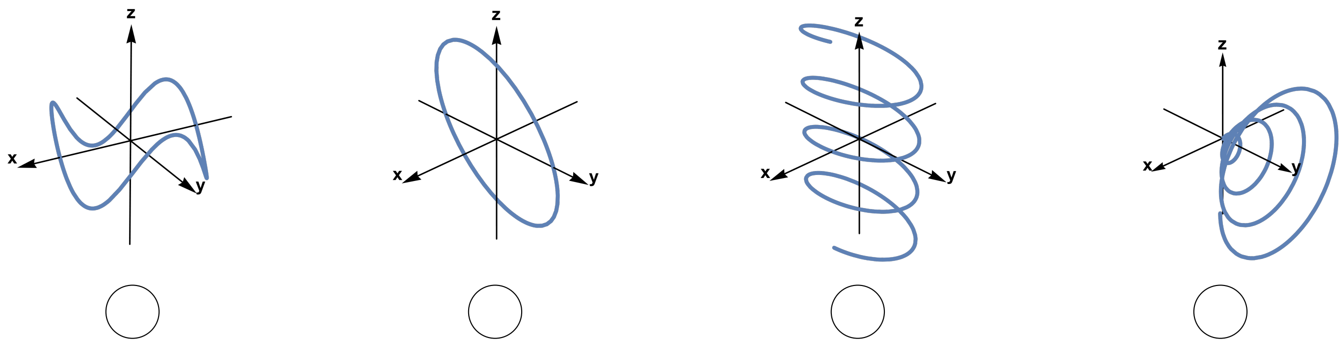
(c) Does the function f have an absolute minimum in the closed unit disk $\{(x, y) \mid x^2 + y^2 \leq 1\}$?
There is no need to determine the value, if it exists.

- ○ Yes
- ○ No
- ○ It is impossible to tell from the given information.

Question 2. Which of the following figures depicts the parameterized curve

$$\mathbf{r}(t) = \langle \cos t, 2\sin t, 3\cos t \rangle, \qquad 0 \le t \le 2\pi?$$

Mark your answer. **(2 points)**



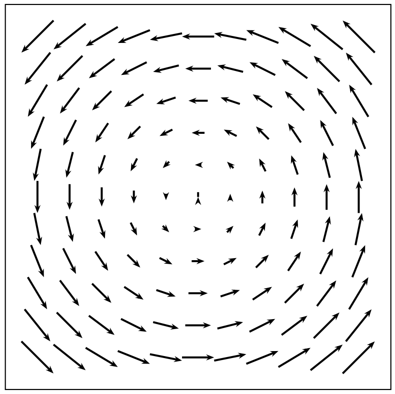
Question 3. Is the vector field $\mathbf{F}(x, y) = \langle 2xy, x^2 - 2y \rangle$ conservative? Circle your answer: **Yes** **No**

If $\mathbf{F}$ is conservative, find a potential function $f(x, y)$ for $\mathbf{F}$.
If there is no potential function, explain why not and leave the answer box blank. **(5 points)**

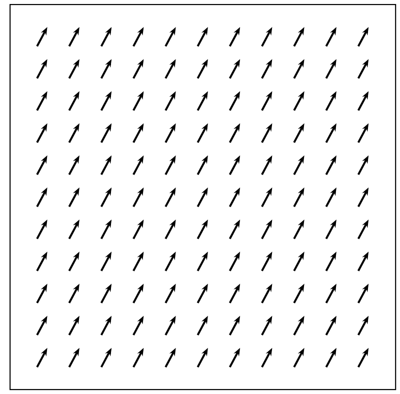
$f(x, y) =$ [ ]

Question 4. Exactly two of the following vector fields are *not* conservative? Which two? **(4 points)**

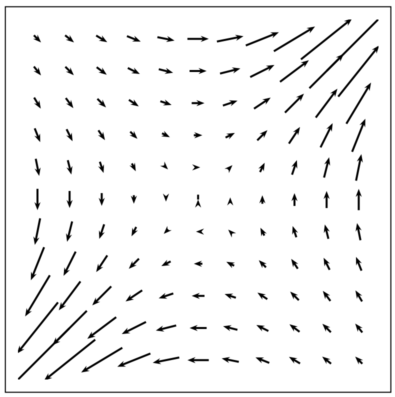
- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ F



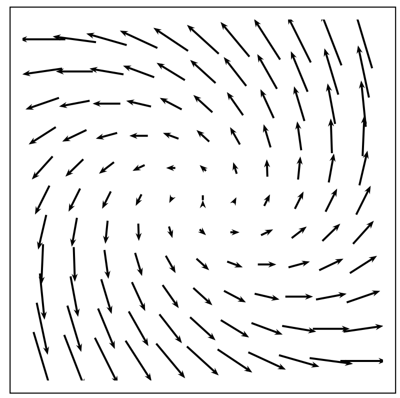
A



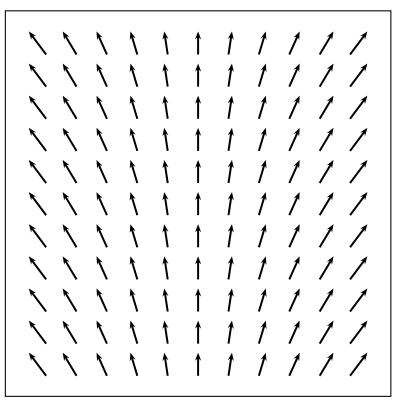
B



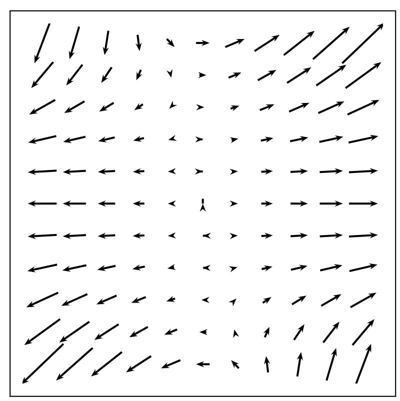
C



D



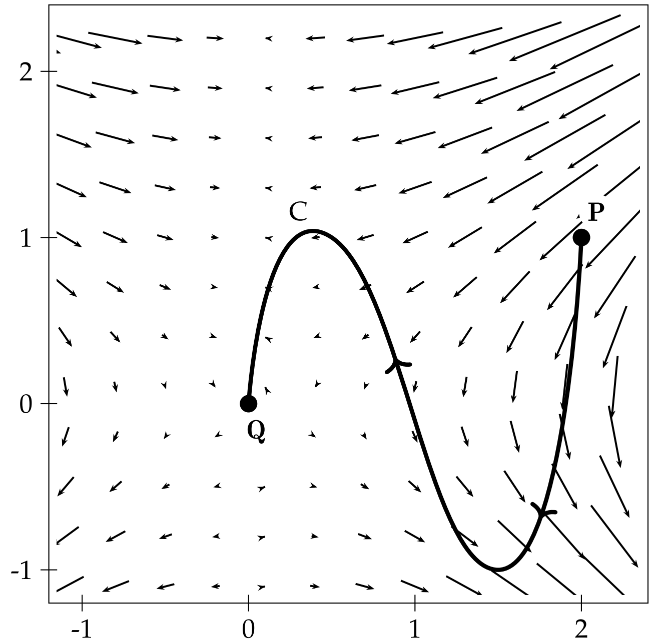
E



F

Question 5. Let $g(x,y) = xy - 3x^2y$. The diagram below depicts the vector field $\mathbf{G} = \nabla g$ and the curve C from the point $P(2,1)$ to the origin $Q(0,0)$. **(4 points)**

Compute $\int_C \mathbf{G} \cdot d\mathbf{r}$.



$\int_C \mathbf{G} \cdot d\mathbf{r} =$

Question 6. Consider the function $f(x,y) = xy$ and the curve C given by $\mathbf{r}(t) = \langle 2\sin(t), 2\cos(t)\rangle$, $0 \leq t \leq \pi/2$. Compute $\int_C f(x,y)\, ds$. **(8 points)**

$\int_C f(x,y) ds =$

Question 7. Use Lagrange multipliers to find the absolute minimum and the absolute maximum of the function

$$f(x,y) = 2x - 4y + 1,$$

subject to the constraint $g(x,y) = x^2 + 2y^2 = 3$. **(8 points)**

minimum value of f =

at the point(s)

maximum value of f =

at the point(s)