Math 241: Exam #3

Write your name **NEATLY**

Name:

Write your NetID **NEATLY**

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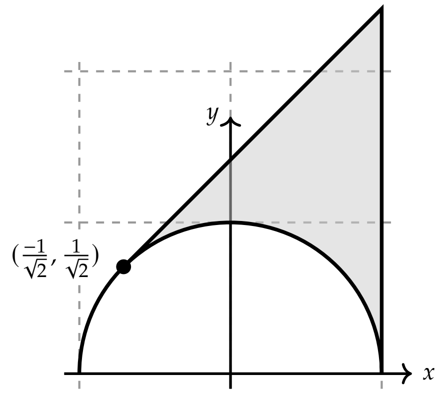
- When space is provided, **show work which justifies your answer**. You do not need to show work on multiple choice questions unless otherwise specified.
- No calculators, notes, books, etc... are permitted.
- You do not need to numerically evaluate expressions such as $\sqrt{7}, 4/13, \cos(\pi/10)$, etc...
- The exam lasts **60 minutes**, has **8 pages** and consists of **7 questions**.

Question 1. Let R be the depicted region above the unit circle $x^2 + y^2 = 1$, below the line $y - x = \sqrt{2}$, and to the left of the line $x = 1$.



(a) **(4 points)** Find the bounds of integration for $\iint_R 2y\ dA$ as an iterated integral

$$\iint_R 2y\ dA = \int_{\square}^{\square} \int_{\square}^{\square} 2y\ dy\ dx.$$

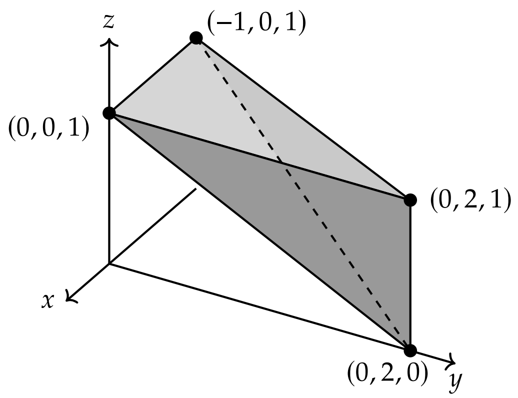
Note: The order of integration is already determined.

(b) **(2 points)** Evaluate the integral $\iint_R 2y\ dA$ as set up in part (a).

$$\iint_R 2y\ dA = \square$$

Scratch Space

Question 2. **(6 points)**



Consider the triple integral $\int_0^1 \int_{2-2z}^2 \int_{(y-2)/2}^0 f(x,y,z)\,dx\,dy\,dz$. Its region of integration is depicted; it is bounded by the yz-plane and the planes with equations $z = 1$, $2x - y + 2 = 0$, and $y + 2z - 2 = 0$.

Determine the limits of integration when changing the order of integration as

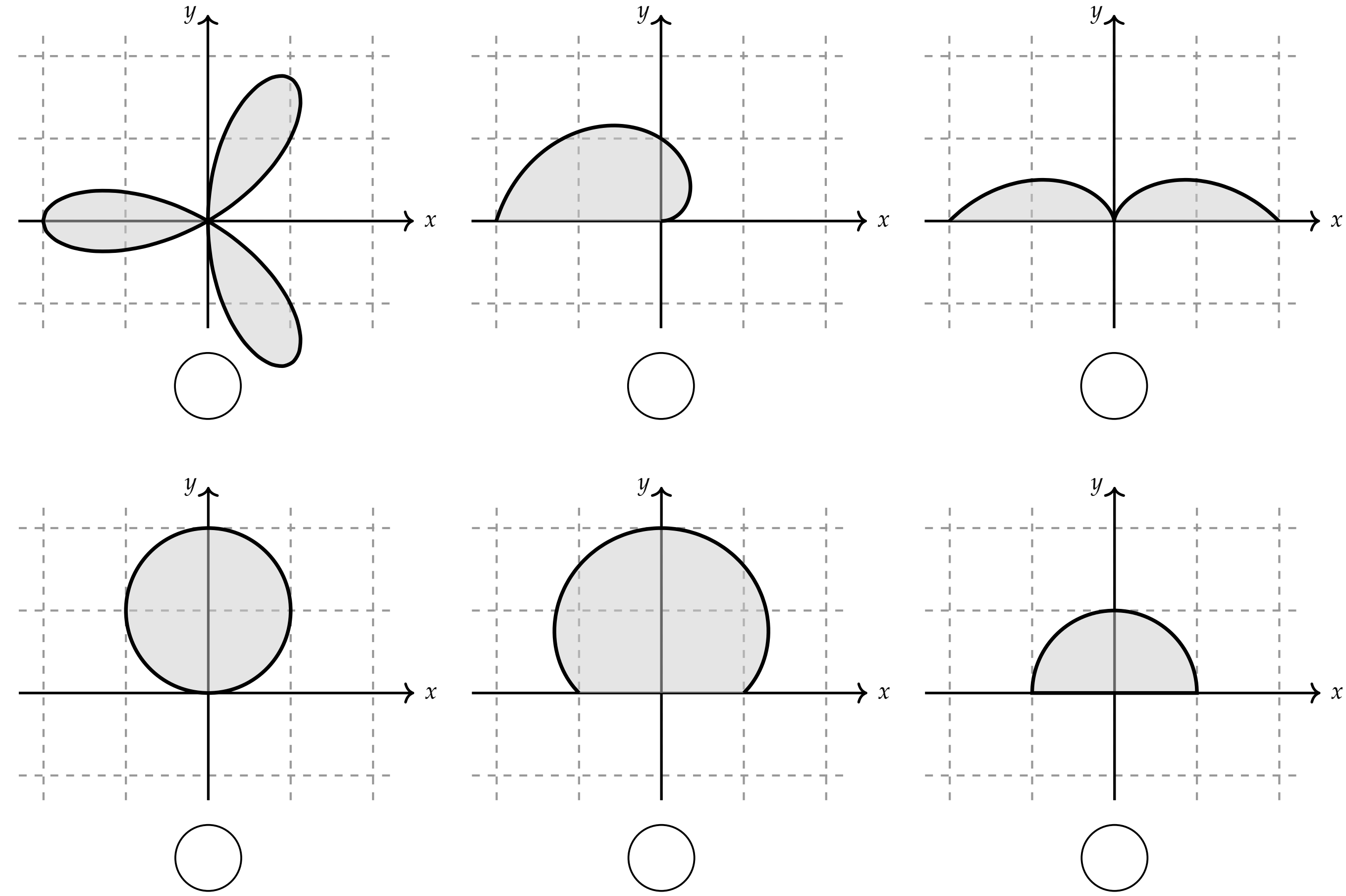
$$\int_0^1 \int_{2-2z}^2 \int_{(y-2)/2}^0 f(x,y,z)\,dx\,dy\,dz$$

$$= \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x,y,z)\,dz\,dy\,dx.$$

Scratch Space

Question 3. The double integral $\iint_R x - y \, dA$ has the form $\int_0^{\pi} \int_0^{2\theta/\pi} ?? \, dr \, d\theta$ when converted into polar coordinates.

(a) **(2 points)** Mark the box of the picture below which depicts the region R in the xy-plane.



(b) **(3 points)** Fill in the missing integrand to convert this integral to polar coordinates. Do **not** compute the integral!

$$\iint_R x - y \, dA = \int_0^{\pi} \int_0^{2\theta/\pi} \boxed{} \, dr \, d\theta$$

Scratch Space

Question 4. **(7 points)** Let R be the region in $\mathbb{R}^3$ that is inside the sphere $x^2 + y^2 + z^2 = 4$, outside of the sphere $x^2 + y^2 + z^2 = 1$, and above the cone $z = -\sqrt{x^2 + y^2}$. Convert the triple integral $\iiint_R z \, dV$ into spherical coordinates. Do **not** compute the integral!

$$\iiint_R z \, dV = \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} \boxed{} \, d\phi \, d\rho \, d\theta.$$

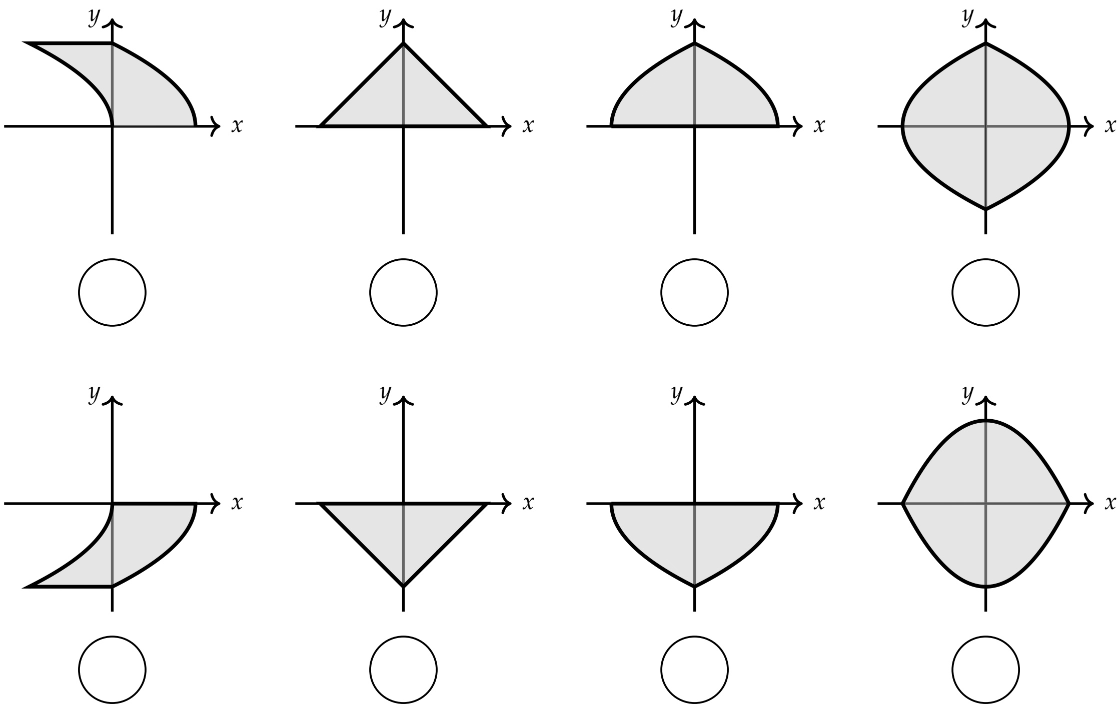
Note: The order of integration is already determined.

Scratch Space

Question 5. Let D be the rectangle $\{(u,v) \mid -1 \le u \le 1 \text{ and } 0 \le v \le 1\}$.
Let $x(u,v) = 2u^2 - 2v^2$ and $y(u,v) = -2uv$. Consider the transformation $T(u,v) = \big(x(u,v), y(u,v)\big) = (2u^2 - 2v^2, -2uv)$.
The transformation T satisfies:

$$T(u,0) = (2u^2, 0)$$
$$T(u,1) = (2u^2 - 2, -2u)$$
$$T(-1,v) = (2 - 2v^2, 2v)$$
$$T(1,v) = (2 - 2v^2, -2v)$$

(a) **(2 points)** Mark the picture which depicts the image $T(D)$.



(b) **(2 points)** Compute the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$.

$$\frac{\partial(x,y)}{\partial(u,v)} = \boxed{}$$

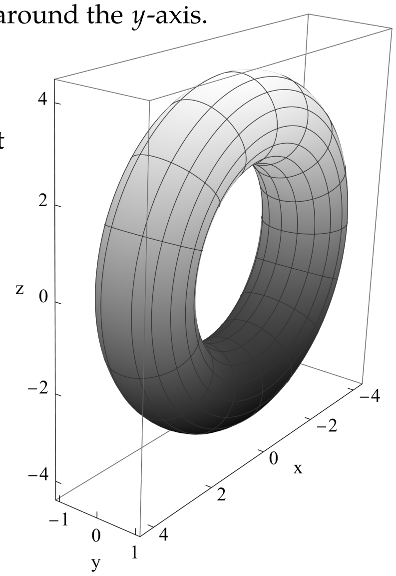
(c) **(3 points)** Set up an iterated integral computing the area of $T(D)$. Do **not** compute the integral!

$$\text{Area}(T(D)) = \int_{\square}^{\square} \int_{\square}^{\square} \boxed{} \, du \, dv$$

Question 6. Let T be the surface parametrized by

$$\mathbf{r}(u,v) = ((3+\cos u)\cos v, \sin u, (3+\cos u)\sin v), \quad \text{for } u, v \text{ in } [0, 2\pi].$$

The surface T can be obtained by revolving the circle $(x-3)^2 + y^2 = 1$ around the y-axis.



(a) **(4 points)** Find the equation for the tangent plane to T at the point $P = \left(-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\right) = \mathbf{r}(\pi, 3\pi/4)$.

Tangent plane

(b) **(2 points)** Which of the following statements is true? You do **not** need to use the parameterization to calculate the integrals.

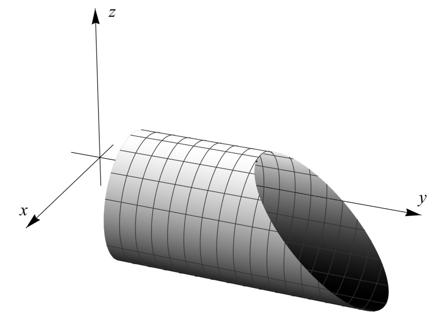
$\iint_T x^2\, dS > \iint_T y^2\, dS$ ◯

$\iint_T x^2\, dS = \iint_T y^2\, dS$ ◯

$\iint_T x^2\, dS < \iint_T y^2\, dS$ ◯

Scratch Space

Question 7. **(3 points)** Let S be the part of the cylinder $(x-1)^2 + z^2 = 1$ between the planes $y = 1$ and the plane $y + z = 4$. Parameterize S with a function $\mathbf{r}(u, v)$. Be sure to specify the domain D of your parameterization.



$$\mathbf{r}(u, v) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$$

$$D = \Big\{(u, v) \;\Big|\; \qquad \le u \le \qquad , \qquad \le v \le \qquad \Big\}$$

Scratch Space