Math 241: Exam #3

Write your name **NEATLY**

Name:

Write your NetID **NEATLY**

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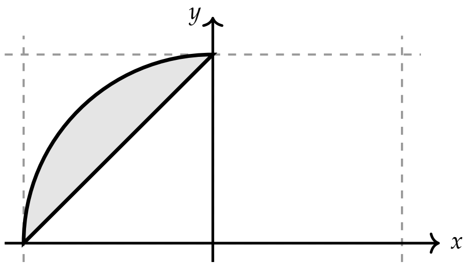
- When space is provided, **show work which justifies your answer**. You do not need to show work on multiple choice questions unless otherwise specified.
- No calculators, notes, books, etc... are permitted.
- You do not need to numerically evaluate expressions such as $\sqrt{7}, 4/13, \cos(\pi/10)$, etc...
- The exam lasts **60 minutes**, has **8 pages** and consists of **7 questions**.

Question 1. Let R be the depicted region above the line $x - y + 1 = 0$ and inside unit circle centered at the origin.



(a) **(4 points)** Find the bounds of integration for $\iint_R 2x \, dA$ as an iterated integral

$$\iint_R 2x \, dA = \int_{\square}^{\square} \int_{\square}^{\square} 2x \, dx \, dy.$$

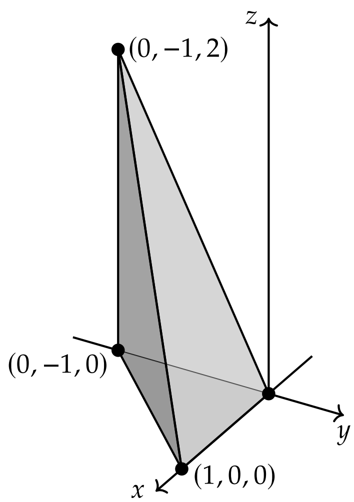
Note: The order of integration is already determined.

(b) **(2 points)** Evaluate the integral $\iint_R 2x \, dA$ as set up in part (a).

$$\iint_R 2x \, dA = \square$$

Scratch Space

Question 2. **(6 points)**



Consider the triple integral $\int_0^2 \int_{-1}^{-z/2} \int_0^{1+y} f(x,y,z)\,dx\,dy\,dz$. Its region of integration is depicted; it is bounded by the xy-plane, the yz-plane, and the planes with equations $x - y - 1 = 0$ and $2y + z = 0$.

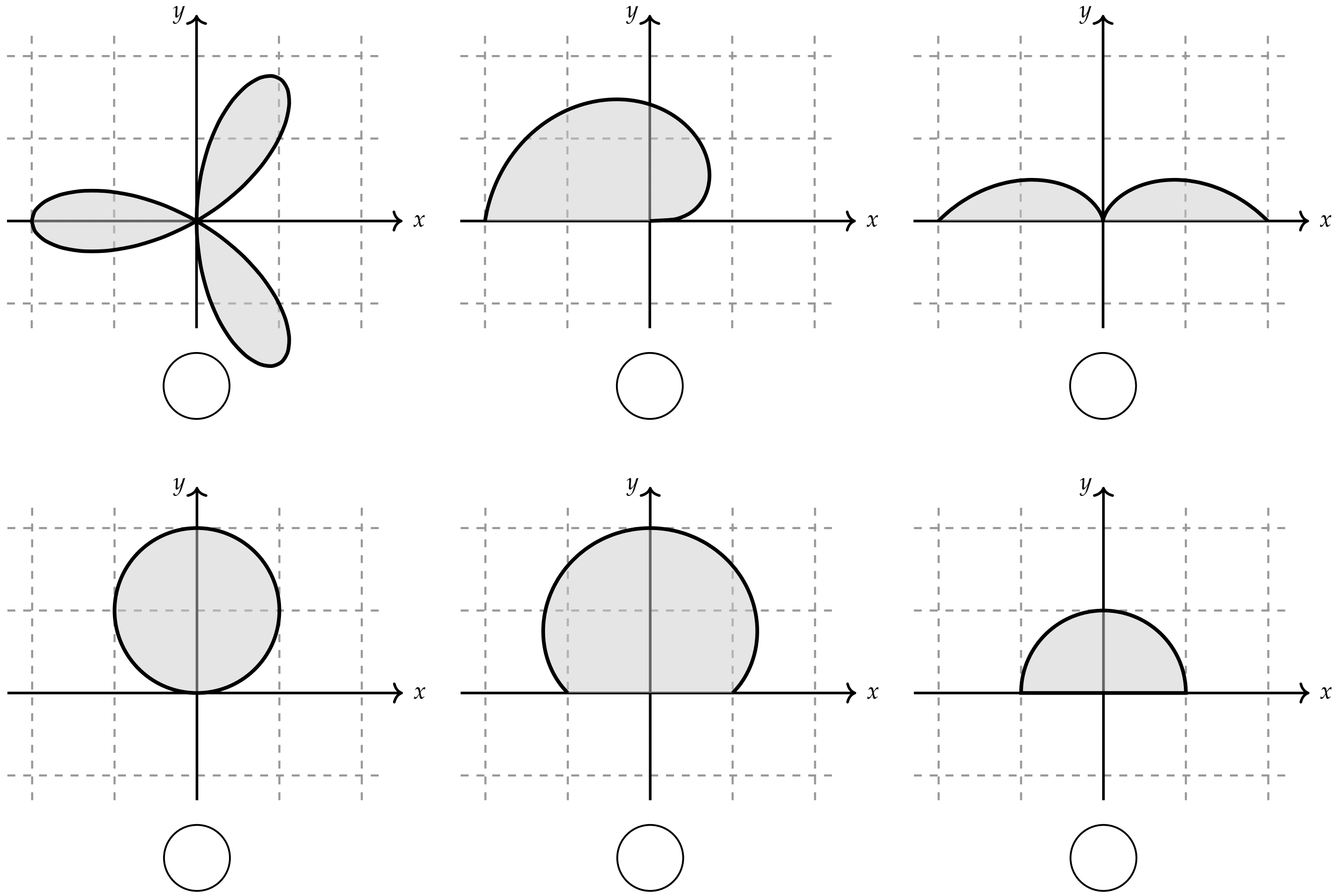
Determine the limits of integration when changing the order of integration as $\int_0^2 \int_{-1}^{-z/2} \int_0^{1+y} f(x,y,z)\,dx\,dy\,dz$

$$= \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x,y,z)\,dz\,dy\,dx.$$

Scratch Space

Question 3. The double integral $\iint_R xy^2\, dA$ has the form $\int_0^{\pi} \int_0^{1+\sin(\theta)} ??\, dr\, d\theta$ when converted into polar coordinates.

(a) **(2 points)** Mark the box of the picture below which depicts the region R in the xy-plane.



(b) **(3 points)** Fill in the missing integrand to convert this integral to polar coordinates. Do **not** compute the integral!

$$\iint_R xy^2\, dA = \int_0^{\pi} \int_0^{1+\sin\theta} \boxed{}\ dr\, d\theta$$

Scratch Space

Question 4. **(7 points)** Let R be the region in $\mathbb{R}^3$ that is inside the sphere $x^2 + y^2 + z^2 = 9$ and below the cone $z = \sqrt{x^2 + y^2}$. Convert the triple integral $\iiint_R x \, dV$ into spherical coordinates. Do **not** compute the integral!

$$\iiint_R x \, dV = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{} \, d\rho \, d\phi \, d\theta.$$

Note: The order of integration is already determined.

Scratch Space

Question 5. Let D be the square $\{(u,v)|\ 0 \le u \le 2 \text{ and } 0 \le v \le 2\}$.
Let $x(u,v) = v^2 - u^2$ and $y(u,v) = uv$. Consider the transformation $T(u,v) = \big(x(u,v), y(u,v)\big) = (v^2 - u^2, uv)$.
The transformation T satisfies:

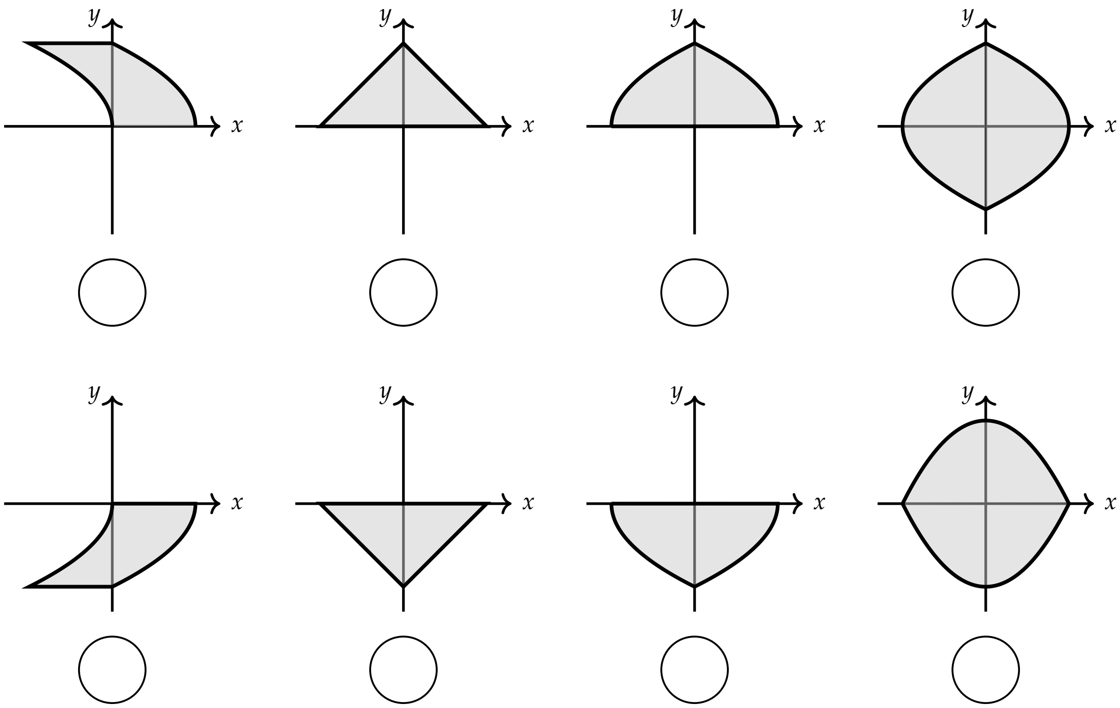
$T(u,0) = (-u^2, 0)$

$T(u,2) = (4 - u^2, 2u)$

$T(0,v) = (v^2, 0)$

$T(2,v) = (v^2 - 4, 2v)$

(a) **(2 points)** Mark the picture which depicts the image $T(D)$.



(b) **(2 points)** Compute the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$.

$$\frac{\partial(x,y)}{\partial(u,v)} = \boxed{}$$

(c) **(3 points)** Set up an iterated integral computing the area of $T(D)$. Do **not** compute the integral!

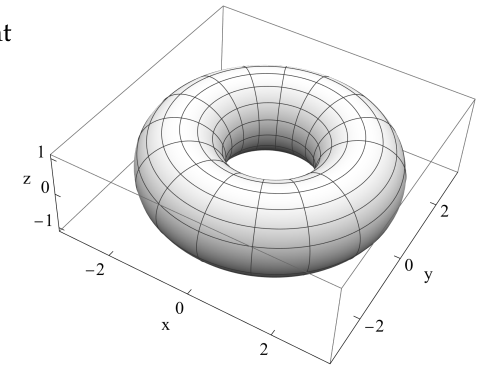
$$\text{Area}(T(D)) = \int_{\square}^{\square} \int_{\square}^{\square} \boxed{}\, du\, dv$$

Question 6. Let T be the surface parametrized by

$$\mathbf{r}(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u), \quad \text{for } u,v \text{ in } [0,2\pi].$$

The surface T can be obtained by revolving the circle $(x-2)^2 + z^2 = 1$ around the z-axis.

(a) **(4 points)** Find the equation for the tangent plane to T at the point $P = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right) = \mathbf{r}\,(\pi, 3\pi/4)$.



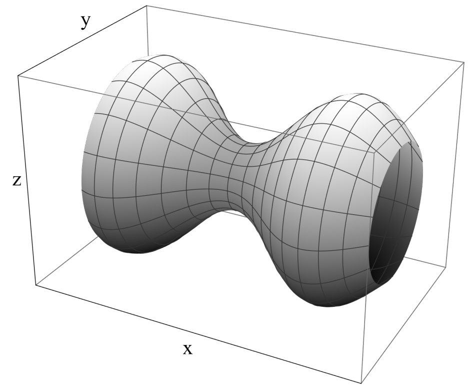
Tangent plane

(b) **(2 points)** Which of the following statements is true? You do **not** need to use the parameterization to calculate the integral.

- ○ $\iint_T x\, dS < \iint_T z\, dS$
- ○ $\iint_T x\, dS = \iint_T z\, dS$
- ○ $\iint_T x\, dS > \iint_T z\, dS$

Scratch Space

Question 7. **(3 points)** A surface of revolution is obtained by rotating the curve $y = 2 + \sin x$ around the x-axis. Let S be the portion of that surface that lies between the planes $x = \pi/3$ and $x = 3\pi$.
Parameterize S with a function $\mathbf{r}(u, v)$. Be sure to specify the domain D of your parameterization.



$\mathbf{r}(u, v) =$ $\Big\langle \qquad , \qquad , \qquad \Big\rangle$

$D =$ $\Big\{(u, v) \;\Big|\; \qquad \le u \le \qquad , \qquad \le v \le \qquad \Big\}$

Scratch Space