Math 241: Exam #3

Write your name **NEATLY**

Name:

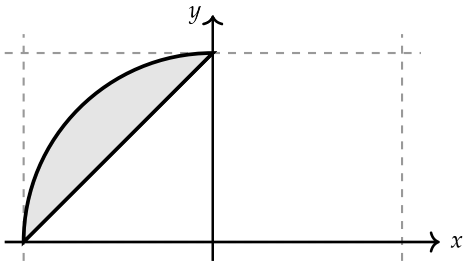
Write your NetID **NEATLY**

NetID:

- When space is provided, **show work which justifies your answer**. You do not need to show work on multiple choice questions unless otherwise specified.
- No calculators, notes, books, etc... are permitted.
- You do not need to numerically evaluate expressions such as $\sqrt{7}, 4/13, \cos(\pi/10)$, etc...
- The exam lasts **60 minutes**, has **8 pages** and consists of **7 questions**.

Question 8.

Let R be the depicted region above the line $x - y + 1 = 0$ and inside unit circle centered at the origin.



(a) **(4 points)** Find the bounds of integration for $\iint_R 2y\ dA$ as an iterated integral

$$\iint_R 2y\ dA = \int_{\square}^{\square} \int_{\square}^{\square} 2y\ dy\ dx.$$

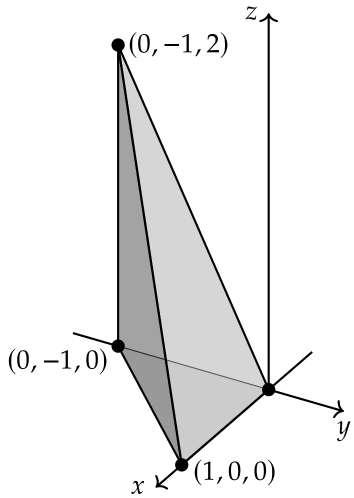
Note: The order of integration is already determined.

(b) **(2 points)** Evaluate the integral $\iint_R 2y\ dA$ as set up in part (a).

$$\iint_R 2y\ dA = \square$$

Scratch Space

Question 9. **(6 points)**



Consider the triple integral $\int_0^2 \int_{-1}^{-z/2} \int_0^{1+y} f(x, y, z)\, dx\, dy\, dz$. Its region of integration is depicted; it is bounded by the xy-plane, the yz-plane, and the planes with equations $x - y - 1 = 0$ and $2y + z = 0$.

Determine the limits of integration when changing the order of integration as

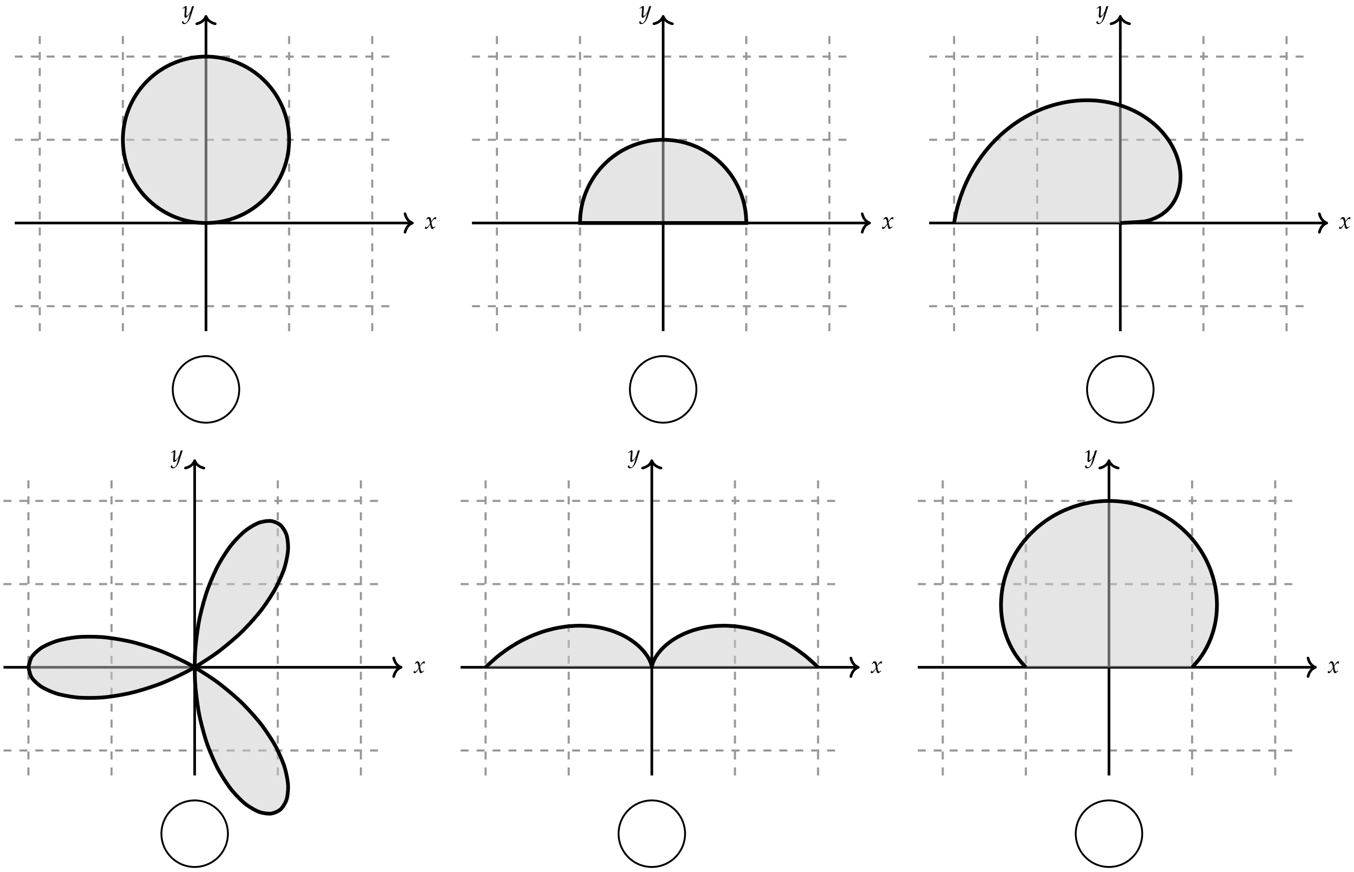
$$\int_0^2 \int_{-1}^{-z/2} \int_0^{1+y} f(x, y, z)\, dx\, dy\, dz$$

$$= \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} f(x, y, z)\, dz\, dy\, dx.$$

Scratch Space

Question 10. The double integral $\iint_R x^2y\ dA$ has the form $\int_0^{\pi}\int_0^{1+\sin(\theta)}$?? $dr\ d\theta$ when converted into polar coordinates.

(a) **(2 points)** Mark the box of the picture below which depicts the region R in the xy-plane.



(b) **(3 points)** Fill in the missing integrand to convert this integral to polar coordinates. Do **not** compute the integral!

$$\iint_R x^2y\ dA = \int_0^{\pi}\int_0^{1+\sin\theta} \boxed{}\ dr\ d\theta$$

Scratch Space

Question 11. **(7 points)** Let R be the region in $\mathbb{R}^3$ that is inside the sphere $x^2+y^2+z^2=4$ and below the cone $z=\sqrt{x^2+y^2}$. Convert the triple integral $\iiint_R y\,dV$ into spherical coordinates. Do **not** compute the integral!

$$\iiint_R y\,dV = \int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} \boxed{}\,d\rho\,d\phi\,d\theta.$$

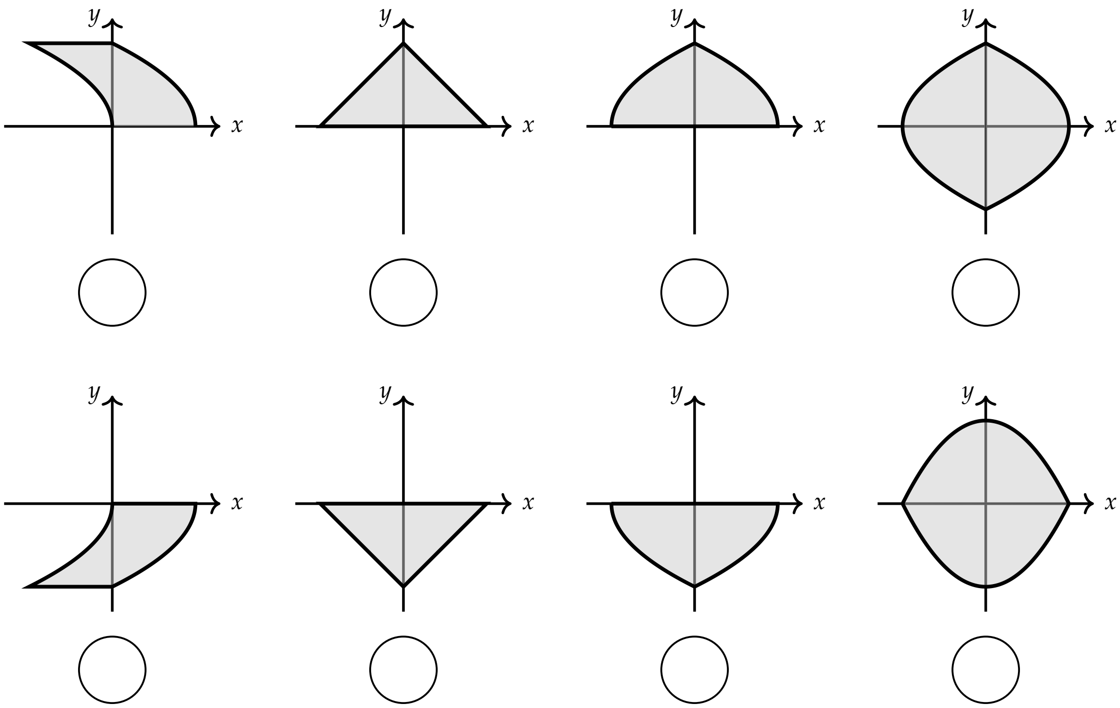
Note: The order of integration is already determined.

Scratch Space

Question 12. Let D be the square $\{(u,v) | -3 \le u \le 0 \text{ and } 0 \le v \le 3\}$.
Let $x(u,v) = v^2 - u^2$ and $y(u,v) = uv$. Consider the transformation $T(u,v) = \big(x(u,v), y(u,v)\big) = (v^2 - u^2, uv)$.
The transformation T satisfies:

$$T(u,0) = (-u^2, 0)$$
$$T(u,3) = (9 - u^2, 3u)$$
$$T(0,v) = (v^2, 0)$$
$$T(-3,v) = (v^2 - 9, -3v)$$

(a) **(2 points)** Mark the picture which depicts the image $T(D)$.



(b) **(2 points)** Compute the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$.

$$\frac{\partial(x,y)}{\partial(u,v)} = \boxed{}$$

(c) **(3 points)** Set up an iterated integral computing the area of $T(D)$. Do **not** compute the integral!

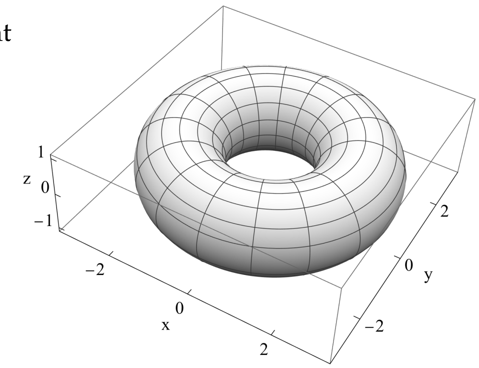
$$\text{Area}(T(D)) = \int_{\square}^{\square} \int_{\square}^{\square} \boxed{} \, du \, dv$$

Question 13. Let T be the surface parametrized by

$$\mathbf{r}(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u), \quad \text{for } u, v \text{ in } [0, 2\pi].$$

The surface T can be obtained by revolving the circle $(x-2)^2 + z^2 = 1$ around the z-axis.

(a) **(4 points)** Find the equation for the tangent plane to T at the point $P = \left(\frac{3\sqrt{3}}{2}, \frac{3}{2}, 0\right) = \mathbf{r}(0, \pi/6)$.



Tangent plane

(b) **(2 points)** Which of the following statements is true? You do **not** need use the parameterization to calculate the integral.

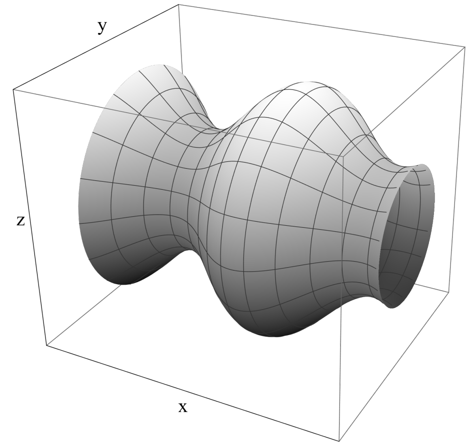
$\iint_T y\, dS > \iint_T z\, dS$ ◯

$\iint_T y\, dS < \iint_T z\, dS$ ◯

$\iint_T y\, dS = \iint_T z\, dS$ ◯

Scratch Space

Question 14. **(3 points)** A surface of revolution is obtained by rotating the curve $y = 3 + \cos x$ around the x-axis. Let S be the portion of that surface that lies between the planes $x = \pi/4$ and $x = 4\pi$.
Parameterize S with a function $\mathbf{r}(u, v)$. Be sure to specify the domain D of your parameterization.



$\mathbf{r}(u, v) = \Big\langle \quad , \quad , \quad \Big\rangle$

$D = \Big\{(u, v) \;\Big|\; \quad \leq u \leq \quad , \quad \leq v \leq \quad \Big\}$

Scratch Space