Question 1.

Let A be the oriented line segment from $(0, -4)$ to $(0, 0)$, B be the oriented line segment from $(0, 0)$, to $(4, 0)$, and C the oriented line segment from $(4, 0)$ to $(0, -4)$.

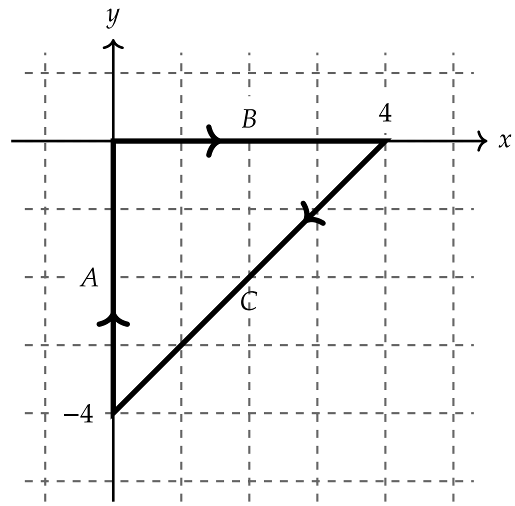
The vector field $\mathbf{F} = \langle P, Q \rangle$ satisfies

$Q_y = P_x$

$Q_x = P_y + 2$.

Suppose that $\int_B \mathbf{F} \cdot d\mathbf{r} = -4$ and $\int_A \mathbf{F} \cdot d\mathbf{r} = -2$.

Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ and mark your answer. Show your work. **(5 points)**



$\int_C \mathbf{F} \cdot d\mathbf{r} =$

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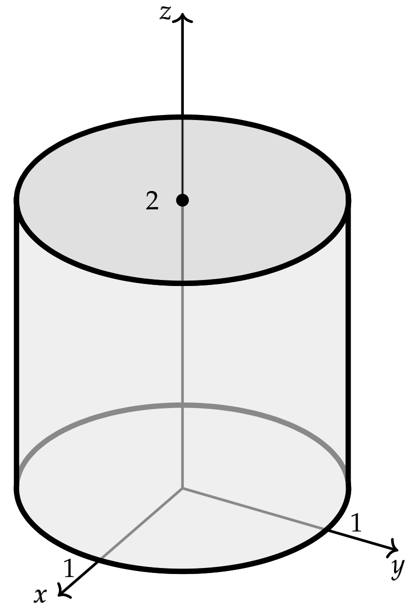
Question 2.

Let $\mathbf{H} = \langle 6x + 3yz^2, -y, z + 2x \rangle$. Let E be the surface of the cylinder of radius 1, where $0 \le z \le 2$, oriented outwards, that is, with an outward pointing unit normal vector. Use the divergence theorem to compute the flux of $\mathbf{H}$ through E. **(5 points)**



flux =

Question 3.

Find real numbers a and b so that $\mathbf{F}(x, y, z) = \langle e^{x^2} + 3az,\ 3zy,\ 9x + 3by^2 \rangle$ is conservative. **(4 points)**

a =

b =

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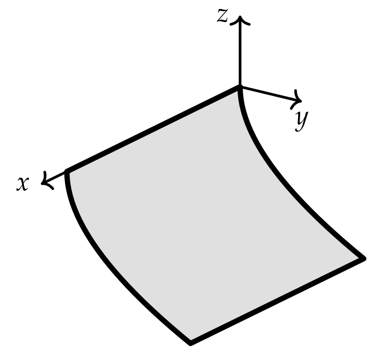
Question 4.
Let S be the surface which is the portion of the graph of $y = z^2$ where $0 \leq x \leq 1$ and $-1 \leq z \leq 0$, oriented upwards, that is, with upward pointing unit normal vector. Let $\mathbf{F} = \langle 3yz, 3x^2 + 2y, z \rangle$. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$.
(6 points)



$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \boxed{}$$

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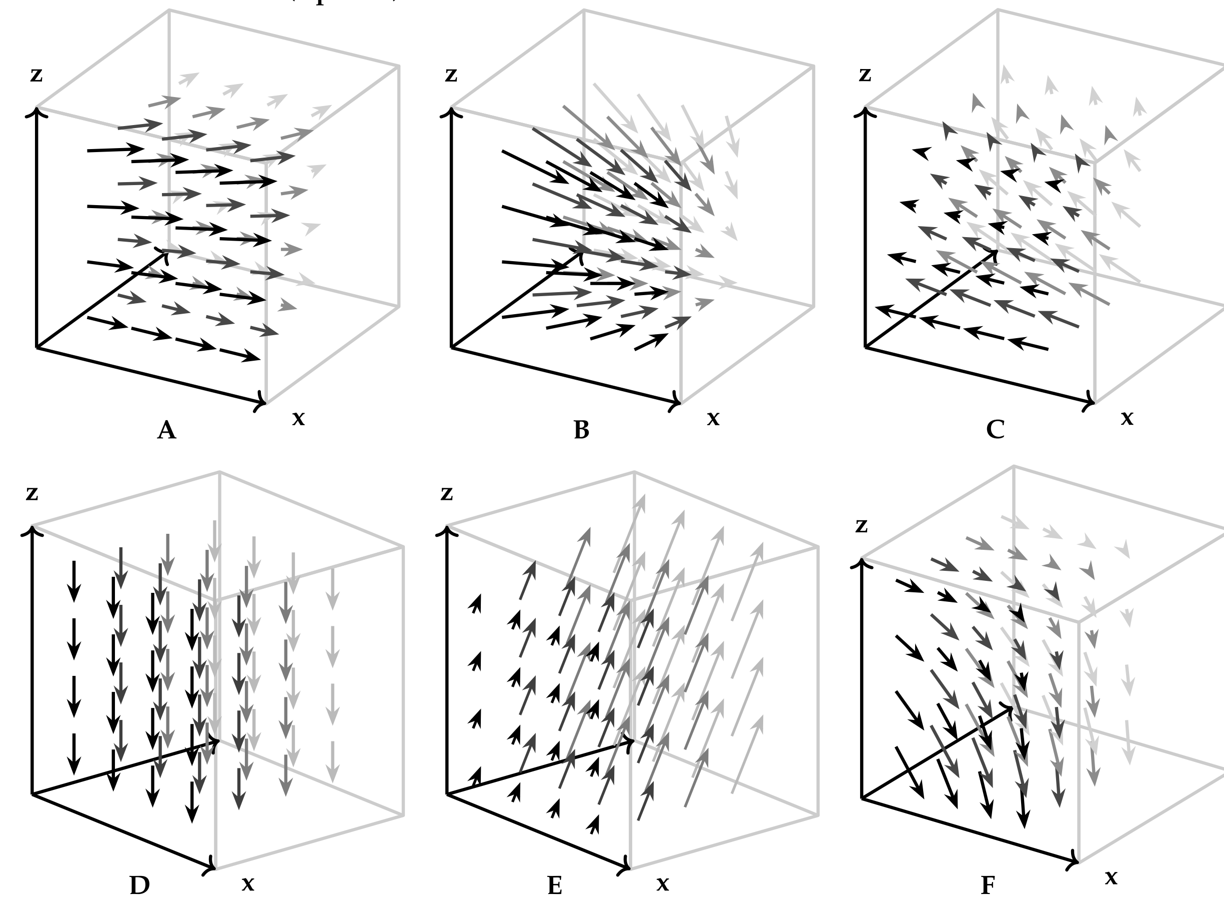
Question 5.
Consider the following vector fields. The x and z axes are labeled; the remaining unlabeled axis is the y-axis. For each part mark the best answer. **(5 points)**



(a) Exactly one of these vector fields has nonzero divergence. Which one?

A ◯ **B** ◯ **C** ◯ **D** ◯ **E** ◯ **F** ◯

What is the sign of the divergence of the vector field you just marked?

◯ positive ◯ zero ◯ negative ◯ positive at some points and negative at some points

(b) Is the vector fi you marked in part (a) equal to curl(**G**) for some vector field **G**?

◯ Yes ◯ No

(c) curl(**F**) is constant and is one of the following. What is its value?

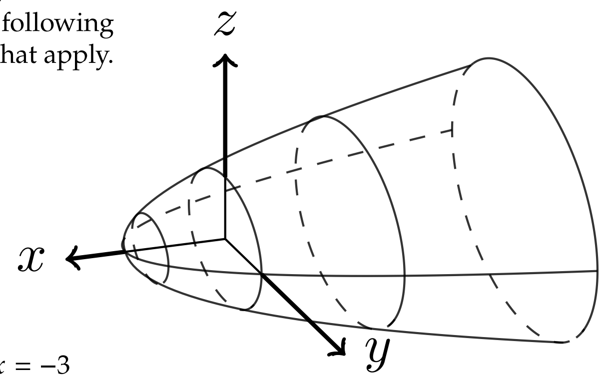
$-\mathbf{i}$ ◯ $-\mathbf{j}$ ◯ $-\mathbf{k}$ ◯ 0 ◯ $\mathbf{i}$ ◯ $\mathbf{j}$ ◯ $\mathbf{k}$ ◯

Question 6. Let $\mathbf{F} = \langle e^{x^2} + 3x^2y,\ 2x + y,\ z - 2x + y \rangle$.

(a) Compute curl($\mathbf{F}$). Mark your answer. **(2 points)**

$\langle 0, 2x+y, 0\rangle$	$\langle 1, -2, 2-3x^2\rangle$	$\langle -1, -2, -2+3x^2\rangle$	0	$\langle 1, 2, 2-3x^2\rangle$	$\langle -1, 2, -2+3x^2\rangle$	$\langle 0, -2x-y, 0\rangle$
◯	◯	◯	◯	◯	◯	◯

(b) Let S be the portion of the graph of $x = 1 - y^2 - z^2$ with $x \geq -3$, oriented outwards, that is, with outward pointing unit normal. Which of the following integrals have value equal to the flux of curl($\mathbf{F}$) through S? Mark all that apply. **(4 points)**



☐ $\int_0^{2\pi} \mathbf{F}($~~−2~~ −3$, \cos(t), \sin(t)) \cdot \langle 0, -\sin(t), \cos(t)\rangle \, dt$

☐ $\int_0^{2\pi} \mathbf{F}($~~−2~~ −3$, \cos(t), \sin(t)) \cdot \langle$~~−2~~ −3$, \cos(t), \sin(t)\rangle \, dt$

☐ $\iint_S \text{curl}(\mathbf{F}) \cdot \langle 1, 0, 0\rangle \, dS$

☐ $\iint_D \text{curl}(\mathbf{F}) \cdot \langle 1, 0, 0\rangle \, dS$, where D is the disc $y^2 + z^2 \leq 4$ with $x = -3$

☐ $\iint_D \text{curl}(\mathbf{F}) \cdot \langle 1, 0, 0\rangle \, dS$, where D is the disc $y^2 + z^2 \leq 1$ with $x = 0$

☐ $\iint_D \text{curl}(\mathbf{F}) \cdot \langle -1, 0, 0\rangle \, dS$, where D is the disc $y^2 + z^2 \leq 4$ with $x = -3$

☐ $\iint_D \text{curl}(\mathbf{F}) \cdot \langle -1, 0, 0\rangle \, dS$, where D is the disc $y^2 + z^2 \leq 1$ with $x = 0$

(c) Use one of the integrals you chose in part (b) to compute the flux of curl($\mathbf{F}$) through S. **(4 points)**

Flux = ☐

Question 7.

Let $\mathbf{F} = \langle P, Q \rangle$ where

$P(x,y) = \dfrac{2y}{x^2+y^2}$ and $Q(x,y) = 3 + \dfrac{-2x}{x^2+y^2}$.

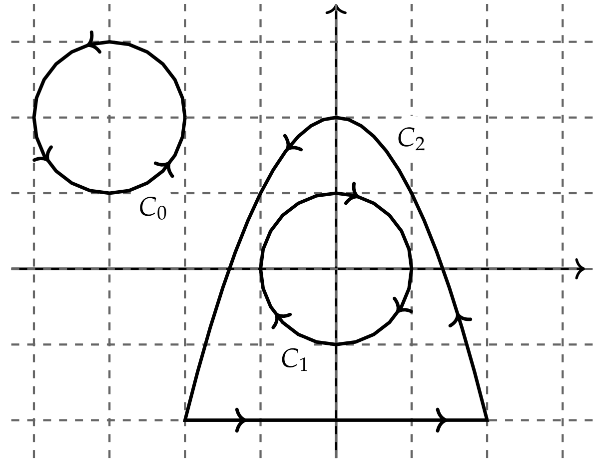
These satisfy

$$P_x = \frac{-4xy}{(x^2+y^2)^2}$$

$$P_y = \frac{2x^2-2y^2}{(x^2+y^2)^2}$$

$$Q_x = \frac{2x^2-2y^2}{(x^2+y^2)^2}$$

$$Q_y = \frac{4xy}{(x^2+y^2)^2}$$



(a) Let C_0 be the circle of radius 1 centered at $(-3,2)$ and oriented counterclockwise. Compute $\int_{C_0} \mathbf{F} \cdot d\mathbf{r}$.

Mark your answer. **(2 points)**

-6π	-4π	$3-2\pi$	0	$3+2\pi$	4π	6π
◯	◯	◯	◯	◯	◯	◯

(b) Let C_1 be the circle of radius 1 centered at $(0,0)$ and oriented clockwise. Compute $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$.

Mark your answer. **(2 points)**

-6π	-4π	$3-2\pi$	0	$3+2\pi$	4π	6π
◯	◯	◯	◯	◯	◯	◯

(c) Let C_2 be the closed curve consisting of the line segment from $(-2,-2)$ to $(2,-2)$ and the graph of $y = 2 - x^2$, oriented as displayed.

Which is the correct answer? **(1 point)**

$\left\lvert \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \right\rvert < \left\lvert \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \right\rvert$	$-\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$	$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$	$\left\lvert \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \right\rvert > \left\lvert \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \right\rvert$
◯	◯	◯	◯

Scratch Space