

Question 1.

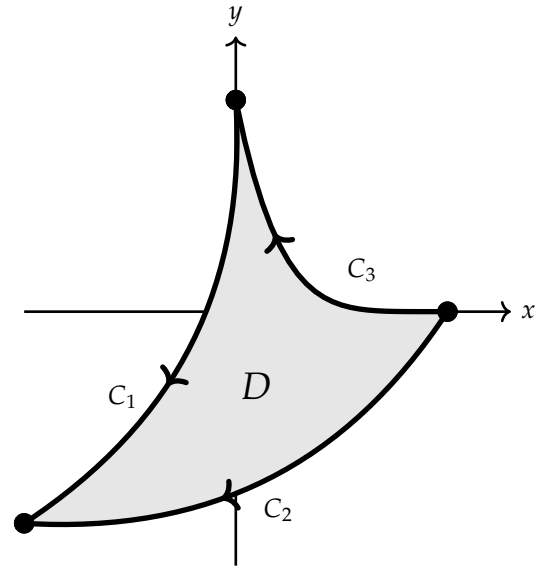
Let C_1 be the displayed path from $(0, 1)$ to $(-1, -1)$, C_2 the displayed path from $(1, 0)$ to $(-1, -1)$, and C_3 the displayed path from $(1, 0)$ to $(0, 1)$. Let D be the region bounded by the curves C_1 , C_2 , and C_3 and consider the vector field $\mathbf{F} = \langle 2 - 2x + 4y, 7x - y \rangle$. We know that

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 5,$$

$$\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 1, \text{ and}$$

$$\int_{C_3} \mathbf{F} \cdot d\mathbf{r} = -1.$$

Use Green's theorem and the vector field \mathbf{F} to compute the area of D . Mark your answer. Show your work. **(5 points)**

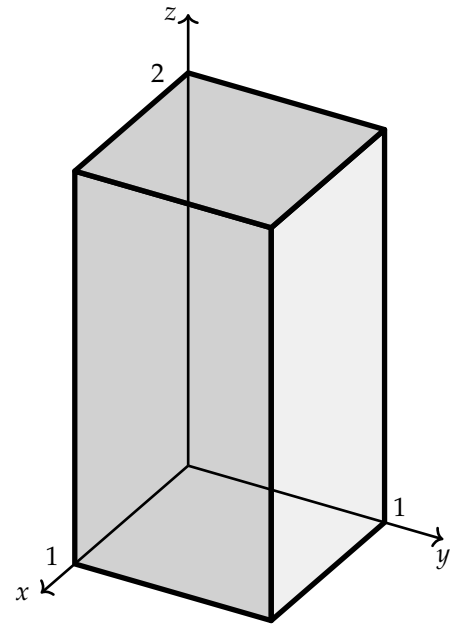


Area(D) =

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Question 2.

Let $\mathbf{H} = \langle 3x + 2z^3, 5, 2z - y \rangle$. Let E be the surface of the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$, oriented outwards, that is, with an outward pointing unit normal vector. Use the divergence theorem to compute the flux of \mathbf{H} through E . **(5 points)**



flux =

Question 3.

Find real numbers a and b so that $\mathbf{F}(x, y, z) = \langle e^{x^2} + ay, 2x + zy, by^2 \rangle$ is conservative. **(4 points)**

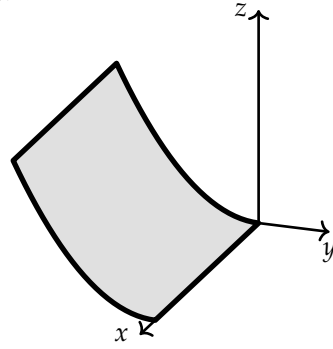
 $a =$ $b =$

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Question 4.

Let S be the surface which is the portion of the graph of $z = y^2$ where $0 \leq x \leq 1$ and $-1 \leq y \leq 0$, oriented in the direction of the positive y -axis, that is, with unit normal vector having non-negative second coordinate.

Let $\mathbf{F} = \langle x^2 + z, y, z \rangle$. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$. (6 points)

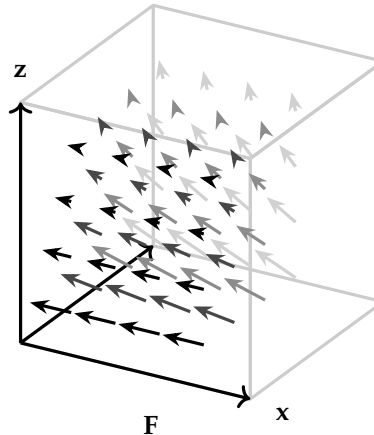
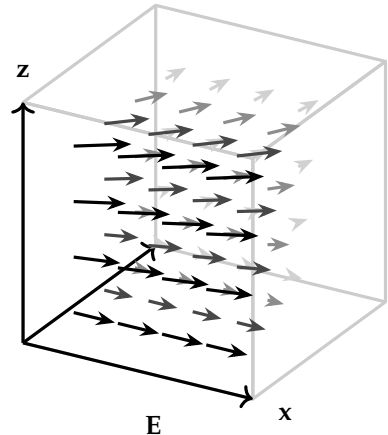
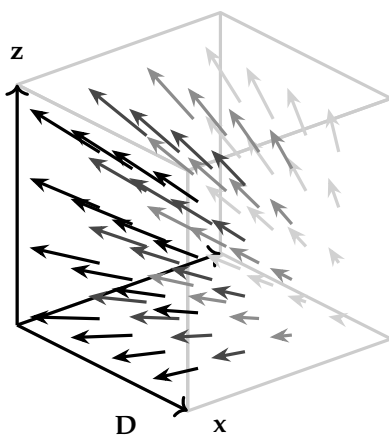
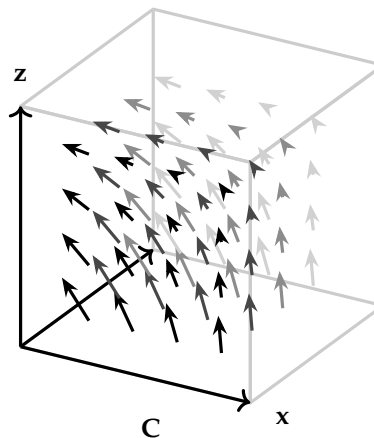
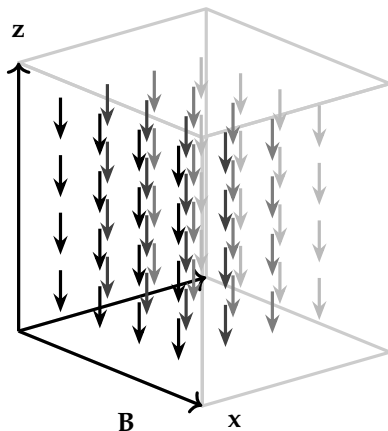
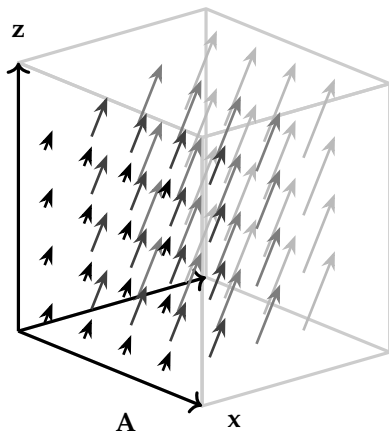


$$\iint_S \mathbf{F} \cdot d\mathbf{S} =$$

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Question 5.

Consider the following vector fields. The x and z axes are labeled; the remaining unlabeled axis is the y -axis. For each part mark the best answer. (5 points)



(a) Exactly one of these vector fields has nonzero divergence. Which one?

- ☐ A
 ☐ B
 ☐ C
 ☐ D
 ☐ E
 ☐ F

What is the sign of the divergence of the vector field you just marked?

- ☐ positive
 ☐ zero
 ☐ negative
 ☐ positive at some points and negative at some points

(b) Is the vector field you marked in part (a) equal to $\text{curl}(\mathbf{G})$ for some vector field \mathbf{G} ?

- ☐ Yes
 ☐ No

(c) $\text{curl}(\mathbf{C})$ is constant and is one of the following. What is its value?

- ☐ $-\mathbf{i}$
☐ $-\mathbf{j}$
☐ $-\mathbf{k}$
☐ 0
☐ \mathbf{i}
☐ \mathbf{j}
☐ \mathbf{k}

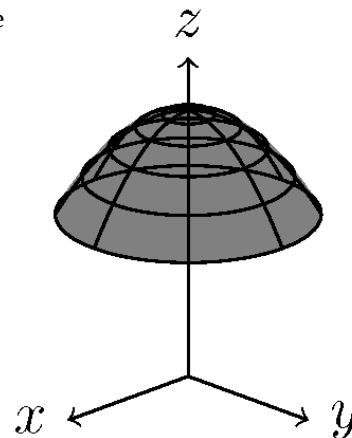
Question 6. Let $\mathbf{F} = \langle e^{x^2} + 2y, 4x, y^2z + x \rangle$.

(a) Compute $\text{curl}(\mathbf{F})$. Mark your answer. **(2 points)**

- ☐ $\langle 0, 0, y^2 \rangle$
☐ $\langle 2yz, -1, 2 \rangle$
☐ $\langle -2yz, -1, -2 \rangle$
☐ 0
☐ $\langle 2yz, 1, 2 \rangle$
☐ $\langle -2yz, 1, -2 \rangle$
☐ $\langle 0, 0, -y^2 \rangle$

(b) Let S be the portion of the graph of $z = 4 - x^2 - y^2$ with $z \geq 3$, oriented upwards, that is, with upward pointing unit normal. Which of the following integrals have value equal to the flux of $\text{curl}(\mathbf{F})$ through S ? Mark all that apply. **(4 points)**

- ☐ $\iint_S \text{curl}(\mathbf{F}) \cdot \langle 0, 0, 1 \rangle dS$
☐ $\iint_D \text{curl}(\mathbf{F}) \cdot \langle 0, 0, 1 \rangle dS$, where D is the disc $x^2 + y^2 \leq 1$ with $z = 3$
☐ $\iint_D \text{curl}(\mathbf{F}) \cdot \langle 0, 0, 1 \rangle dS$, where D is the disc $x^2 + y^2 \leq 2$ with $z = 0$
☐ $\iint_D \text{curl}(\mathbf{F}) \cdot \langle 0, 0, -1 \rangle dS$, where D is the disc $x^2 + y^2 \leq 1$ with $z = 3$
☐ $\int_0^{2\pi} \mathbf{F}(\cos(t), \sin(t), 3) \cdot \langle -\sin(t), \cos(t), 0 \rangle dt$
☐ $\int_0^{2\pi} \mathbf{F}(\cos(t), \sin(t), 3) \cdot \langle \cos(t), \sin(t), 3 \rangle dt$



(c) Use one of the integrals you chose in part (b) to compute the flux of $\text{curl}(\mathbf{F})$ through S . **(4 points)**

Flux =

Question 7.

Let $\mathbf{F} = \langle P, Q \rangle$ where

$$P(x, y) = \frac{y}{x^2 + y^2} \text{ and } Q(x, y) = 3 + \frac{-x}{x^2 + y^2}.$$

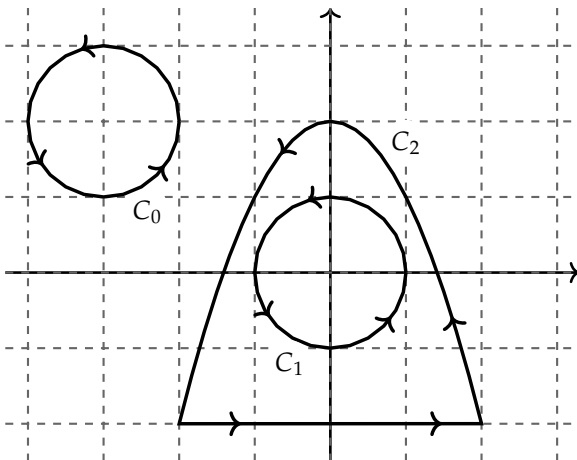
These satisfy

$$P_x = \frac{-2xy}{(x^2 + y^2)^2}$$

$$P_y = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$Q_x = \frac{x^2 - y^2}{(x^2 + y^2)^2}$$

$$Q_y = \frac{2xy}{(x^2 + y^2)^2}$$



- (a) Let C_0 be the circle of radius 1 centered at $(-3, 2)$ and oriented counterclockwise. Compute $\int_{C_0} \mathbf{F} \cdot d\mathbf{r}$.

Mark your answer. (2 points)

- ☐ 3π
☐ 2π
☐ $3 + \pi$
☐ 0
☐ $3 - \pi$
☐ -2π
☐ -3π

- (b) Let C_1 be the circle of radius 1 centered at $(0, 0)$ and oriented counterclockwise. Compute $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$.

Mark your answer. (2 points)

- ☐ 3π
☐ 2π
☐ $3 + \pi$
☐ 0
☐ $3 - \pi$
☐ -2π
☐ -3π

- (c) Let C_2 be the closed curve consisting of the line segment from $(-2, -2)$ to $(2, -2)$ and the graph of $y = 2 - x^2$, oriented as displayed.

Which is the correct answer? (1 point)

- ☐ $\left| \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \right| > \left| \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \right|$
☐ $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$
☐ $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = - \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$
☐ $\left| \int_{C_1} \mathbf{F} \cdot d\mathbf{r} \right| < \left| \int_{C_2} \mathbf{F} \cdot d\mathbf{r} \right|$

Scratch Space

