Question 1.

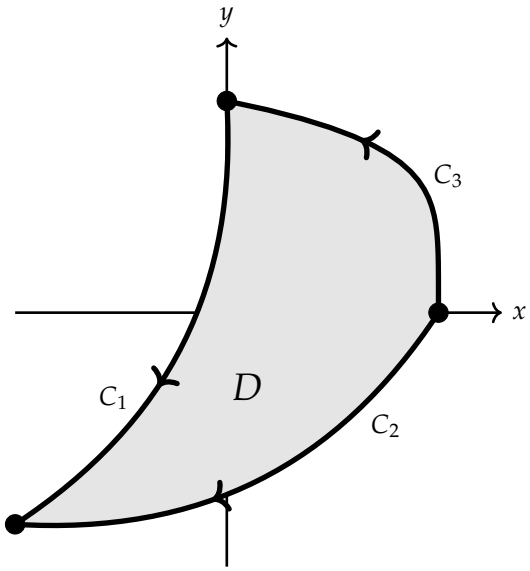
Let C_1 be the displayed path from $(0,1)$ to $(-1,-1)$, C_2 the displayed path from $(1,0)$ to $(-1,-1)$, and C_3 the displayed path from $(1,0)$ to $(0,1)$. Let D be the region bounded by the curves C_1, C_2, and C_3 and consider the vector field $\mathbf{F} = \langle 1 - 4x + 4y,\ 6x - 3y \rangle$. We know that

$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = 3,$

$\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = 1,$ and

$\int_{C_3} \mathbf{F} \cdot d\mathbf{r} = 2.$

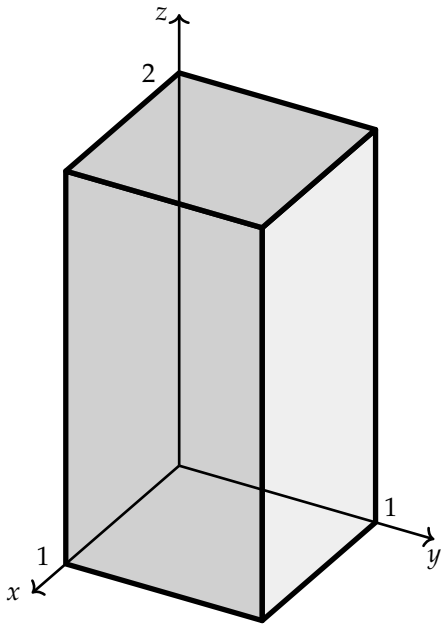
Use Green's theorem and the vector field $\mathbf{F}$ to compute the area of D. Mark your answer. Show your work. **(5 points)**



Area(D) =

Scratch Space

Question 2.
Let $\mathbf{H} = \langle 2x + 3z^2, -y, 3z + 2x \rangle$. Let E be the surface of the box bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, and $z = 2$, oriented outwards, that is, with an outward pointing unit normal vector. Use the divergence theorem to compute the flux of $\mathbf{H}$ through E. **(5 points)**



flux = $\boxed{}$

Question 3.
Find real numbers a and b so that $\mathbf{F}(x, y, z) = \langle e^{x^2} + az^2, 4z, 6xz + 2by \rangle$ is conservative. **(4 points)**

$a =$ $\boxed{}$

$b =$ $\boxed{}$

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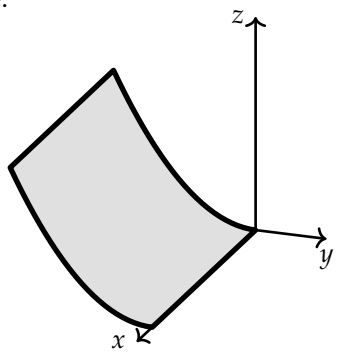
Question 4.

Let S be the surface which is the portion of the graph of $z = y^2$ where $0 \le x \le 1$ and $-1 \le y \le 0$, oriented in the direction of the positive y-axis, that is, with unit normal vector having non-negative second coordinate.

Let $\mathbf{F} = \langle z - x^2, -2y, -3z \rangle$. Compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$. **(6 points)**



$\iint_S \mathbf{F} \cdot d\mathbf{S} =$

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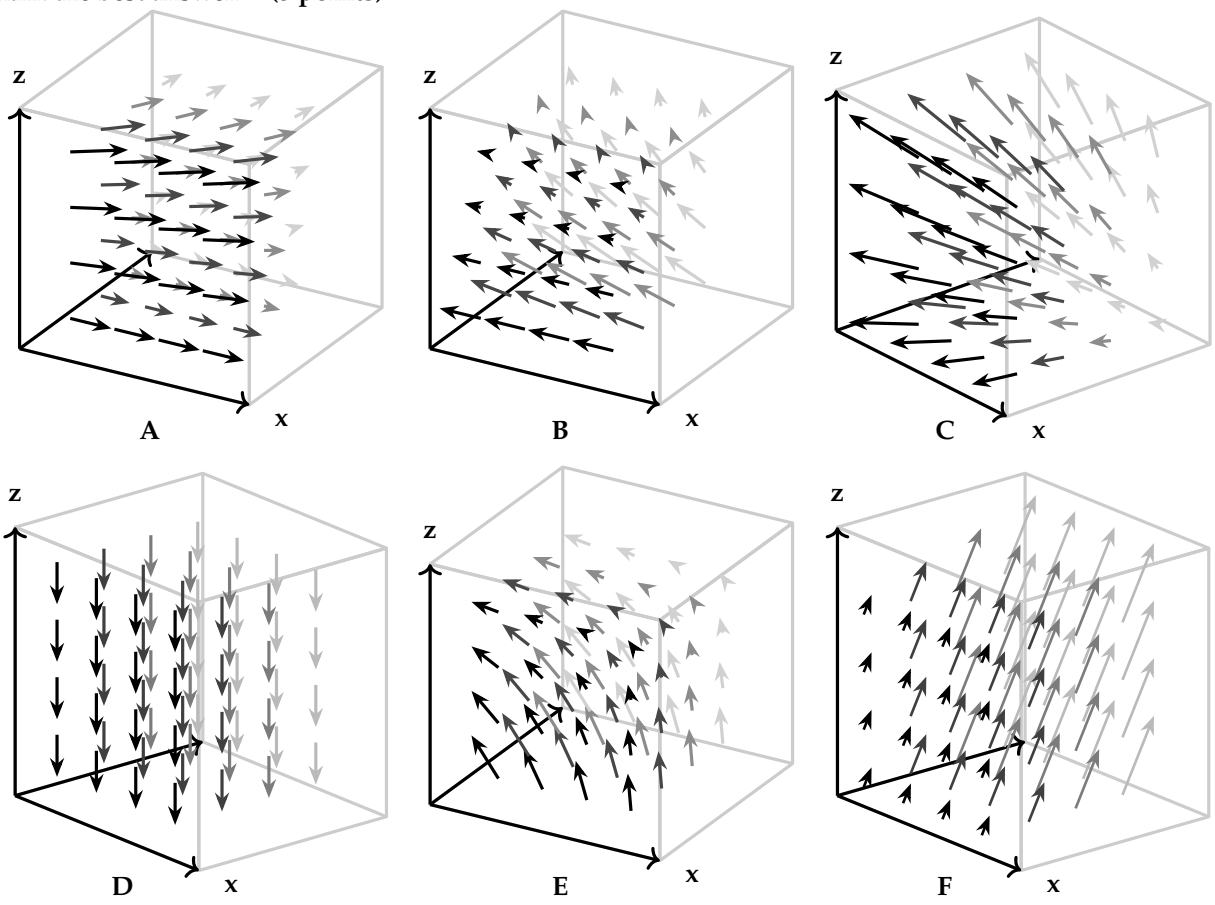
Question 5.

Consider the following vector fields. The x and z axes are labeled; the remaining unlabeled axis is the y-axis. For each part mark the best answer. **(5 points)**



(a) Exactly one of these vector fields has nonzero divergence. Which one?

A ◯ **B** ◯ **C** ◯ **D** ◯ **E** ◯ **F** ◯

What is the sign of the divergence of the vector field you just marked?

◯ positive ◯ zero ◯ negative ◯ positive at some points and negative at some points

(b) Is the vector field you marked in part (a) equal to curl(**G**) for some vector field **G**?

◯ Yes ◯ No

(c) curl(**E**) is constant and is one of the following. What is its value?

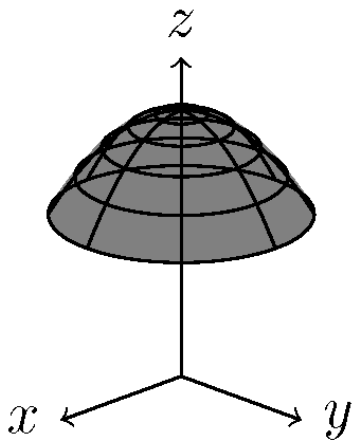
i ◯ **j** ◯ **k** ◯ 0 ◯ **−i** ◯ **−j** ◯ **−k** ◯

Question 6. Let $\mathbf{F} = \langle e^{x^2} + 2y + z,\ 5x,\ y^2 z \rangle$.

(a) Compute curl($\mathbf{F}$). Mark your answer. **(2 points)**

$\langle 0, 0, y^2 \rangle$ ◯ $\quad \langle 2yz, -1, 3 \rangle$ ◯ $\quad \langle -2yz, -1, -3 \rangle$ ◯ $\quad 0$ ◯ $\quad \langle 2yz, 1, 3 \rangle$ ◯ $\quad \langle -2yz, 1, -3 \rangle$ ◯ $\quad \langle 0, 0, -y^2 \rangle$ ◯

(b) Let S be the portion of the graph of $z = 4 - x^2 - y^2$ with $z \geq 3$, oriented upwards, that is, with upward pointing unit normal. Which of the following integrals have value equal to the flux of curl($\mathbf{F}$) through S? Mark all that apply. **(4 points)**



- ☐ $\displaystyle\int_0^{2\pi} \mathbf{F}\big(\cos(t),\ \sin(t),\ 3\big) \cdot \langle -\sin(t), \cos(t), 0 \rangle \, dt$
- ☐ $\displaystyle\int_0^{2\pi} \mathbf{F}\big(\cos(t),\ \sin(t),\ 3\big) \cdot \langle \cos(t),\ \sin(t),\ 3 \rangle \, dt$
- ☐ $\displaystyle\iint_S \text{curl}(\mathbf{F}) \cdot \langle 0,\ 0,\ 1 \rangle \, dS$
- ☐ $\displaystyle\iint_D \text{curl}(\mathbf{F}) \cdot \langle 0,\ 0,\ 1 \rangle \, dS$, where D is the disc $x^2 + y^2 \leq 1$ with $z = 3$
- ☐ $\displaystyle\iint_D \text{curl}(\mathbf{F}) \cdot \langle 0,\ 0,\ 1 \rangle \, dS$, where D is the disc $x^2 + y^2 \leq 2$ with $z = 0$
- ☐ $\displaystyle\iint_D \text{curl}(\mathbf{F}) \cdot \langle 0,\ 0,\ -1 \rangle \, dS$, where D is the disc $x^2 + y^2 \leq 1$ with $z = 3$

(c) Use one of the integrals you chose in part (b) to compute the flux of curl($\mathbf{F}$) through S. **(4 points)**

Flux = $\boxed{}$

Question 7.

Let $\mathbf{F} = \langle P, Q \rangle$ where

$P(x,y) = \dfrac{y}{x^2+y^2}$ and $Q(x,y) = 3 + \dfrac{-x}{x^2+y^2}$.

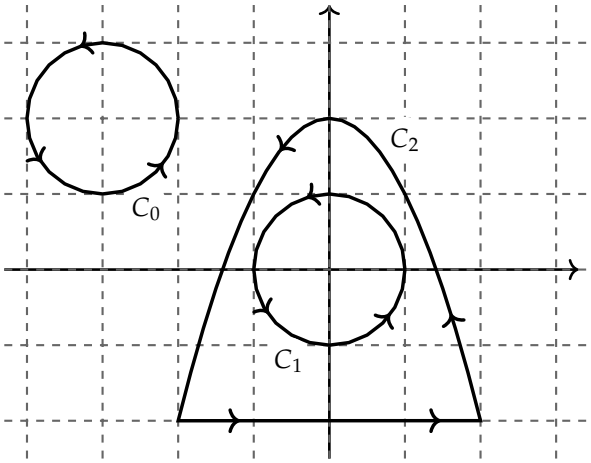
These satisfy

$$P_x = \frac{-2xy}{(x^2+y^2)^2}$$

$$P_y = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$Q_x = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$Q_y = \frac{2xy}{(x^2+y^2)^2}$$



(a) Let C_0 be the circle of radius 1 centered at $(-3,2)$ and oriented counterclockwise. Compute $\displaystyle\int_{C_0} \mathbf{F} \cdot d\mathbf{r}$.

Mark your answer. **(2 points)**

-3π ◯ -2π ◯ $3-\pi$ ◯ 0 ◯ $3+\pi$ ◯ 2π ◯ 3π ◯

(b) Let C_1 be the circle of radius 1 centered at $(0,0)$ and oriented counterclockwise. Compute $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$.

Mark your answer. **(2 points)**

-3π ◯ -2π ◯ $3-\pi$ ◯ 0 ◯ $3+\pi$ ◯ 2π ◯ 3π ◯

(c) Let C_2 be the closed curve consisting of the line segment from $(-2,-2)$ to $(2,-2)$ and the graph of $y = 2 - x^2$, oriented as displayed.

Which is the correct answer? **(1 point)**

$\left|\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r}\right| < \left|\displaystyle\int_{C_2} \mathbf{F} \cdot d\mathbf{r}\right|$ ◯ $\quad -\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ ◯ $\quad \displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ ◯ $\quad \left|\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r}\right| > \left|\displaystyle\int_{C_2} \mathbf{F} \cdot d\mathbf{r}\right|$ ◯

Scratch Space