Question 1 **(6 points)** Consider the points $A = (2, 1, 0)$, $B = (1, 1, 2)$, and $C = (2, 0, 1)$.

(a) A normal vector to the plane containing A, B, and C is:

- ○ $\langle -1, 2, -1\rangle$
- ○ $\langle 1, -2, 1\rangle$
- ✓ $\langle 2, 1, 1\rangle$
- ○ $\langle 2, -1, 1\rangle$
- ○ $\langle -1, 1, 2\rangle$
- ○ $\langle -1, 1, 2\rangle$

(b) The area of the triangle with vertices A, B, C is:

- ○ 2
- ○ $\sqrt{6}$
- ✓ $\sqrt{\frac{3}{2}}$
- ○ $\frac{\sqrt{2}}{2}$
- ○ $\frac{\sqrt{21}}{2}$
- ○ $\frac{\sqrt{5}}{2}$

(c) The angle θ between $\overrightarrow{AB}$ and $\overrightarrow{AC}$ satisfies:

- ○ $\theta = 0$
- ✓ $0 < \theta < \pi/2$
- ○ $\theta = \pi/2$
- ○ $\pi/2 < \theta < \pi$
- ○ $\theta = \pi$

Scratch Space

(a) a normal vector is given by $\overrightarrow{AB} \times \overrightarrow{AC}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 2 \\ 0 & -1 & 1 \end{vmatrix} = \langle 2, 1, 1\rangle$$

(b) $\text{area} = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\sqrt{4+1+1} = \sqrt{\frac{6}{4}} = \sqrt{\frac{3}{2}}$

(c) $\overrightarrow{AB} \cdot \overrightarrow{AC} = |\overrightarrow{AB}||\overrightarrow{AC}| \cos\theta = 0 + 0 + 2 > 0$

$\Rightarrow \cos\theta > 0$, so $0 \le \theta < \frac{\pi}{2}$.

$\theta \neq 0$ because $|\overrightarrow{AB} \times \overrightarrow{AC}| \neq 0$.

Question 2 **(4 points)** Consider the function $f(x, y)$ defined by

$$f(x,y) = \begin{cases} \dfrac{3x^2y}{x^4+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0). \end{cases}$$

Is f continuous at $(0, 0)$? Choose the best answer.

- [ ] $f(x, y)$ is continuous at $(0, 0)$.
- [x] $f(x, y)$ is **not** continuous at $(0, 0)$.

Select the reason which best supports your claim.

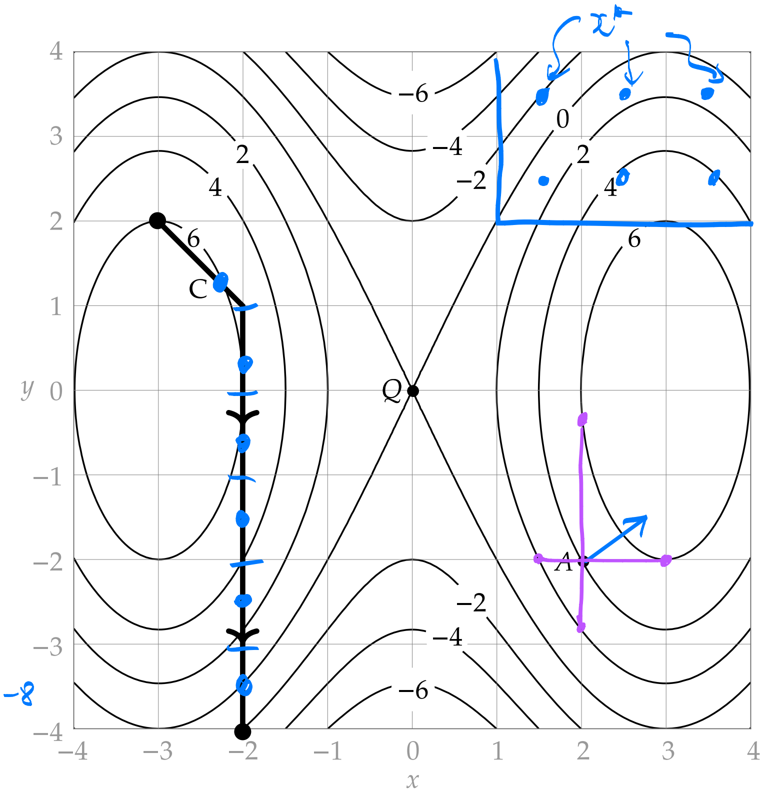
- [ ] The limit $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+y^2} = 0$, which can be seen by checking on any line of the form $ax = by$.
- [ ] The limit $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+y^2} = 0$, which can be seen by converting to polar coordinates.
- [ ] The limit $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+y^2}$ does not exist because the limits along the lines $x = 0$ and $y = 0$ are different.
- [x] The limit $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+y^2}$ does not exist because the limits along the curves $y = x^2$ and $y = 0$ are different.
- [ ] The limit $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^4+y^2}$ does not exist because the limits along the lines $y = x$ and $y = 0$ are different.

Scratch Space

$$\lim_{(x,0)\to(0,0)} \frac{3x^2 \cdot 0}{x^4+0^2} = 0$$

$$\lim_{(x,x^2)\to(0,0)} \frac{3x^2 \cdot x^2}{x^4+x^4} = \lim_{x\to 0} \frac{3x^4}{2x^4} = \frac{3}{2}.$$

Question 3 **(12 points)**
The contour plot of a differentiable function f is shown below. Additionally, a curve C from $(-3,2)$ to $(-2,-4)$ is drawn below. For each part, circle the best answer.



(a) Estimate $\int_2^4 \int_1^4 f(x,y)\,dx\,dy$:

−22   −9   −2   0   2   9   22

(b) Estimate $\int_C f\,ds$:

−29   −17   −6.5   0   6.5   17   29

(c) Let $\mathbf{F}$ be a force field. If $\mathbf{F} = \nabla f$, evaluate the work done by $\mathbf{F}$ to move a particle from $(-3,2)$ to $(-2,-4)$ along the curve C.

−8   −6   −4   0   4   6   8

work $= \int_C \nabla f \cdot d\vec{r} \overset{\text{FTLI}}{=} f(-2,-4) - f(-3,2) = -2-6 = -8$

(d) Evaluate $\int_C \operatorname{curl}(\nabla f)\cdot d\mathbf{r}$:

−16   −8   −4   0   4   8   16

(e) Estimate $\nabla f(A)$:

$\langle\sqrt2,1\rangle$ $\langle\sqrt2,-1\rangle$ $\langle-\sqrt2,-1\rangle$ $\langle-\sqrt2,1\rangle$ $\mathbf{0}$ $\langle1,\sqrt2\rangle$ $\langle1,-\sqrt2\rangle$ $\langle-1,-\sqrt2\rangle$ $\langle-1,\sqrt2\rangle$

(e) $\nabla f \perp$ to level curve, pointing in direction of greatest increase $\Rightarrow f_x(A), f_y(A) > 0$. restricts to options $\langle\sqrt2,1\rangle$ or $\langle 1,\sqrt2\rangle$. But f grows faster in x-direction $\Rightarrow f_x > f_y$

(f) At the point Q, the product $f_{xx}(Q)f_{yy}(Q)$ is

positive   zero   negative

$f_{xx}(Q) > 0$
$f_{yy}(Q) < 0$

or $f_{xy}(Q) = 0$ and $f_{xx}f_{yy} - f_{xy}^2 < 0$

Scratch Space

(a) $\int_2^4\int_1^4 f(x,y)\,dx\,dy \sim \sum_{1\times1\text{-squares}} f(x^*) = (-2)+1+1+1+4+4 = 9$

x^* sample point in square

(b) Split C into segments; on each, $\int_{C_i} f\,ds = (\text{length of } C_i)\cdot(\text{average of } f \text{ on } C_i) \sim (\text{length of } C_i)\cdot f(\text{sample point})$

$\rightsquigarrow \int_C f\,ds \sim \sqrt2\cdot 6 + 1\cdot6 + 1\cdot6 + 1\cdot5 + 1\cdot3 + 1\cdot0 = 29$

Question 4 (4 points) The function $g(x,y)$ describes the barometric pressure (in pounds per square inch, or psi) in a given region D in the plane, so that $g(x,y)$ is the pressure at position (x,y). A few values of g together with its rates of change are given in the following table. Assuming that g is differentiable, use this data and linear approximation to estimate the pressure at $(2.1, 3.7)$.

(x,y)	$g(x,y)$	$g_x(x,y)$	$g_y(x,y)$	$g_{xx}(x,y)$	$g_{xy}(x,y)$	$g_{yy}(x,y)$
$(2,4)$	2	-2	1	5	9	11
$(0.1,-0.3)$	8	-3	-6	7	-2	-4

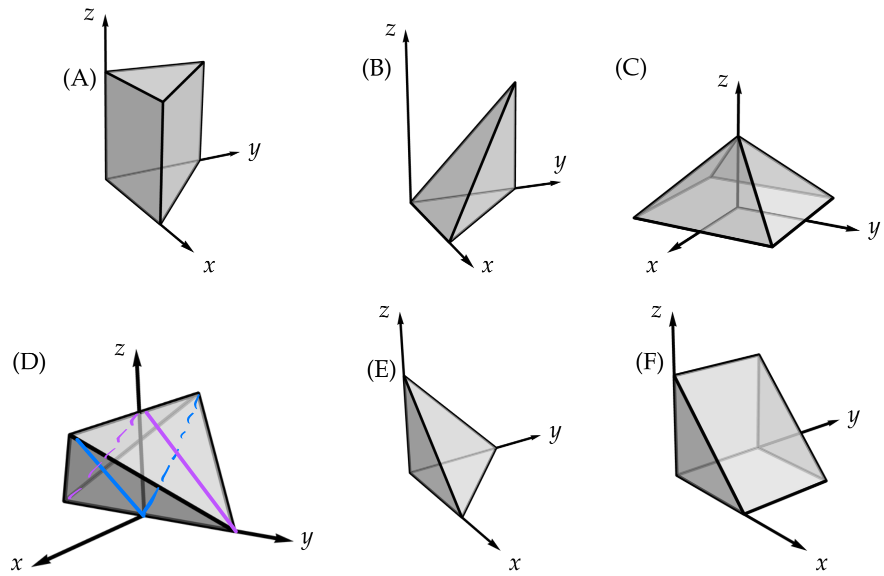
$$g(2.1,3.7) \sim g(2,4) + g_x(2,4)\cdot(2.1-2) + g_y(2,4)(3.7-4)$$

$$= 2 - 2\cdot 0.1 + 1\cdot(-0.3) = 2 - 0.2 - 0.3 = 1.5$$

$g(2.1, 3.7) \approx$ $\boxed{1.5}$

Scratch Space

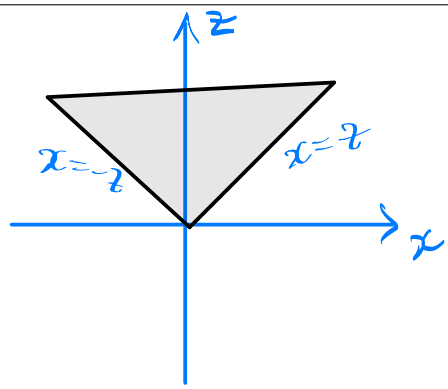
Question 5 (4 points)
Label each integral with its corresponding region of integration. Write your answer in the box next to the integral.

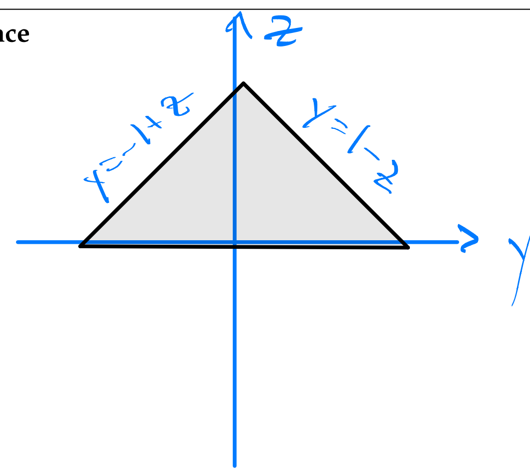


$\boxed{D}$ $\displaystyle\int_0^1 \int_{-z}^{z} \int_{-1+z}^{1-z} f(x,y,z)\, dy\, dx\, dz$

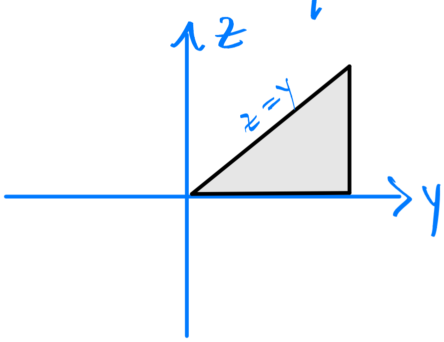
$\boxed{B}$ $\displaystyle\int_0^1 \int_0^{1-x} \int_z^{1-x} g(x,y,z)\, dy\, dz\, dx$

Scratch Space





$0 \le z \le y \le 1-x \le 1$



Question 6 **(5 points)**

Consider the differentiable function $f(x,y)$. The table lists values of partial derivatives of f at several points. For each of the listed points below determine whether it is a local minimum, local maximum, saddle point, or none of these. Mark your answer below.

(x,y)	$f_x(x,y)$	$f_y(x,y)$	$f_{xx}(x,y)$	$f_{yy}(x,y)$	$f_{xy}(x,y)$
$(0,0)$	0	0	2	4	0
$(-1,0)$	0	1	-6	-6	0
$(0,1)$	0	0	0	0	-5

$(0,0)$ is: ○ not a critical point ✓ a local minimum ○ a local maximum ○ a saddle point

$(-1,0)$ is: ✓ not a critical point ○ a local minimum ○ a local maximum ○ a saddle point

$(0,1)$ is: ○ not a critical point ○ a local minimum ○ a local maximum ✓ a saddle point

---

Scratch Space

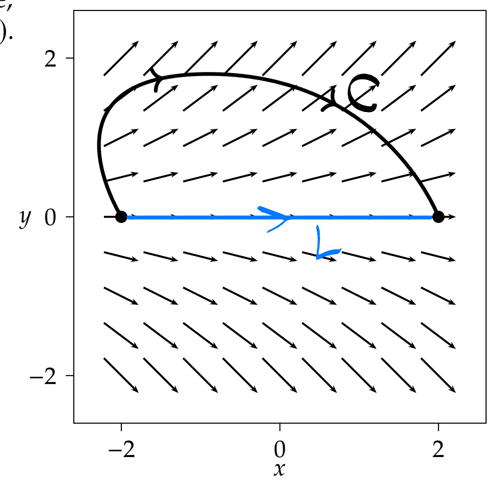
$D(0,0) = f_{xx}(0,0)\, f_{yy}(0,0) - f_{xy}(0,0)^2 = 2\cdot 4 - 0 > 0$

$f_{xx} > 0$

$D(0,1) = f_{xx}(0,1)\, f_{yy}(0,1) - f_{xy}(0,1)^2 = 0 - (-5)^2 < 0$

Question 7 **(2 points)**

A conservative vector field $\mathbf{F}$ is shown on the right. For scale, $|\mathbf{F}(0,0)| = \frac{1}{2}$. Let C be the indicated path from $(-2,0)$ to $(2,0)$. Estimate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.



$-\pi$	-2	$\frac{\pi}{2}$	0	$\frac{\pi}{2}$	2	π
○	○	○	○	○	✓	○

Scratch Space

let L = line segment $(-2,0)$ to $(2,0)$.

FTLI $\Rightarrow \displaystyle\int_C \vec{F} \cdot d\vec{r} = \int_L \vec{F} \cdot d\vec{r} = \int_L \vec{F} \cdot \vec{T}\, ds =$

unit tangent to L, $\vec{T} = \langle 1, 0 \rangle$

along L, $\vec{F}$ is constant & equals $\vec{F}(0,0) = \langle \frac{1}{2}, 0 \rangle$

$$\Rightarrow \int_C \vec{F} \cdot d\vec{r} = \int_L \langle \tfrac{1}{2}, 0 \rangle \cdot \langle 1, 0 \rangle\, ds = \frac{1}{2}\underbrace{(\text{length of } L)}_{4} = 2$$

Question 8 (4 points)

Let S be the sphere $u^2 + v^2 + w^2 = 4$ of radius 2 and let E be the ellipsoid $x^2 + \frac{(y-2)^2}{4} + \frac{z^2}{9} = 1$. Find a transformation T that takes the sphere S to the ellipsoid E.

$$u^2 + v^2 + w^2 = 4 \iff \left(\frac{u}{2}\right)^2 + \left(\frac{v}{2}\right)^2 + \left(\frac{w}{2}\right)^2 = 1$$

$$\rightsquigarrow \quad x = \frac{u}{2}, \quad \frac{y-2}{2} = \frac{v}{2}, \quad \frac{z}{3} = \frac{w}{2}$$

$$T(u, v, w) = \boxed{\left\langle \frac{u}{2}, \quad v+2, \quad \frac{3}{2}w \right\rangle}$$

Scratch Space

Question 9 **(4 points)** Let C be the curve parameterized by $\mathbf{r}(t) = t^2\mathbf{i} + 4e^{3(t-1)}\mathbf{j} + t\mathbf{k}$. Let P be the point of intersection of C and the plane $z = 1$. Find a vector equation for the tangent line to C at the point P.

P on C has $z = t = 1$ ⤳ $P = \vec{r}(1) = \langle 1, 4, 1 \rangle$.

$r'(t) = \langle 2t, 4 \cdot 3e^{t-1}, 1 \rangle$

$r'(1) = \langle 2, 4, 1 \rangle$ ⟵ tangent vector

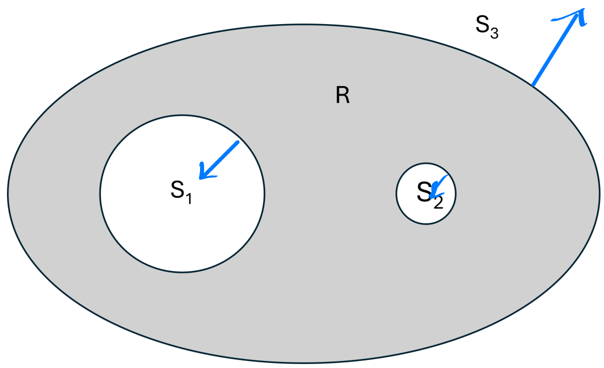
Tangent line: $\boxed{\langle 1 + 2t \; , 4 + 4t \; , 1 + t \rangle}$

Scratch Space

Question 10 **(2 pts)** Consider the following surfaces:

- S_1 is the sphere centered at the origin with radius 3; $S_1 = \{\langle x, y, z\rangle : x^2 + y^2 + z^2 = 9\}$, oriented outwards.
- S_2 is the sphere centered at $\langle 5, 0, 0\rangle$ with radius 1; $S_2 = \{\langle x, y, z\rangle : (x-5)^2 + y^2 + z^2 = 1\}$, oriented outwards.
- $S_3 = \left\{\langle x, y, z\rangle : (x-2)^2 + 4y^2 + 4z^2 = 400\right\}$, oriented outwards.

Suppose that R is the region inside the ellipsoid S_3 but outside the spheres S_1 and S_2.
See the figure for a cross section (not to scale).



Suppose that $\mathbf{E}$ is a vector field on R with continuous first derivatives such that $\operatorname{div}(\mathbf{E}) = 0$, and that

- The flux of $\mathbf{E}$ through S_1 is 6
- The flux of $\mathbf{E}$ through S_3 is 10.

What is the flux of $\mathbf{E}$ through S_2?

-16	-10	-4	0	4	10	16
○	○	○	○	✓	○	○

Scratch Space

$\partial R = S_3(\text{outward orient.}) + S_1(\text{inward orient.}) + S_2(\text{inward orient.})$

$= S_3 - S_1 - S_2$

Divergence theorem $\Rightarrow$

$$0 = \iiint_R \operatorname{div}\vec{E}\, dV = \iint_{\partial R} \vec{E}\cdot d\vec{S} = \iint_{S_3} \vec{E}\cdot d\vec{S} - \iint_{S_2} \vec{E}\cdot d\vec{S} - \iint_{S_1} \vec{E}\cdot d\vec{S} =$$

$$= 10 - \iint_{S_2} \vec{E}\cdot d\vec{S} - 6$$

$$\Rightarrow \iint_{S_2} \vec{E}\cdot d\vec{S} = 4.$$

Question 11 **(4 points)** Let D be all of $\mathbb{R}^2$ **except** the points $(1,2)$ and $(3,4)$. Assume $\mathbf{F}(x,y) = \langle P(x,y), Q(x,y)\rangle$ is a vector field defined on D. Assume that P and Q have continuous first partial derivatives on D.

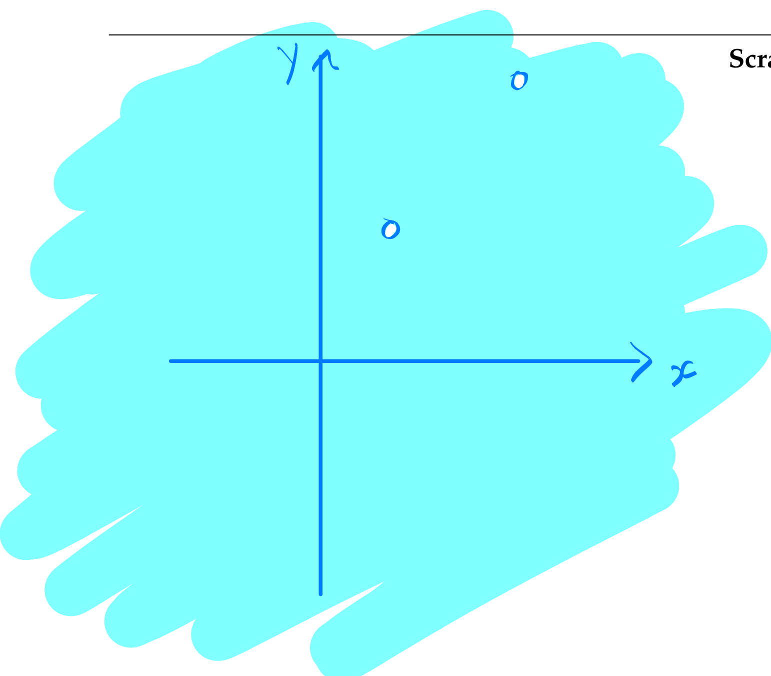
(a) **(2 points)** Which of the following statements about D is correct? (Mark all that apply.)

- [x] D is open
- [x] D is connected
- [ ] D is simply connected
- [ ] D is bounded

(b) **(2 points)** Assume $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$. Which of the following statements about $\mathbf{F}$ on the region D are correct? (Mark exactly one option.)

- ○ $\mathbf{F}$ must be conservative
- ○ $\mathbf{F}$ cannot be conservative
- ● $\mathbf{F}$ may or may not be conservative

Scratch Space



Question 12 (4 points) Let V be the solid lying *above* the surface $z = 9(x^2 + y^2)$, *below* the plane $z = 49$ and *outside* of the cylinder $x^2 + y^2 = 4$. In cylindrical coordinates, the mass of V is computed by an integral of the form

$$\int_f^e \int_d^c \int_b^a g(r, \theta, z)\, d?\, d?\, d?$$

(a) Mark the integral below with the correct bounds of integration. Pay attention to the given orders of integration.

- ○ $\int_0^{2\pi} \int_{36}^{49} \int_4^{z/3} g(r,\theta,z)\, dr\, dz\, d\theta$
- ○ $\int_0^{2\pi} \int_0^{49} \int_0^{z/3} g(r,\theta,z)\, dz\, dr\, d\theta$
- ○ $\int_0^{2\pi} \int_0^{49} \int_0^{z/3} g(r,\theta,z)\, dr\, dz\, d\theta$
- ○ $\int_0^{2\pi} \int_0^{7} \int_4^{\sqrt{z}/3} g(r,\theta,z)\, dz\, dr\, d\theta$
- ✓ $\int_0^{2\pi} \int_{36}^{49} \int_2^{\sqrt{z}/3} g(r,\theta,z)\, dr\, dz\, d\theta$
- ○ $\int_0^{2\pi} \int_4^{z/\sqrt{3}} \int_{9r^2}^{4} g(r,\theta,z)\, dz\, dr\, d\theta$
- ○ $\int_0^{2\pi} \int_{36}^{49} \int_0^{\sqrt{z}/9} g(r,\theta,z)\, dr\, dz\, d\theta$
- ○ $\int_0^{2\pi} \int_0^{z/9} \int_0^{4} g(r,\theta,z)\, dz\, dr\, d\theta$

(b) Mark the integral with correct integrand if the mass density is $\rho(x, y, z) = x^2 y$.

- ○ $\int_f^e \int_d^c \int_b^a r \cos\theta \sin\theta\, d\theta\, dr\, dz$
- ○ $\int_f^e \int_d^c \int_b^a r^2 \cos\theta \sin\theta\, d\theta\, dr\, dz$
- ○ $\int_f^e \int_d^c \int_b^a \cos^2\theta \sin\theta\, d\theta\, dr\, dz$
- ✓ $\int_f^e \int_d^c \int_b^a r^4 \cos^2\theta \sin\theta\, d\theta\, dr\, dz$
- ○ $\int_f^e \int_d^c \int_b^a r^3 \cos^2\theta \sin\theta\, d\theta\, dr\, dz$
- ○ $\int_f^e \int_d^c \int_b^a r^4 \cos^2\theta \sin^2\theta\, d\theta\, dr\, dz$

Scratch Space

(a) $z \geq 9r^2$ (above), $z \leq 49$, $r^2 \geq 4$ (outside)

$\leadsto$ $36 \leq 9r^2 \leq z \leq 49$

$2 \leq r \leq \frac{\sqrt{z}}{3}$

(b) $\rho = x^2 y = r^2\cos^2\theta \cdot r \sin\theta$

integrand $= r^3 \cos^2\theta \sin\theta \cdot r$

Question 13 **(9 points)** Let $\mathbf{F} = \langle z - xy,\ z^2,\ yz - 3x\rangle$.

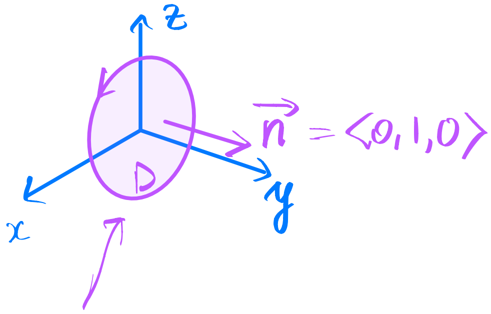
(a) Compute curl($\mathbf{F}$). Mark your answer.

- ○ $\langle 3z, 2, -x\rangle$
- ○ $\langle -z, -4, x\rangle$
- ○ $\langle -y, 2, 0\rangle$
- ○ $\langle 0, 0, 0\rangle$
- ✓ $\langle -z, 4, x\rangle$
- ○ $\langle z, -2, x\rangle$
- ○ $\langle 0, 2, -x\rangle$

(b) Compute div($\mathbf{F}$).

- ○ $-2y$
- ○ $-y - z$
- ○ $1 - x - z$
- ✓ 0
- ○ $x + y + z$
- ○ $xy + yz$
- ○ $2y$

(c) Let C be the curve $x^2 + z^2 = 4$ oriented counterclockwise in the xz-plane, as viewed from the positive y-axis. Use Stokes's Theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$.



D = disk $x^2+z^2 \le 4$
in x-z plane,
with $\vec{n} = \langle 0,1,0\rangle$

$$\int_C \vec{F}\cdot d\vec{r} = \iint_D \operatorname{curl}\vec{F}\cdot\vec{n}\, dS$$

$$= \iint_D \langle -z, 4, x\rangle \cdot \langle 0,1,0\rangle\, dS =$$

$$= 4 \cdot \text{area}(D) = 4 \cdot 2^2 \cdot \pi$$

$\int_C \mathbf{F} \cdot d\mathbf{r} =$ $\boxed{16\pi}$

Scratch Space

(a) $\operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ z-xy & z^2 & yz-3x \end{vmatrix} = \langle z - 2z, -(-3-1), 0-(-x)\rangle$

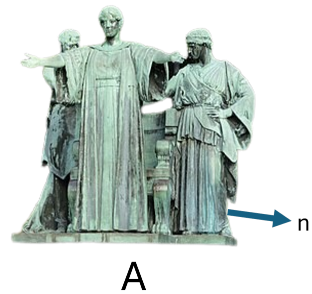
$= \langle -z, 4, x\rangle$

(b) $\operatorname{div}\vec{F} = -y + 0 + y = 0$

Question 14 (8 points)

Let A be the surface of the Alma Mater and let C be the outside of a Coffee Cup, with unit normals oriented outwards, as pictured. The boundary of C is the unit circle D in the yz-plane and A has no boundary.

Let $\mathbf{F} = \langle e^{-x}, z, x \rangle$

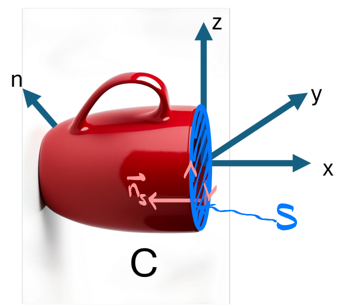


(a) The integral $\iint_A \mathbf{F} \cdot d\mathbf{S}$ is: $\text{div}\,\vec{F} = -e^{-x} < 0$

positive ◯ zero ◯ negative ✓

(b) The flux of curl($\mathbf{F}$) = $\langle -1, -1, 0 \rangle$ across A is: $\text{div}(\text{curl}\,\vec{F}) = 0$

positive ◯ zero ✓ negative ◯



(c) Orient the circle D in a *clockwise* direction as viewed from the right.

The integral $\int_D \mathbf{F} \cdot d\mathbf{r}$ is:

-2π ◯ $-\pi$ ◯ -2 ◯ 0 ◯ 2 ◯ π ✓ 2π ◯

(d) Let $\mathbf{G} = \langle 4, x + z, -2 \rangle$. The flux $\iint_C \mathbf{G} \cdot d\mathbf{S}$ is:

-4π ✓ -2π ◯ $-\pi$ ◯ -4 ◯ -2 ◯ 0 ◯ 2 ◯ 4 ◯ π ◯ 2π ◯ 4π ◯

Scratch Space

(c) let S = unit disk in y-z-plane, oriented to the left, i.e. with normal $\vec{n}_S = \langle -1, 0, 0 \rangle$. Then by Stokes,

$$\int_D \vec{F} \cdot d\vec{r} = \iint_S \text{curl}\,\vec{F} \cdot d\vec{S} = \iint_S \langle -1, -1, 0 \rangle \cdot \langle -1, 0, 0 \rangle \, dS = \text{area}(S) = \pi$$

(d)
$$\iint_C \vec{G} \cdot d\vec{S} \underset{\text{Stokes (or divergence) thm}}{=} \iint_S \vec{G} \cdot d\vec{S} = \iint_S \langle 4, x+z, -2 \rangle \cdot \langle -1, 0, 0 \rangle \, dS = -4\,\text{area}(S)$$

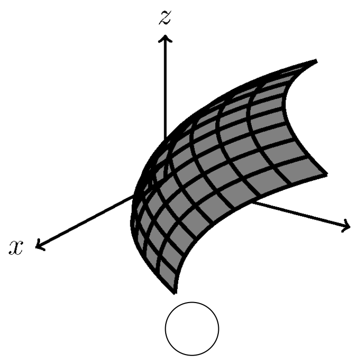
Question 15 **(8 points)** Consider the surface S parameterized by $\mathbf{r}(u,v) = \langle u, u^2 + 2v, 2v \rangle$ with domain

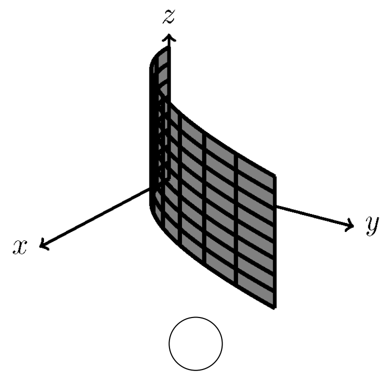
$$D = \{0 \le u \le 1,\ 0 \le v \le 1\}.$$

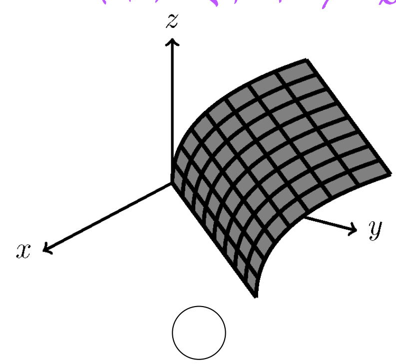
$\vec{r}(u,0) = \langle u, u^2, 0 \rangle$: parabola $y = x^2, z=0$

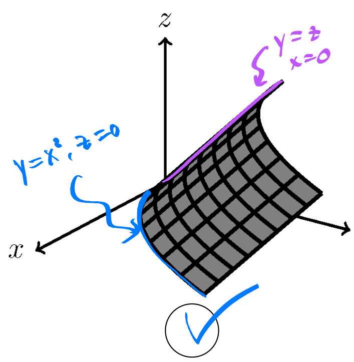
$\vec{r}(0,v) = \langle 0, 2v, 2v \rangle$: line $x=0, y=z$

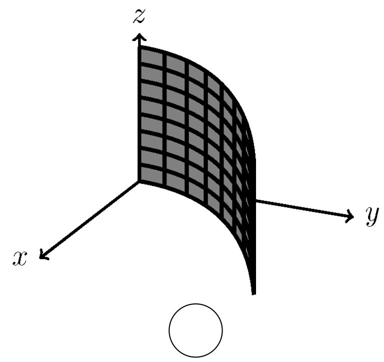
(a) Mark the picture below which corresponds to S.

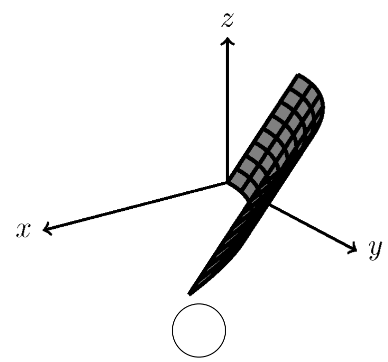












(b) Which one of the following integrals computes the surface area of S?

$\iint |\vec{r}_u \times \vec{r}_v| \, du\, dv$ (domain of parametrization)

- ○ $\int_0^1 \int_0^1 1\, du\, dv$
- ○ $\int_0^1 \int_0^1 \sqrt{10}\, du\, dv$
- ○ $\int_0^1 \int_0^1 \sqrt{4u^2+4}\, du\, dv$
- ○ $\int_0^1 \int_0^1 2u\, du\, dv$
- ✓ $\int_0^1 \int_0^1 2\sqrt{4u^2+2}\, du\, dv$
- ○ $\int_0^1 \int_0^1 \sqrt{16u^2+4}\, du\, dv$

$\vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2u & 0 \\ 0 & 2 & 2 \end{vmatrix} = \langle 4u, -2, 2 \rangle$

$|\vec{r}_u \times \vec{r}_v| = \sqrt{16u^2+4+4} = 2\sqrt{4u^2+2}$

(c) Orient S in the direction of the positive y-axis, that is, with a unit normal vector $\mathbf{n}$ whose second component is positive. Which one of the following integrals computes the flux of $\mathbf{F} = \langle x, 3, z^2 \rangle$ across S? Mark the correct answer.

- ○ $\int_0^1 \int_0^1 (4u^2 + 3v^2)\, du\, dv$
- ○ $\int_0^1 \int_0^1 (4u^2 - 6 + 8v^2)\, du\, dv$
- ✓ $\int_0^1 \int_0^1 (-4u^2 + 6 - 8v^2)\, du\, dv$
- ○ $\int_{\partial S} \langle x^2, 3, z^2 \rangle \cdot d\mathbf{r}$
- ○ $\int_0^1 \int_0^1 (-4u^2 - 3v^2)\, du\, dv$
- ○ $-\int_{\partial S} \langle x^2, 3, z^2 \rangle \cdot d\mathbf{r}$

(d) For the vector field $\mathbf{G} = \langle x^2, 3, z^2 \rangle$, what is the sign of $\iint_S \operatorname{div} \mathbf{G}\, dS$? Mark your answer.

$\operatorname{div} \vec{G} = 2x + 2z \ge 0$ since $x, z \ge 0$ on S.

$\iint_S \operatorname{div} \mathbf{F}\, dS$ is: positive ✓ ○ zero ○ negative ○

Scratch Space

$\vec{r}_u \times \vec{r}_v = \langle 4u, -2, 2 \rangle$. negative y-component

$\text{Flux} = \int_0^1\int_0^1 \underbrace{\langle x(u,v), 3, z(u,v)^2 \rangle}_{\langle u, 3, 4v^2 \rangle} \cdot \underbrace{(-\vec{r}_u \times \vec{r}_v)}_{\langle -4u, 2, -2 \rangle} \, du\, dv = \int_0^1\int_0^1 -4u^2 + 6 - 8v^2$