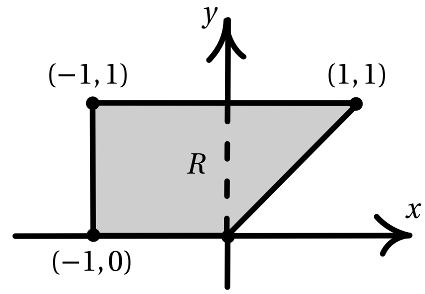
1. For the region R at right, evaluate $\iint_R 2x\,dA$. **(4 points)**



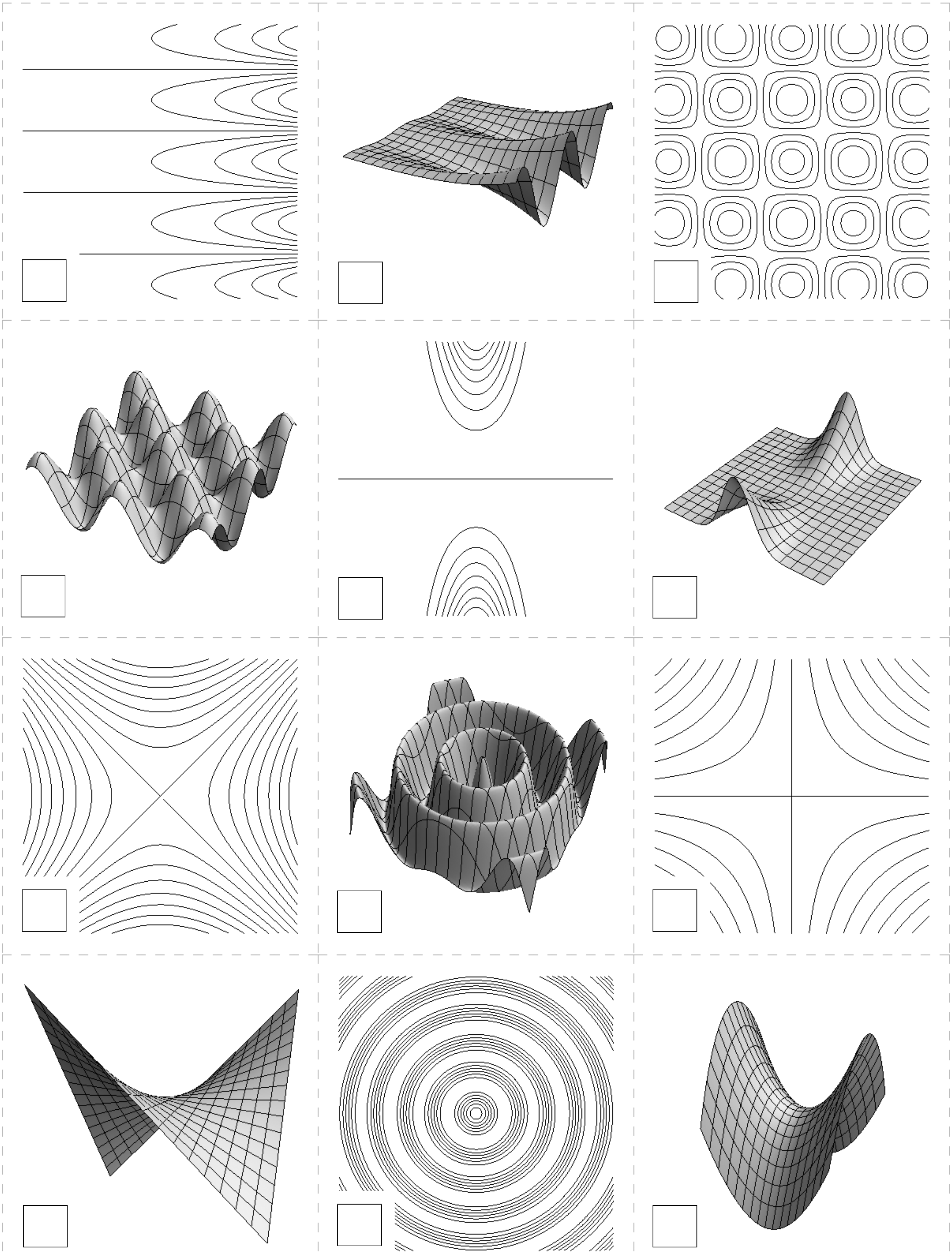
$\iint_R 2x\,dA =$

2. Consider the ellipse C given by $x^2 - xy + y^2 = 1$. Find all the points on C which are closest to the origin. **(6 points)**

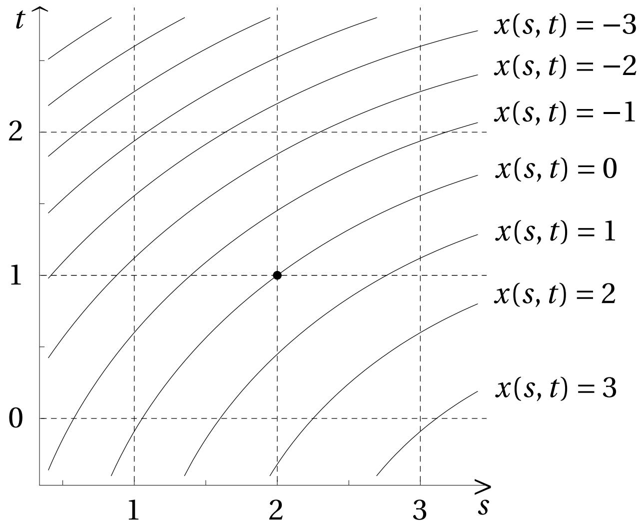
Closest points:

3. For each function: (a) xy (b) $\cos\left(\sqrt{x^2+y^2}\right)$ (c) $e^x \cos y$

label its graph and its level set diagram from among the options below. Here each level set diagram consists of level sets $\{f(x,y)=c_i\}$ drawn for evenly spaced c_i. **(9 points)**



4. Let $x(s,t)$ be the function whose contour plot is shown at right.



(a) Estimate $\dfrac{\partial x}{\partial t}(2,1)$ and circle the closest number below. **(2 points)**

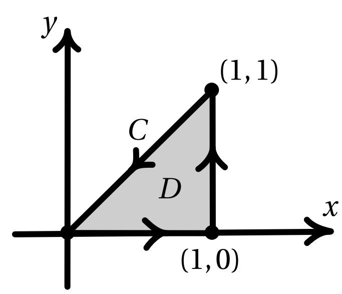
$\dfrac{\partial x}{\partial t}(2,1)$:	-6	-4	-2	0	2	4	6

(b) Let $g(x,y)$ be a function with the table of values and partial derivatives shown below and let $y(s,t) = s+t$. For $G(s,t) = g\big(x(s,t), y(s,t)\big)$, compute $\dfrac{\partial G}{\partial t}(2,1)$. **(4 points)**

(x,y)	$g(x,y)$	$\dfrac{\partial g}{\partial x}$	$\dfrac{\partial g}{\partial y}$
$(0,3)$	0	3	6
$(0,1)$	2	-3	-1
$(2,1)$	3	4	7
$(3,3)$	1	3	5

$\dfrac{\partial G}{\partial t}(2,1) =$

5. (a) For the curve C at right, directly compute $\displaystyle\int_C y\,dx + 3x\,dy$. **(5 points)**

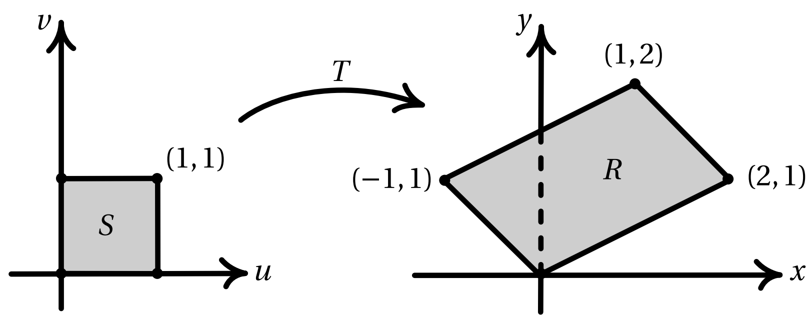


$\displaystyle\int_C y\,dx + 3x\,dy =$

(b) Check your answer in (a) using Green's Theorem. **(2 points)**

6. Let E be the tetrahedron in $\mathbb{R}^3$ with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,2)$. Setup but do not evaluate a triple integral that computes the volume of E. **(5 points)**

7. (a) Let R be the region shown. Find a transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ taking $S = [0,1] \times [0,1]$ to R. **(3 points)**



$T(u, v) = \Big(\qquad , \qquad \Big)$

(b) Use your change of coordinates to evaluate $\iint_R x\,dA$ via an integral over S. **(5 points)**

Emergency backup transformation: If you can't do (a), pretend you got the answer $T(u,v) = (uv, v)$ and do part (b) anyway.

$\iint_R x\,dA =$

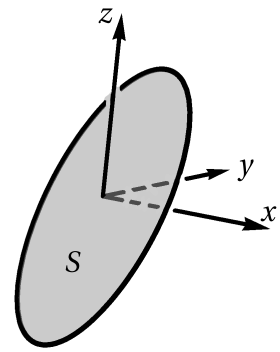
8. Let R be the portion of the cylinder $x^2 + y^2 \leq 1$ which lies in the octant where $\{x \geq 0, y \geq 0, z \geq 0\}$ and lies below the cone $z = 1 + \sqrt{x^2 + y^2}$. For the density $\rho = 6z$, compute the total mass of R. **(7 points)**

Mass =

9. Let $\mathbf{F}(x, y, z) = \left\langle \frac{x^3}{3}, x^2\cos(z) + \frac{y^3}{3}, \frac{z^3}{3} \right\rangle$ and let S denote the surface defined by $x^2 + y^2 + z^2 = 1$, equipped with the inward-pointing unit normal vector field $\mathbf{n}$. Compute $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by any valid method. **(6 points)**

$\iint_S \mathbf{F} \cdot \mathbf{n}\, dA =$

10. Consider the surface S which is the portion of the plane $z = x + y$ which lies inside the cylinder $x^2 + z^2 = 4$. Give a parameterization $\mathbf{r}\colon\ D \to S$ where D is a rectangle in plane with coordinates u and v. **(5 points)**



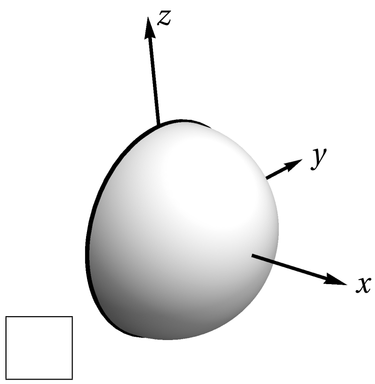
$D = \Big\{ \qquad \le u \le \qquad \text{and} \qquad \le v \le \qquad \Big\}$

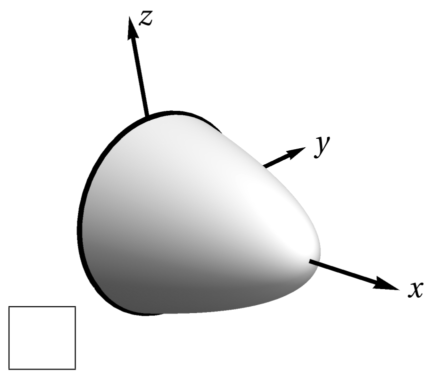
$\mathbf{r}(u, v) = \Big\langle \qquad\qquad , \qquad\qquad , \qquad\qquad \Big\rangle$

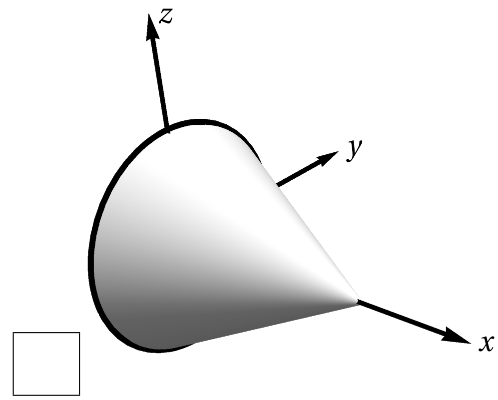
11. Consider the portion S of the surface $z = 1 - x^2$ where $0 \le y \le 1$ and $z \ge 0$. Completely setup but do not evaluate $\iint_S x^2\, dA$. **(5 points)**

12. Let S be the surface in $\mathbb{R}^3$ parameterized by $\mathbf{r}(u,v) = \langle 2-2v^2,\ v\cos u,\ v\sin u\rangle$ for $0 \le u \le 2\pi$ and $0 \le v \le 1$.

(a) Mark the correct picture of S below. **(2 points)**





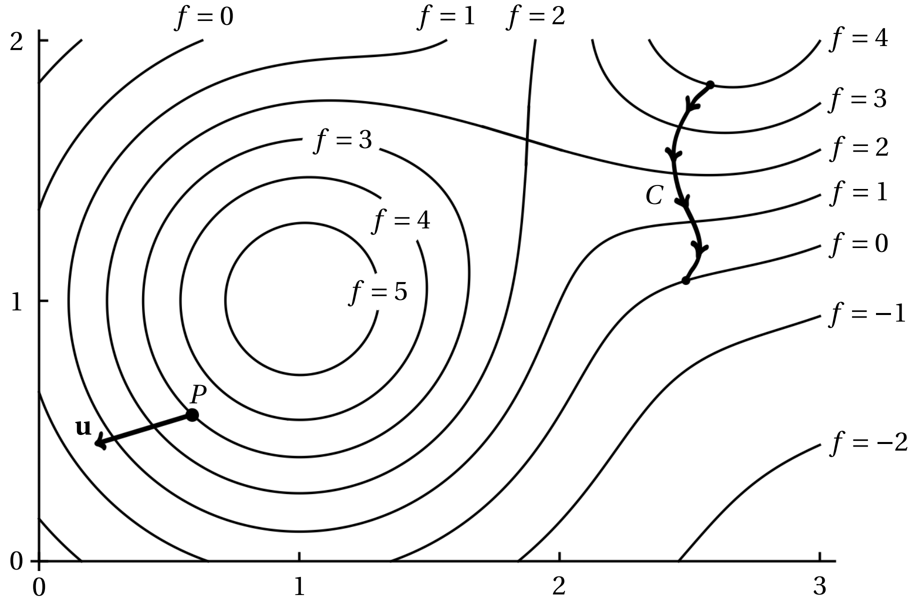


(b) For the vector field $\mathbf{F} = \langle 0,\ -z,\ y\rangle$, directly evaluate $\iint_S (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dA$ where $\mathbf{n}$ is unit normal vector field that points in the positive x-direction. **(5 points)**

$\iint_S (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dA =$

(c) Check your answer in (b) using Stokes' Theorem. **(3 points)**

13. Consider the function $f(x, y)$ on the rectangle $D = \{0 \le x \le 3 \text{ and } 0 \le y \le 2\}$ whose contours are shown below right. For each part, circle the best answer. **(1 point each)**



(a) The maximum value of f on D is:

0	3	6	9	DNE

(b) At P, the derivative $\dfrac{\partial^2 f}{\partial^2 y}$ is:

negative	zero	positive

(c) The value of $D_{\mathbf{u}} f(P)$ is:

negative	zero	positive

(d) The number of critical points of f in D which are saddles is:

0	1	2	3

(e) The integral $\int_C f \, ds$ is:

negative	zero	positive

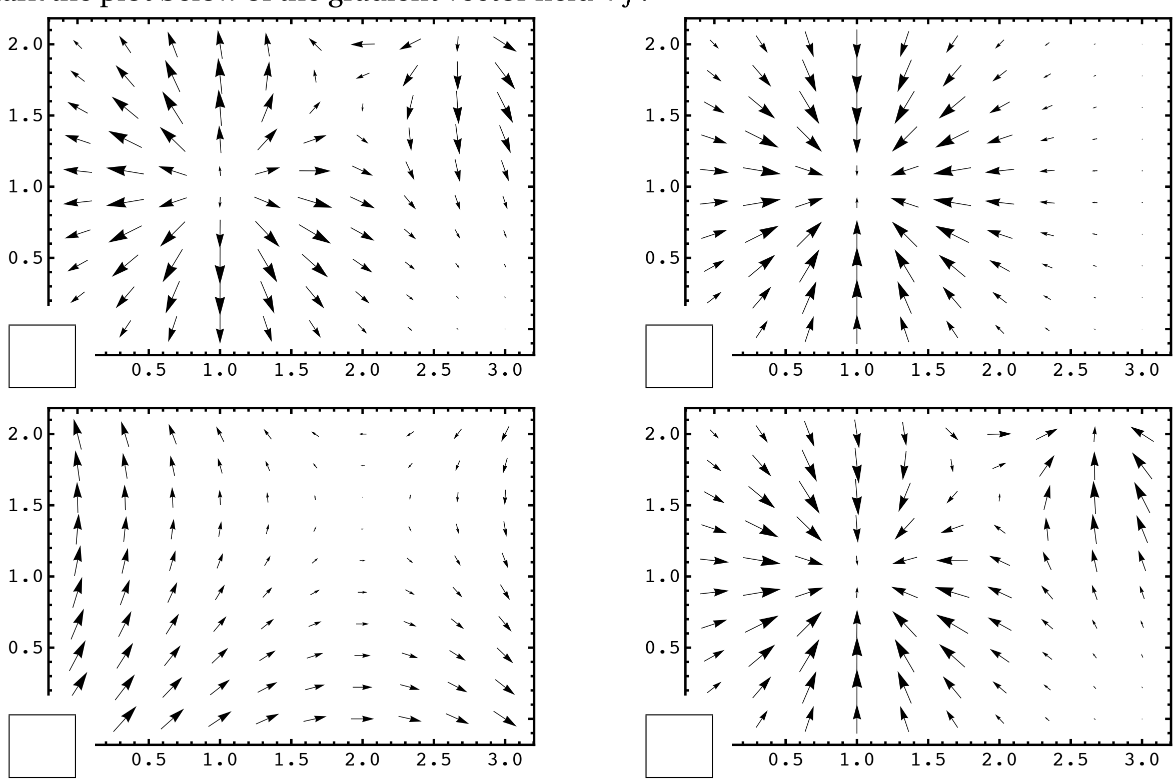
(f) The integral $\int_C \nabla f \cdot d\mathbf{r}$ is:

−4	−2	0	2	4

(g) The integral $\iint_D f \, dA$ is:

−27	−18	−9	0	9	18	27

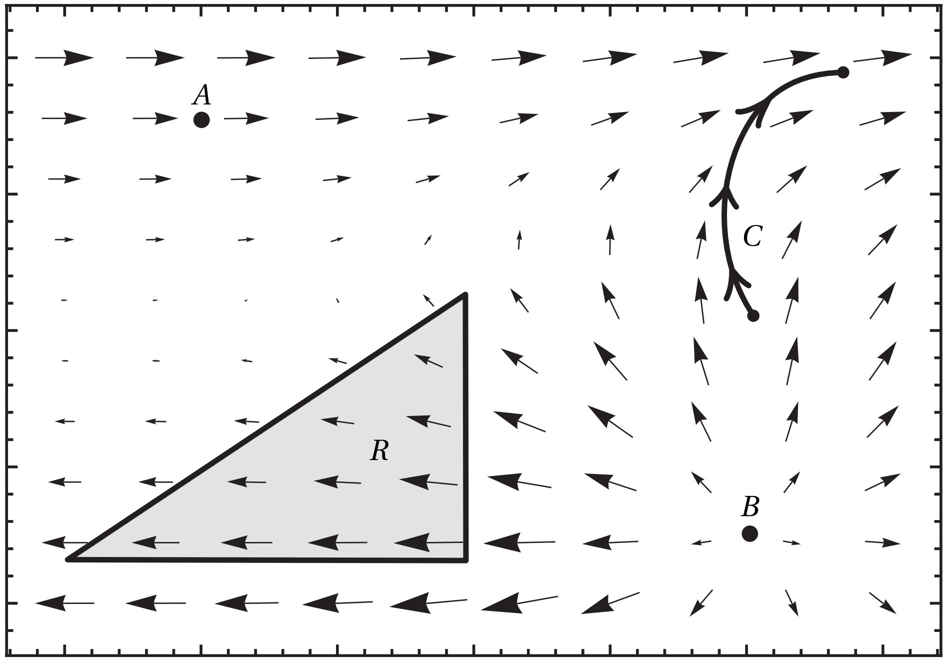
(h) Mark the plot below of the gradient vector field ∇f.



14. Consider the vector field $\mathbf{F}$ on $\mathbb{R}^2$ shown below right. For each part, circle the best answer. **(1 point each)**

(a) The line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ is:

negative	zero	positive



(b) At A, the vector $\operatorname{curl}\mathbf{F}$ is:

$\langle 1,0,0\rangle$	$\langle 0,0,-1\rangle$	$\langle 0,0,1\rangle$

(c) At B, the divergence $\operatorname{div}\mathbf{F}$ is:

negative	zero	positive

(d) If $\mathbf{F} = \langle P, Q\rangle$, then $\iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$ is:

negative	zero	positive

(e) The vector field $\mathbf{F}$ is conservative:

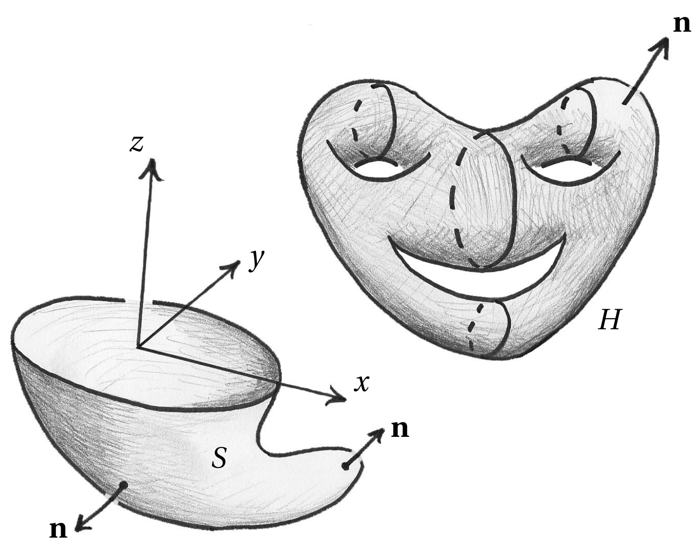
True	False

15. Consider the surfaces S and H show below right; the boundary of S is the unit circle in the xy-plane, and H has no boundary. For each part, circle the best answer.

(a) For $\mathbf{F} = \langle yz,\ xz + x,\ z\rangle$, the integral $\iint_H \mathbf{F}\cdot\mathbf{n}\, dA$ is:

negative	zero	positive

(1 point)



(b) The flux of $\operatorname{curl}\mathbf{F} = \langle -x, y, 1\rangle$ through H is:

negative	zero	positive

(1 point)

(c) The integral $\iint_S (\operatorname{curl}\mathbf{F})\cdot d\mathbf{S}$ is:

-2π	$-\pi$	0	π	2π

(2 points)

(d) For $\mathbf{G} = \langle y,\ z,\ 2\rangle$, the integral $\iint_S \mathbf{G}\cdot\mathbf{n}\, dA$ is:

-2π	$-\pi$	0	π	2π

(2 points)