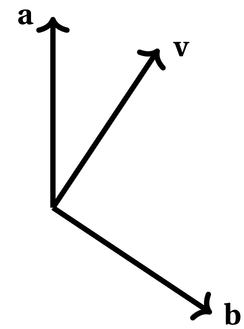
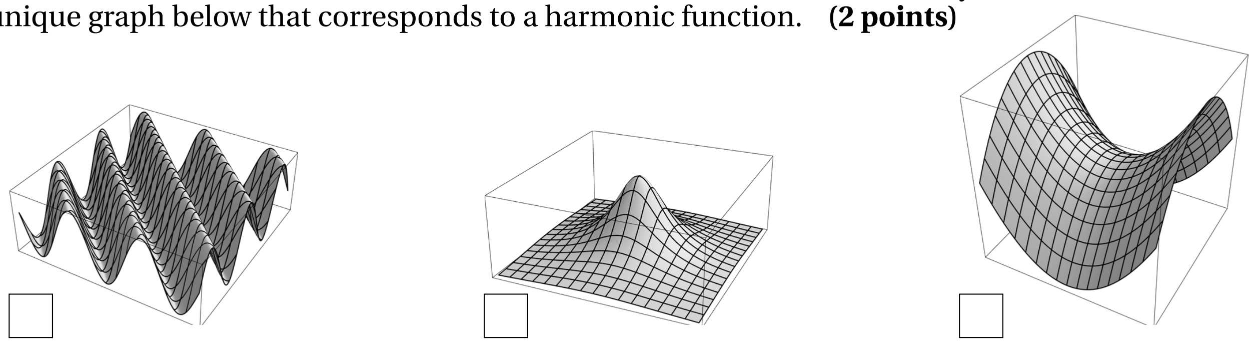
1. Let **v**, **a**, and **b** be the vectors (in the plane of the paper) drawn at right, all of which have length 1. Let **w** be a vector of length 2 pointing directly out of the paper. Which of the following vectors is $\mathbf{v} \times \mathbf{w}$? **(2 points)**



$-2\mathbf{b}$	$-\mathbf{b}$	$\mathbf{b}$	$2\mathbf{b}$	$\langle 0,0,0\rangle$	$-2\mathbf{a}$	$-\mathbf{a}$	$\mathbf{a}$	$2\mathbf{a}$

2. A function $u\colon \mathbb{R}^2 \to \mathbb{R}$ is *harmonic* if it satisfies Laplace's equation: $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$. Check the box next to the unique graph below that corresponds to a harmonic function. **(2 points)**



3. Let f be a function from $\mathbb{R}^2$ to $\mathbb{R}$. Suppose that $f(x,y) \to 3$ as (x,y) approaches $(0,1)$ along every line of the form $y = kx + 1$. What can you say about the limit $\lim_{(x,y)\to(0,1)} f(x,y)$? Check the box next to the correct statement.
(2 points)

- ☐ It exists and is equal to 3.
- ☐ We cannot determine if the limit exists, but if it does, the limit is 3.
- ☐ It does not exist.

4. Suppose we know the following data about $g : \mathbb{R}^3 \to \mathbb{R}$.

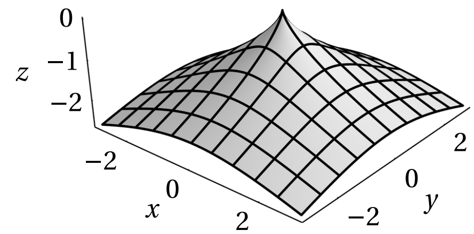
(x,y,z)	$g(x,y,z)$	$g_x(x,y,z)$	$g_y(x,y,z)$	$g_z(x,y,z)$
$(3,3,1)$	30	6	4	5
$(0.5,0.1,0)$	1	2	6	4

Circle the best estimate for $g(3.5, 3.1, 1)$:

30.6	31.0	32.6	33.4	34.1

(2 points)

5. Let S be the surface $x^2 + y^2 = -z^3$ shown at right. Let f be a function on $\mathbb{R}^3$ with continous partial derivatives such that



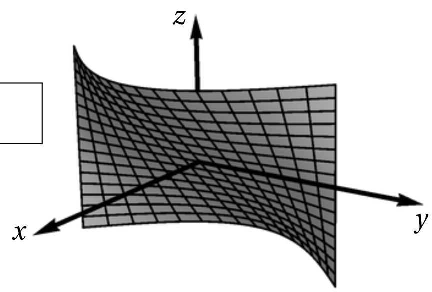
$$\nabla f(0,0,0) = \langle 1,1,3\rangle \qquad \nabla f(2,2,-2) = \langle 1,1,3\rangle$$

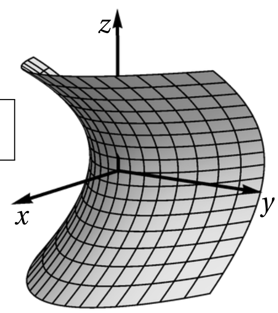
$$\nabla f(-2,-2,-2) = \langle 1,1,3\rangle \qquad \nabla f(2,-2,-2) = \langle 0,0,0\rangle$$

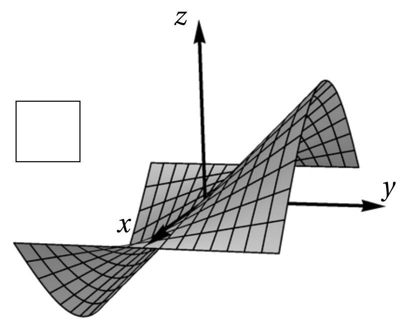
Circle every point below that can **not** be the point at which f achieves its minimum value on S. **(4 points)**

$(0,0,0)$	$(2,2,-2)$	$(-2,-2,-2)$	$(2,-2,-2)$

6. Let S be the surface parametrized by $\mathbf{r}(u,v) = \langle\, v\cos u,\ u,\ v\rangle$ for $-\pi/2 \le u \le \pi/2$ and $-1 \le v \le 1$. Check the box next to the picture of S below: **(2 points)**

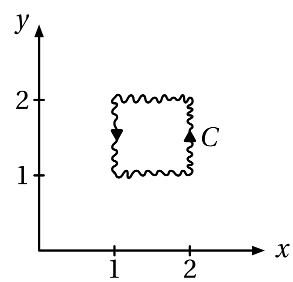
☐ 

☐ 

☐ 

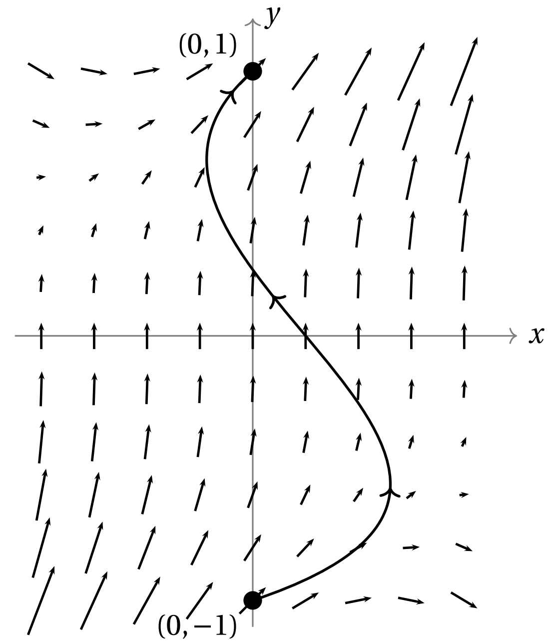
7. Let $\mathbf{F}(x,y) = \left\langle e^{y^2},\ 3x + 2xye^{y^2}\right\rangle$, and let C be the oriented curve at right. Estimate the value of $\int_C \mathbf{F}\cdot d\mathbf{r}$. **(2 points)**

-9	-6	-3	0	3	6	9



Scratch Space

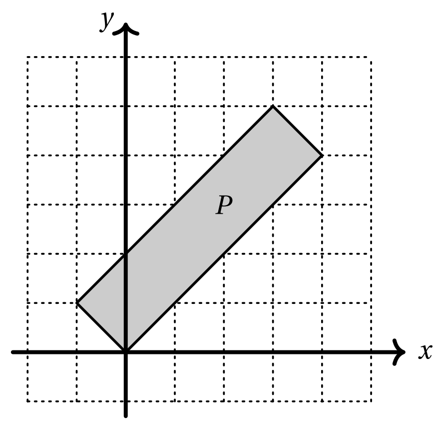
8. A vector field $\mathbf{F}$ is shown at right; for scale, here $\mathbf{F}(0,0) = \langle 0,\ 0.1\rangle$. Assuming that $\mathbf{F}$ is conservative, circle the value of $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is the curve shown from $(0,-1)$ to $(0,1)$.



−0.3	−0.2	−0.1	0	0.1	0.2	0.3

(2 points)

9. Consider the transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(u,v) = \big(2u - v,\ 2u + v\big)$. Let P be the rectangle shown below in the (x,y)–plane, drawn against a unit-square grid. Check the box next to the region D in the (u,v)–plane below that is mapped to P by T. **(2 points)**



☐ $D = \{0 \le u \le 1 \text{ and } 0 \le v \le 1\}$

☐ $D = \{0 \le u \le 2 \text{ and } 0 \le v \le 1\}$

☐ $D = \{0 \le u \le 1 \text{ and } 0 \le v \le 4\}$

☐ $D = \{0 \le u \le 4 \text{ and } 0 \le v \le 1\}$

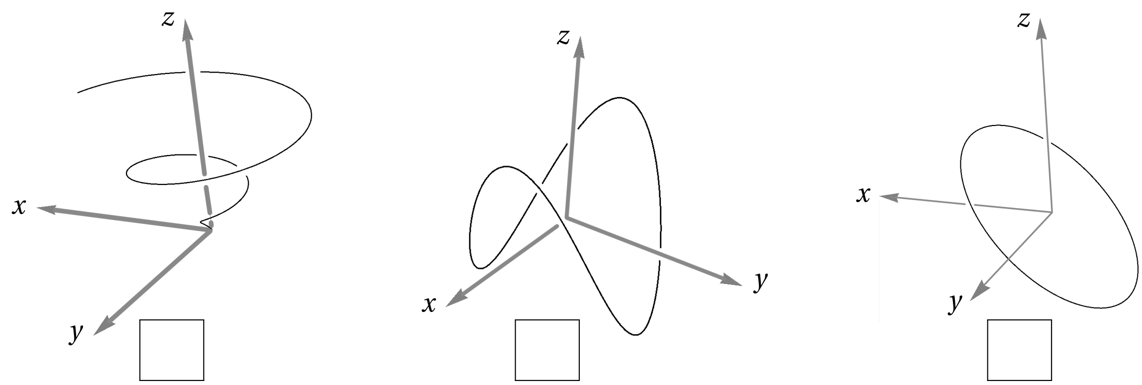
☐ $D = \{-1 \le u \le 0 \text{ and } 0 \le v \le 1\}$

☐ $D = \{0 \le u \le 1 \text{ and } 0 \le v \le 2\}$

Scratch Space

10. Consider the curve C parametrized by $\mathbf{r}(t) = \left(\cos t, \sin t, \cos 2t\right)$, where $0 \le t \le 2\pi$.

(a) Check the box next to the correct sketch of C. **(2 points)**



(b) Find the work done if a particle travels along path $\mathbf{r}(t)$ under the force field given by $\mathbf{F}(x, y, z) = \langle -2y, 2x, 0\rangle$. **(4 points)**

Total work done =

11. Let $S = \{u^2 + v^2 + w^2 = 1\}$ be the unit sphere around the origin, and let $E = \{4x^2 + (y-1)^2 + (z-3)^2 = 1\}$, which is an ellipsoid with center $(0,1,3)$. Find a transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(S) = E$. **(3 points)**

$T(u, v, w) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$

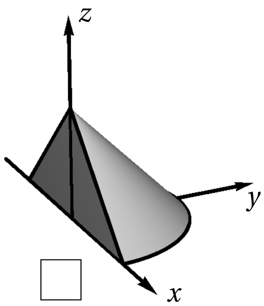
12. Find the volume of the solid that lies below the cone $z = -\sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 4$. **(5 points)**

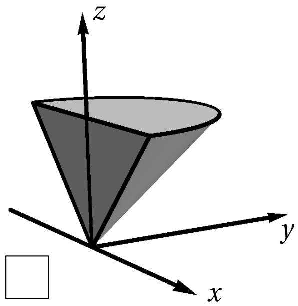
Write your volume integral here:

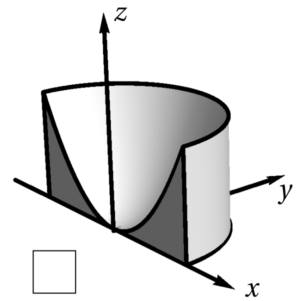
$$\int \int \int \qquad d \quad d \quad d$$

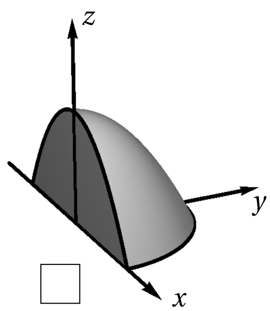
Your final answer: Volume =

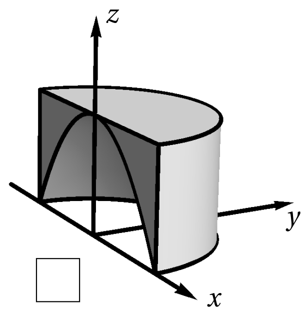
13. For each of the integrals: **(A)** $\int_0^{\pi}\int_0^1\int_{1-r^2}^1 f(r,\theta,z)\, r\; dz\,dr\,d\theta$ and **(B)** $\int_0^{\pi}\int_0^1\int_0^{1-z} f(r,\theta,z)\, r\; dr\,dz\,d\theta$ label the solid corresponding to the region of integration below. **(2 points each)**

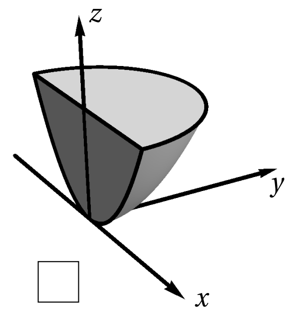






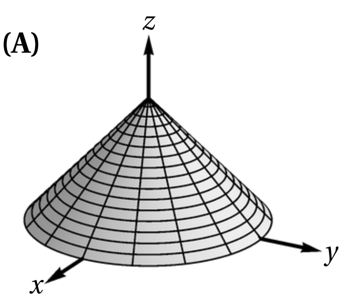




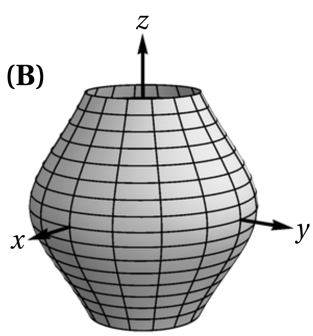


14. Label the boxes next to the parametrizations that correspond to the following two surfaces: **(2 points each)**

(A)



(B)



☐ $\mathbf{r}(u,v) = \left\langle \dfrac{\cos v}{1+u^2}, \dfrac{\sin v}{1+u^2}, u \right\rangle$ for $-1 \le u \le 1$, $0 \le v \le 2\pi$

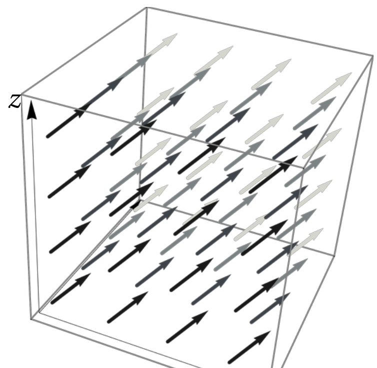
☐ $\mathbf{r}(u,v) = \langle \cos v, \sin v, u\rangle$ for $-1 \le u \le 1$, $0 \le v \le 2\pi$

☐ $\mathbf{r}(u,v) = \langle u\cos v, u\sin v, 1-u\rangle$ for $0 \le u \le 1$, $0 \le v \le 2\pi$

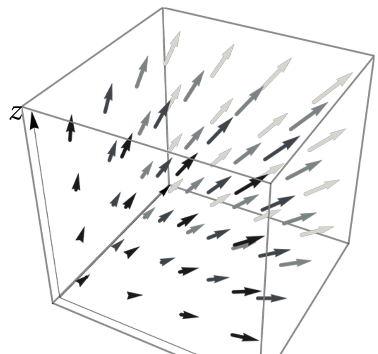
☐ $\mathbf{r}(u,v) = \left\langle u\cos v, u\sin v, 1-u^2\right\rangle$ for $0 \le u \le 1$, $0 \le v \le 2\pi$

☐ $\mathbf{r}(u,v) = \langle \sin u\cos v, \sin u\sin v, \cos u\rangle$ for $0 \le u \le \pi/2$, $0 \le v \le 2\pi$

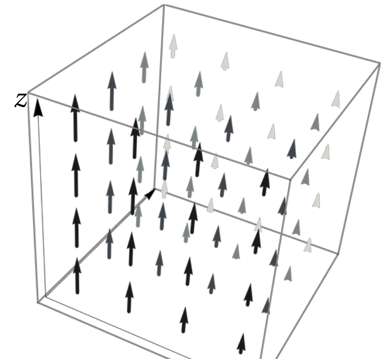
15. Here are plots of five vector fields on the box where $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$. **(2 points each)**



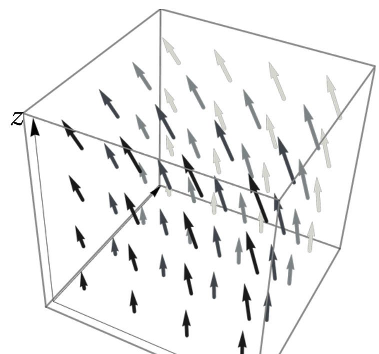
A



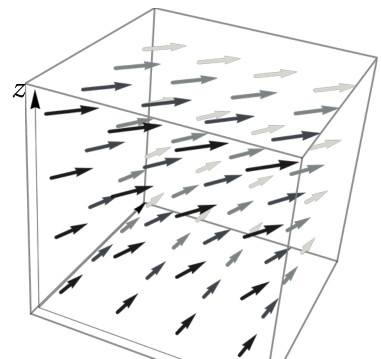
B



C



D



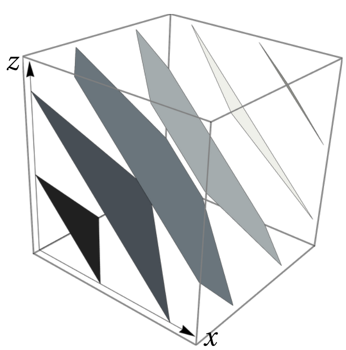
E

(a) Circle the name of the vector field that is given by $\langle -z,\ 0,\ 1+x \rangle$: **A B C D E**

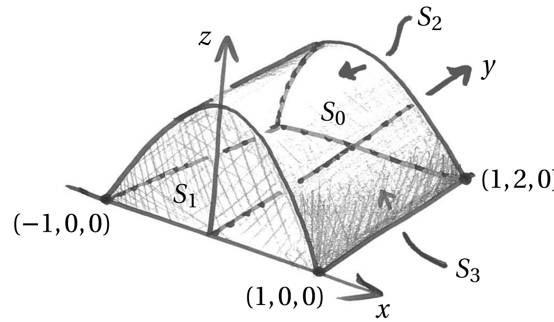
(b) Exactly one of these vector fields has nonzero divergence. Circle it: **A B C D E**

(c) The vector field **C** is conservative: true false

(d) Which vector field is the gradient of a function f whose level sets are shown below? **A B C D E**



16. Let E be the solid region shown below, where ∂E is decomposed into the four subsurfaces S_i indicated; here the top S_0 is where $z + x^2 = 1$, the front S_1 is in the xz-plane, the back is S_2, and the bottom is S_3.



(a) Use a triple integral to compute the volume of E. **(4 points)**

Vol(E) =

(a) Give a parameterization of S_0 and use it to directly compute the flux of $\mathbf{F} = \langle 1,\ 0,\ z+2 \rangle$ through S_0 with respect to the upwards normals. **(5 points)**

$\iint_{S_0} \mathbf{F} \cdot d\mathbf{S} =$

(b) The flux of **F** through exactly two of S_1, S_2, and S_3 is zero. Circle the one where the flux is **nonzero**:

S_1	S_2	S_3

(1 point)

17. Consider the vector field $\mathbf{F} = \left\langle -y,\ x+z,\ x^2+z \right\rangle$ on $\mathbb{R}^3$.

(a) Circle the curl of **F**: **(2 points)**

curl **F** =	$\left\langle z,\ -y,\ x \right\rangle$	$\langle -1,\ 2x,\ 2 \rangle$	$\langle 0,\ 1,\ 2x \rangle$	$\langle -1,\ -2x,\ 2 \rangle$	$\left\langle -y,\ 2x,\ 2z \right\rangle$

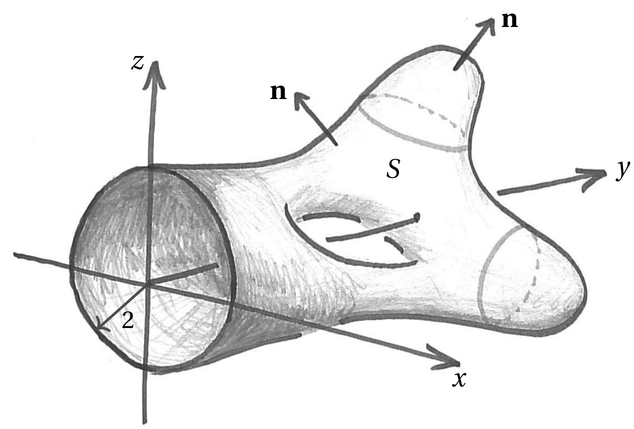
(b) Suppose C is a closed curve in the plane P given by $x - z = 1$. Assuming C bounds a region R of area 10 in P, determine the absolute value of $\int_C \mathbf{F} \cdot d\mathbf{r}$. **(4 points)**

$$\left| \int_C \mathbf{F} \cdot d\mathbf{r} \right| =$$

Scratch Space

18. The surface S shown below has boundary the circle of radius 2 in the xz-plane. With respect to the normal vector field indicated, compute the flux of $\mathbf{G} = \langle 0, 3, 0 \rangle$ through S. **(5 points)**

Backup question: If you can't find the exact answer, determine whether the flux is positive, zero, or negative and write that in the answer box for partial credit.



$$\iint_S \mathbf{G} \cdot d\mathbf{S} =$$

Scratch Space

19. Suppose $g(x, y, z) = e^x + y\cos z$ and $\mathbf{G} = \nabla g = \langle e^x, \cos z, -y\sin z\rangle$ is its gradient vector field. **(2 points each)**

(a) Let C denote the parametric curve $\mathbf{r}(t) = \langle 2,\ t,\ \pi t\rangle$ for $0 \le t \le 1$. The integral $\int_C \mathbf{G}\cdot d\mathbf{r}$ is:

e^2	π	-1	0	1	$e^2 - 1$

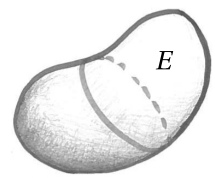
(b) Let S denote the hemisphere defined by $x^2 + y^2 + z^2 = 1$ and $z \ge 0$; let $\mathbf{n}$ denote the upward unit normal. The integral $\iint_S \operatorname{curl}\mathbf{G}\cdot\mathbf{n}\, dA$ is:

positive	zero	negative

(c) Consider the vector field $\mathbf{F} = \langle -y,\ x + z,\ y\rangle$. The vector field $\mathbf{F}$ is:

conservative	not conservative

20. Suppose E is a solid region in $\mathbb{R}^3$ looking like the picture at right.



(a) Suppose E has volume 10 and is made of material of constant density. Check the box next to the integral that must compute the x-coordinate of its center of mass. **(2 points)**

- ☐ $\frac{1}{30}\iint_{\partial E} \mathbf{A}\cdot d\mathbf{S}$ for $\mathbf{A} = \langle x,\ y,\ z\rangle$
- ☐ $\iiint_E x\, dV$
- ☐ $\frac{1}{10}\iiint_E y\, dV$
- ☐ $\frac{1}{20}\iint_{\partial E} \mathbf{B}\cdot d\mathbf{S}$ for $\mathbf{B} = \langle z,\ xy,\ xz\rangle$
- ☐ None of these.

(b) Assuming the origin lies inside of E, determine the flux of $\mathbf{H} = \dfrac{-3}{(x^2 + y^2 + z^2)^{3/2}}\langle x,\ y,\ z\rangle$ through ∂E.

(2 points)

$$\iint_{\partial E} \mathbf{H}\cdot d\mathbf{S} = \boxed{}$$

Scratch Space