1. Consider the three points $A = (0,1,0)$, $B = (1,1,1)$, and $C = (0,2,1)$ in $\mathbb{R}^3$. For each part, circle the best answer. **(1 point each)**

(a) The projection of the vector $\overrightarrow{AC}$ onto $\overrightarrow{AB}$ is:

$\langle 0,\ 2,\ 2\rangle$	$\left\langle \frac{1}{\sqrt{2}},\ 0,\ \frac{1}{\sqrt{2}} \right\rangle$	$\left\langle \frac{1}{2},\ 0,\ \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2},\ \frac{1}{2},\ 0 \right\rangle$

(b) The area of the triangle formed by these three points is:

$\sqrt{3}$	$\frac{\sqrt{3}}{2}$	3	$\frac{3}{2}$

(c) $\overrightarrow{AB} \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) =$

-3	-2	-1	0	1	2	3

(d) For the vector $\mathbf{v} = \langle 0,\ 2,\ 2\rangle$, the angle between $\overrightarrow{AC}$ and $\mathbf{v}$ is:

0	$\pi/4$	$\pi/2$	π	$3\pi/2$

2. Suppose $f\colon \mathbb{R}^2 \to \mathbb{R}$ has the table of values and partial derivatives shown at right. For $x(r,s) = 2r - s$ and $y(r,s) = -4r + s^2$, let $F(r,s) = f\big(x(r,s), y(r,s)\big)$ be their composition with f.

Circle the value of $\frac{\partial F}{\partial r}(1,2)$:

24	-24	40	-40	-11	11

(2 points)

(x,y)	$f(x,y)$	$\frac{\partial f}{\partial x}$	$\frac{\partial f}{\partial y}$
$(0,0)$	3	4	8
$(1,2)$	6	2	5
$(3,3)$	19	-8	5
$(4,3)$	7	3	2

Scratch Space

3. **Exactly one** of the limits at right exists.

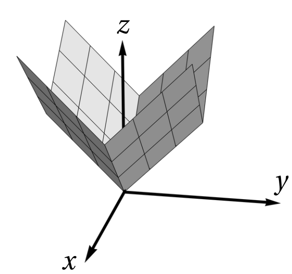
Circle the limit **that exists:** | *A* *B* |

Justify your answer either by showing that the limit you circled exists and computing its value, or by showing that the other limit does not exist. **(4 points)**

(A) $$\lim_{(x,y)\to(0,0)} \frac{\sin\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}$$

(B) $$\lim_{(x,y)\to(0,0)} \frac{x^2+x}{\sqrt{x^2+y^2}}$$

4. Consider the function $f(x,y)=|x|+|y|$, whose graph is shown at right. Circle the phrase that best completes each sentence. **(1 point each)**



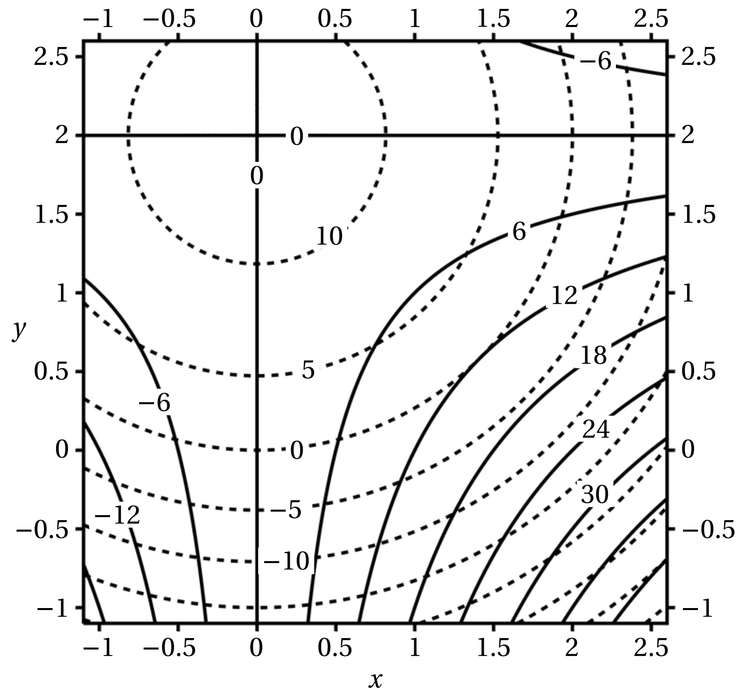
(a) At the point $(1,1)$, the function f is | continuous differentiable both continuous and differentiable |

(b) At the point $(0,0)$, the function f is | continuous differentiable both continuous and differentiable |

5. Suppose that $g(x,y,z)$ is a function and $g(1,0,3)=6$ and $\nabla g(1,0,3)=2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$. Use linear approximation to estimate the value of $g(1.2,-0.1,2.9)$. **(2 points)**

$g(1.2,-0.1,2.9) \approx$ 5.5   5.7   5.8   5.9   6   6.1   6.2   6.3   6.4   6.5

6. The level curves of the partial derivatives f_x (solid lines) and f_y (dashed lines) of a function $f(x,y)$ are shown at right. There are **exactly two** critical points of f in the domain shown in the picture. Find both of them and classify each as a local minimum, local maximum, or saddle. **(4 points)**



(,) which is a local min   local max   saddle

(,) which is a local min   local max   saddle

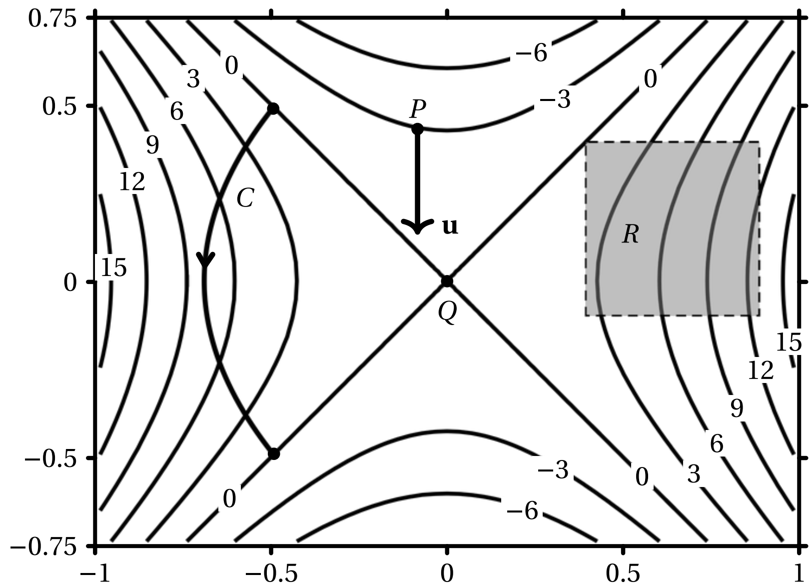
Scratch Space

7. Find the absolute maximum and minimum values of the function $f(x, y) = 2x^2 + y^2 + 5$ subject to the constraint $x^2 + y^2 \le 4$. **(6 points)**

Absolute max value =

Absolute min value =

8. The contour map of a differentiable function f is shown at right. For each part, circle the best answer. **(2 points each)**



(a) The directional derivative $D_{\mathbf{u}} f(P)$ is:

positive	negative	zero

(b) Estimate $\int_C f\, ds$:

−8.1	−5.4	−2.7	0	2.7	5.4	8.1

(c) Estimate $\iint_R f\, dA$:

−4.8	−3.2	−1.6	0	1.6	3.2	4.8

(d) The point Q is:

a local maximum	a local minimum	a saddle	not a critical point

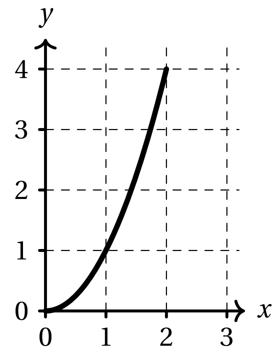
(e) Find $\int_C \nabla f \cdot d\mathbf{r}$:

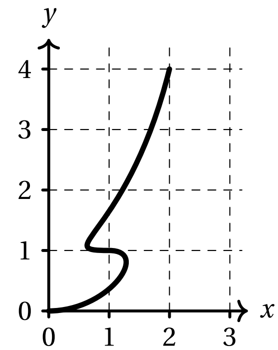
−12	−9	−6	−3	0	3	6	9	12

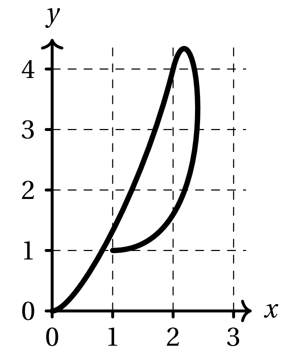
Scratch Space

9. Suppose that $\mathbf{r}: [0,2] \to \mathbb{R}^2$ is a parametric curve in the plane and that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ have the values given in the table at left. Check the box below the picture that could be a plot of $\mathbf{r}(t)$. **(2 points)**

t	$\mathbf{r}(t)$	$\mathbf{r}'(t)$
0	$(0,0)$	$\mathbf{i}$
1	$(1,1)$	$-\mathbf{i}$
2	$(2,4)$	$\mathbf{i}+4\mathbf{j}$



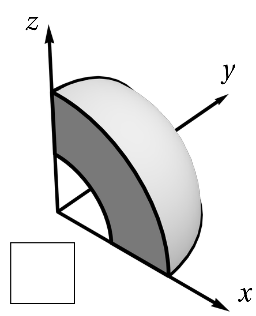


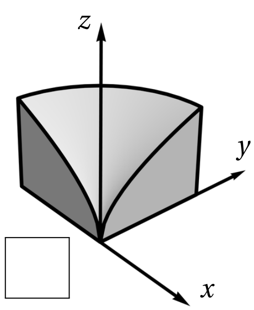


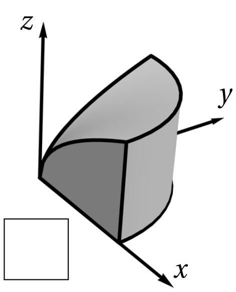
10. For each of the integrals below, label the picture of the corresponding region of integration. **(2 points each)**

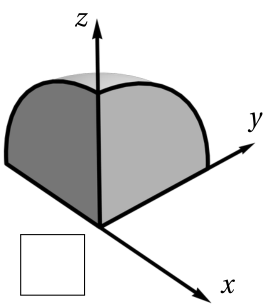
(A) $\displaystyle\int_1^2 \int_0^{\pi/2} \int_{\pi/2}^{\pi} f(\rho,\phi,\theta)\,\rho^2 \sin\phi \; d\theta\, d\phi\, d\rho$

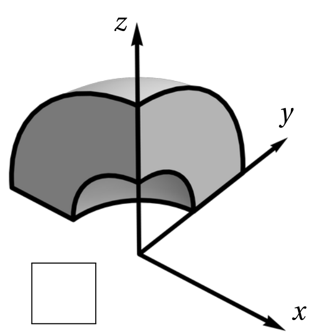
(B) $\displaystyle\int_{\pi/2}^{\pi} \int_0^{\pi/2} \int_0^{\phi} g(\rho,\phi,\theta)\,\rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$

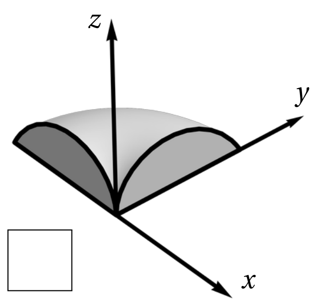












11. (a) Consider the vector field $\mathbf{F} = \langle\, yz,\ -xz,\ yx \,\rangle$ on $\mathbb{R}^3$. Compute the curl of $\mathbf{F}$. **(2 points)**

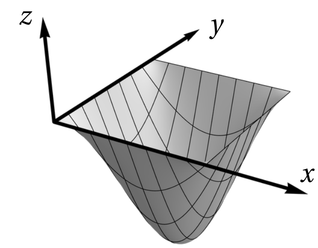
curl $\mathbf{F} = \langle \quad , \quad , \quad \rangle$

(b) Let S be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \leq 3$. Find the flux of curl $\mathbf{F}$ through S with respect to the outward pointing unit normal vector field. **(6 points)**

flux =

(c) Let D be the portion of the sphere $x^2 + y^2 + z^2 = 13$ where $x \geq 3$. Find the flux of curl $\mathbf{F}$ through D with respect to the outward pointing unit normal vector field. flux = **(1 point)**

12. (a) Let S be the portion of the surface $z = -\sin(x)\sin(y)$ where $0 \le x \le \pi$ and $0 \le y \le \pi$ which is shown at right. Use a parameterization to find the flux of $\mathbf{F} = \langle 0,\ 0,\ 2z+1 \rangle$ through S with respect to the downward normals. **(5 points)**



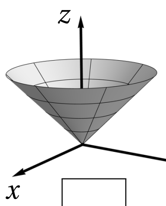
flux =

(b) Let E be the region below the xy-plane and above S. Use an integral theorem to compute the flux of $\mathbf{F}$ through ∂E with respect to the outward normals. **(4 points)**

flux =

(c) Your answers in (a) and (b) should differ. Explain what accounts for the difference. **(1 point)**

13. (a) Let S be the surface parameterized by $\mathbf{r}(u,v) = \langle v\cos u,\ v,\ v\sin u\rangle$ for $0 \le u \le 2\pi$ and $0 \le v \le 2$. Mark the box next to its picture. **(2 points)**



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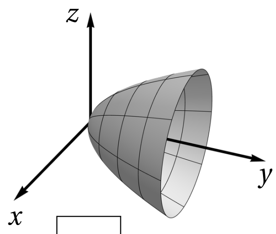

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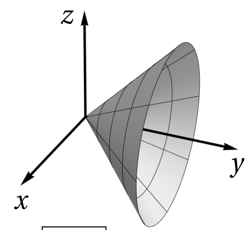

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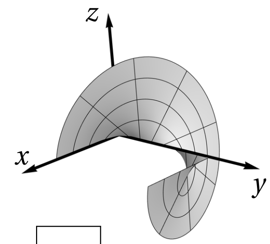

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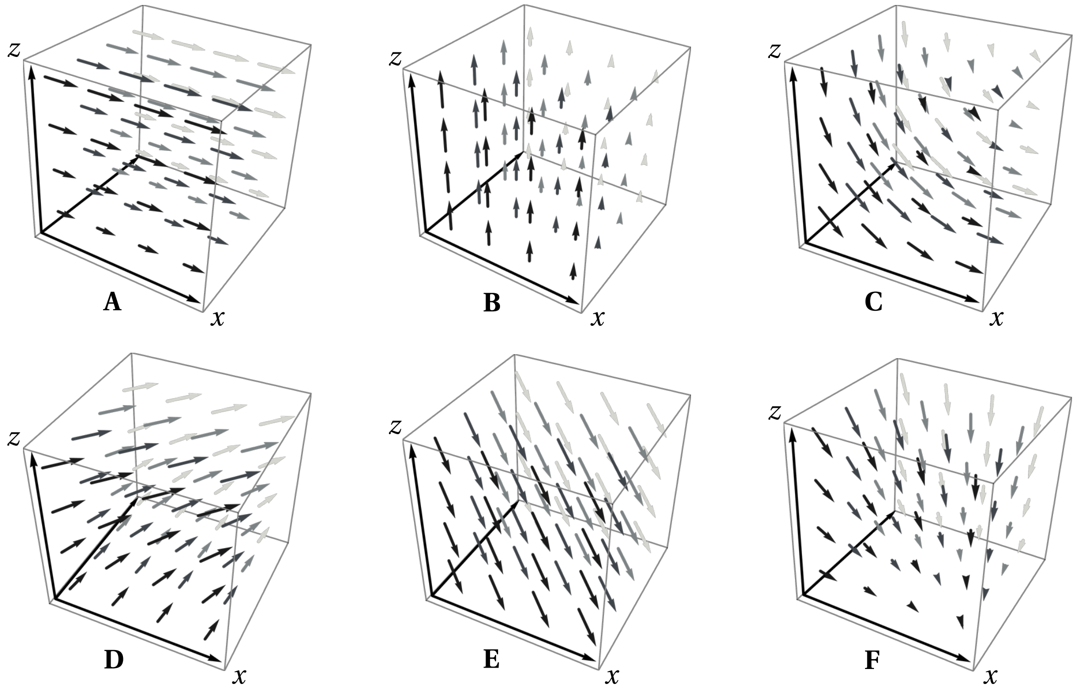
(b) Use the parameterization to find the tangent plane to S at $\left(1, \sqrt{2}, 1\right)$. **(4 points)**

Equation: $\square\, x + \square\, y + \square\, z = \square$

(c) The surface S has area $4\sqrt{2}\pi$. Find the average of $f(x,y,z) = y$ on S. **(4 points)**

Average =

14. Here are plots of six vector fields on the box where $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$. For each part, circle the best answer. **(1 point each)**



(a) The vector field given by $\langle z,\ 1,\ 0\rangle$ is: **A B C D E F**

(b) Exactly one of these vector fields has nonzero divergence. It is: **A B C D E F**

For this vector field, the divergence is generally: negative positive

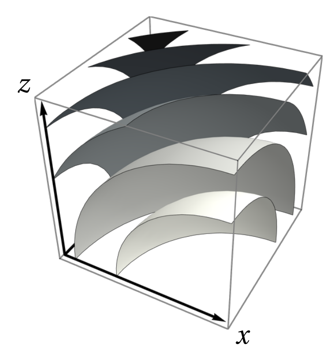
(c) The vector field **A** is conservative: true false

(d) Exactly one of the vector fields is constant, that is, independent of position. It is: **A B C D E F**

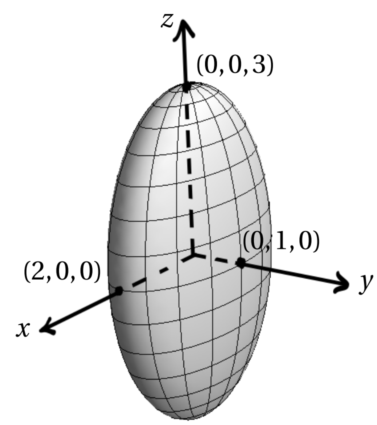
(e) The vector field curl **C** is constant. The value of curl **C** is:

i $-$**i** **j** $-$**j** **k** $-$**k** **0**

(f) The vector field that is the gradient of a function f whose level sets are shown at right is: **A B C D E F**



15. Let S be the ellipsoid $\dfrac{x^2}{4} + y^2 + \dfrac{z^2}{9} = 1$ which is shown at right. Give a parameterization $\mathbf{r}\colon D \to \mathbb{R}^3$ for S, being sure to specify the domain D of the parameterization in the (u, v)-plane. **(4 points)**



$D = \Big\{ \qquad \le u \le \qquad , \qquad \le v \le \qquad \Big\}$

$\mathbf{r}(u, v) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$

16. For each transformation $\mathbb{R}^2 \to \mathbb{R}^2$ below, circle "yes" or "no" depending on whether or not it takes the rectangle $0 \le u \le 1$, $0 \le v \le 2$ in the (u, v)-plane to the parallelogram in the (x, y)-plane with vertices $(0,0), (4,2), (2,-2)$, and $(6,0)$. **(1 point each)**

yes no $T(u, v) = (2u + 4v,\ -2u + 2v)$

yes no $T(u, v) = (4u + v,\ 2u - v)$

yes no $T(u, v) = (4u + 2v,\ 2u - 2v)$

yes no $T(u, v) = (2u + 2v,\ -2u + v)$

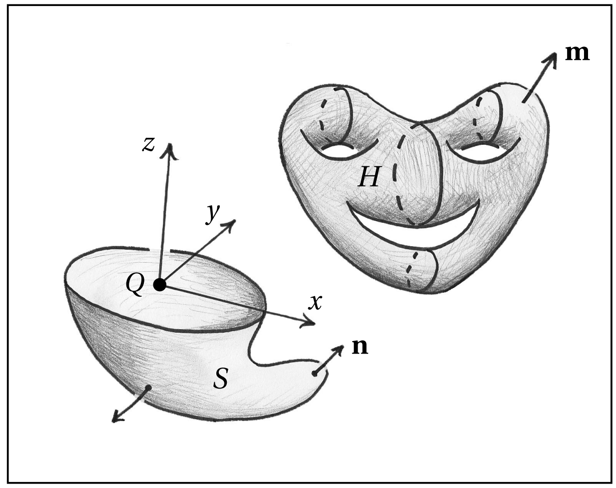
Scratch Space

17. Let $\mathbf{F} = \langle -x^3\cos^3(x) + y^3,\ y^3\sin^3(y) - x^3 \rangle$ and let C be the unit circle in the plane $\mathbb{R}^2$ oriented counter-clockwise. Determine whether the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ is:

negative	zero	positive

(2 points)

18. Let S and H be the surfaces at right; the boundary of S is the unit circle in the xy-plane, and H has no boundary. Suppose there is a positive charge Q placed at the origin and let $\mathbf{E}$ be the resulting electrical field. For each part, circle the correct answer. **(2 points each)**



(a) The flux $\iint_H \mathbf{E}\cdot\mathbf{m}\, dS$ is:

negative	zero	positive

(b) The flux $\iint_S \mathbf{E}\cdot\mathbf{n}\, dS$ is:

negative	zero	positive

(c) For $\mathbf{G} = \langle xy^2,\ yz,\ x+z \rangle$, the flux $\iint_H (\operatorname{curl}\mathbf{G})\cdot\mathbf{m}\, dS$ is:

negative	zero	positive

Scratch Space