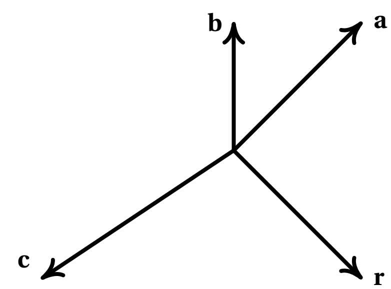
1. Circle the vector **n** that is normal to the plane containing the point $P = (1,2,2)$ and the line L parameterized by $x = 2,\ y = 1 + t$, and $z = 1 - t$. **(2 points)**

$\langle -2,\ 1,\ 1\rangle$	$\langle 0,\ 1,\ -1\rangle$	$\langle -2,\ 0,\ -1\rangle$	$\langle -2,\ -1,\ -1\rangle$

2. Consider the vectors **a**, **b**, **c** and **r** in the plane of this piece of paper. For each part, circle the best answer. **(1 point each)**



(a) $\mathbf{r} =$ $\mathbf{a}-\mathbf{b}$ $\quad$ $\mathbf{a}-2\mathbf{b}$ $\quad$ $\mathbf{b}-\mathbf{c}$ $\quad$ $2\mathbf{b}$ $\quad$ $2\mathbf{a}$

(b) $\text{proj}_{\mathbf{b}}(\mathbf{a}+\mathbf{b}) =$ $\mathbf{a}+\mathbf{b}$ $\quad$ $\mathbf{a}$ $\quad$ $\mathbf{b}$ $\quad$ $2\mathbf{b}$ $\quad$ $2\mathbf{a}$

(c) $\mathbf{b}\cdot\mathbf{c} =$ positive $\quad$ negative $\quad$ zero

3. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $f(x,\ y) = \begin{cases} \dfrac{xy+y^3}{x^2+y^2} & (x,\ y) \neq (0,\ 0) \\ 0 & (x,\ y) = (0,\ 0) \end{cases}$ **(2 points each)**

(a) Check the box next to the only true statement below. The limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist because

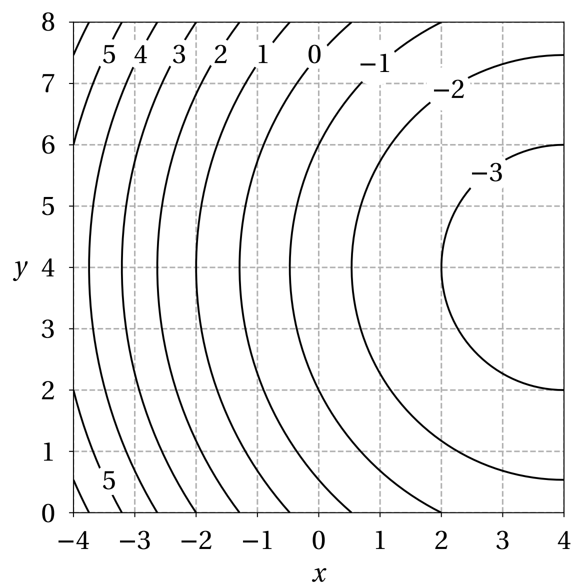
- ☐ the numerator and denominator are both zero at $(0,0)$.
- ☐ the limit as one approaches $(0,0)$ along the lines $y = 0$ and $x = 0$ are different.
- ☐ the limit as one approaches $(0,0)$ along the the paths $y = x^2$ and $x = 0$ are different.
- ☐ the limit as one approaches $(0,0)$ along the lines $y = x$ and $x = 0$ are different.

(b) Compute $\dfrac{\partial f}{\partial x}(0,0)$ and $\dfrac{\partial f}{\partial y}(0,0)$; if a partial derivative does not exist, write "DNE".

$\dfrac{\partial f}{\partial x}(0,0) =$ ☐ $\qquad$ $\dfrac{\partial f}{\partial y}(0,0) =$ ☐

Scratch Space

4. The level curves of a differentiable function $f(x,y)$ on $[-4,4]\times[0,8]$ are shown below.



(a) Circle the best estimate for $\int_{-4}^{-1}\int_{6}^{8} f(x,y)\,dy\,dx$.

−30	−24	−18	−12	−6	0	6	12	18	24	30

(2 points)

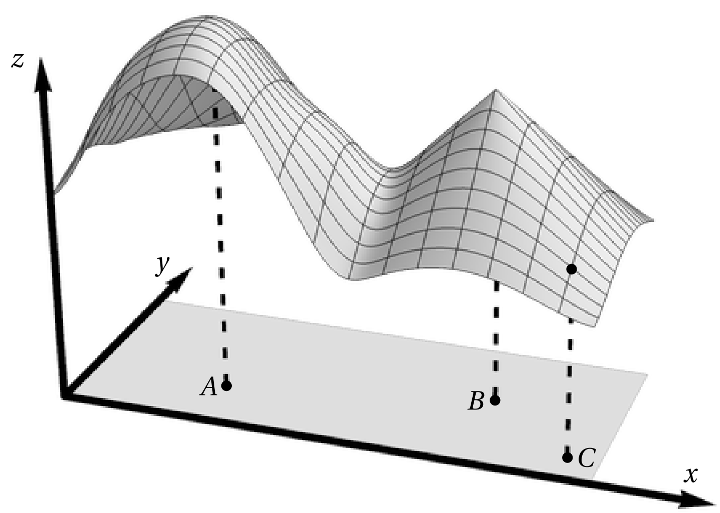
(b) Find the points on the curve $x^2+(y-4)^2=4$ where f attains its absolute maximum and minimum values.

Max value = ______ at the point(s) ______ **(1 point)**

Min value = ______ at the point(s) ______ **(1 point)**

(c) The absolute minimum value of f on the region $D=\{x^2+(y-4)^2\le 4\}$ is: ______ **(1 point)**

5. Consider the function $g\colon \mathbb{R}^2\to\mathbb{R}$ whose graph is shown at right. Let A and B be the points in $\mathbb{R}^2$ corresponding to the two "peaks" of the graph, and C be the point in $\mathbb{R}^2$ corresponding to the dot on the graph. For each part, circle the answer that is most consistent with the picture. **(1 point each)**

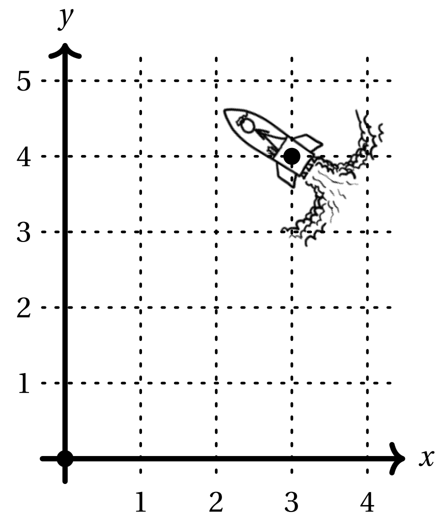


(a) At the point A, the function g is: continuous | differentiable | both | neither

(b) At the point B, the function g is: continuous | differentiable | both | neither

(c) At the point C, the function $\frac{\partial g}{\partial x}$ is: negative | zero | positive

6. An exceptionally tiny spaceship positioned as shown is traveling so that its x-coordinate *decreases* at a rate of $1/3$ m/s and y-coordinate *increases* at a rate of $1/2$ m/s. Use the Chain Rule to calculate the rate at which the distance between the spaceship and the point $(0,0)$ is increasing. **(5 points)**

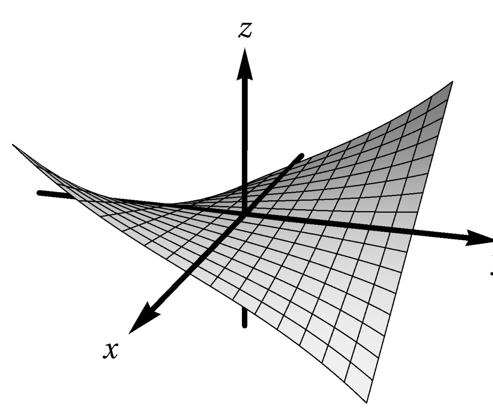


Distances in meters

Rocket courtesy of xkcd.com

rate = $\boxed{\quad\quad}$ m/s

7. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the function whose graph is shown at right.



(a) Find the tangent plane to the graph at $(0,0,0)$. **(1 point)**

Equation: $\boxed{\quad}\,x + \boxed{\quad}\,y + \boxed{\quad}\,z = \boxed{\quad}$

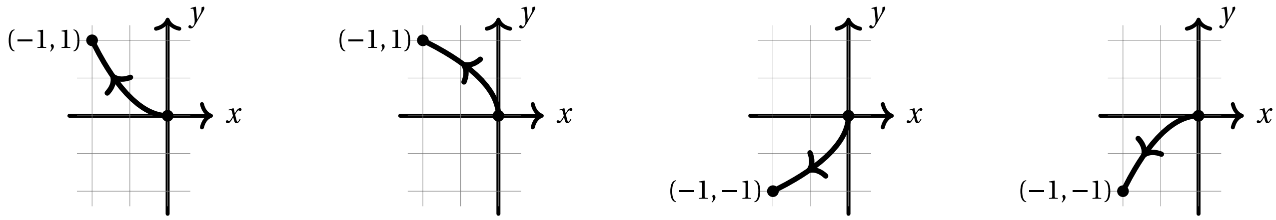
(b) The partial derivative $\dfrac{\partial^2 f}{\partial x \partial y}(0,0)$ is (circle your answer):

negative	zero	positive

(1 point)

8. Let C be the curve in $\mathbb{R}^2$ parameterized by $\mathbf{r}(t) = \langle -t^2,\ t \rangle$ for $0 \le t \le 1$.

(a) Mark the picture of C from among the choices below. **(1 point)**



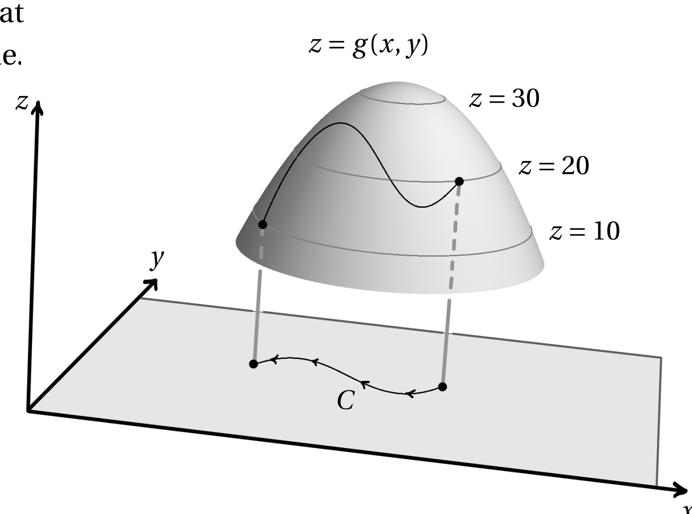
☐ ☐ ☐ ☐

(b) For the vector field $\mathbf{F} = \langle y,\ -x \rangle$, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. **(3 points)**

$\int_C \mathbf{F} \cdot d\mathbf{r} =$ ☐

9. Let $g\colon \mathbb{R}^2 \to \mathbb{R}$ be the function whose graph is shown at right, and let C be the indicated curve in the xy-plane. Evaluate the line integral:

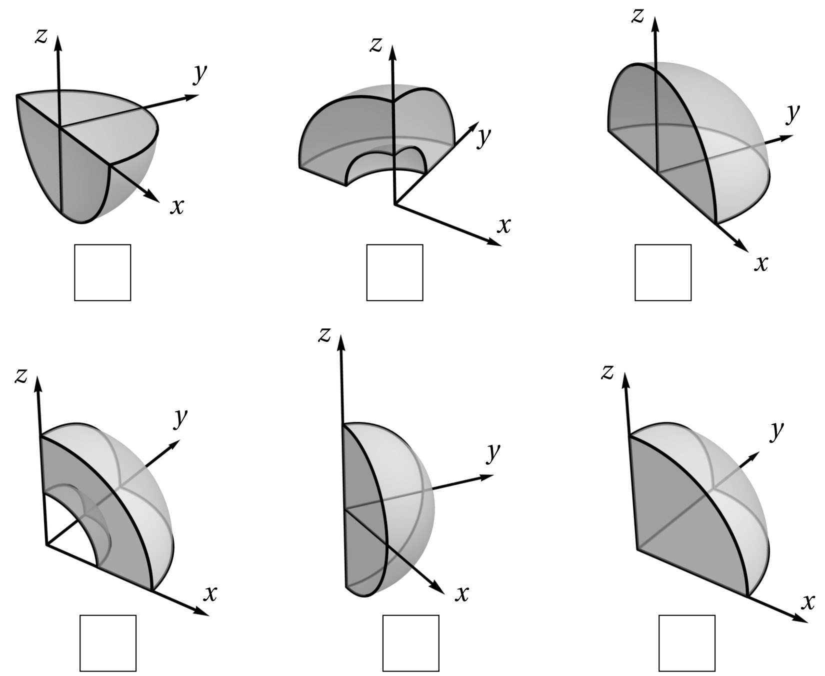
$\int_C \nabla g \cdot d\mathbf{r} =$ ☐ **(2 points)**



10. For each of the given integrals, label the box below the picture of the corresponding region of integration in spherical coordinates. **(2 points each)**

(A) $\displaystyle\int_0^{\pi/2}\int_0^{\pi/2}\int_1^{2} \rho^2 \sin\phi \; d\rho \, d\phi \, d\theta$

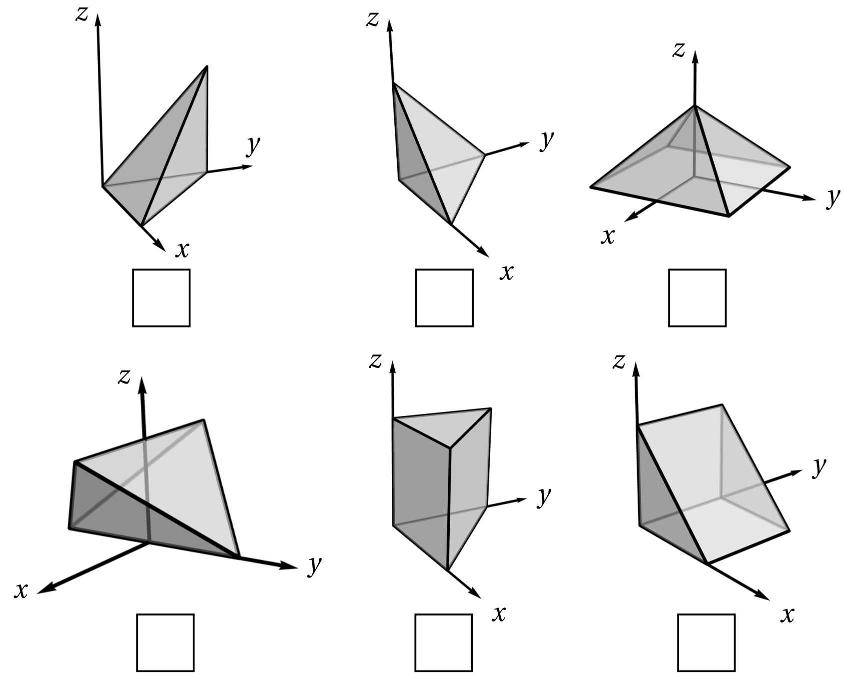
(B) $\displaystyle\int_0^{\pi}\int_{\pi/2}^{\pi}\int_0^{2} \rho^2 \sin\phi \; d\rho \, d\phi \, d\theta$



11. For each of the given integrals, label the box below the picture of the corresponding region of integration.
(2 points each)

(A) $\displaystyle\int_0^{1}\int_0^{1-x}\int_0^{1} f(x,y,z)\; dz\, dy\, dx$

(B) $\displaystyle\int_0^{1}\int_{-1+z}^{1-z}\int_{-z}^{z} g(x,y,z)\; dx\, dy\, dz$



Scratch Space

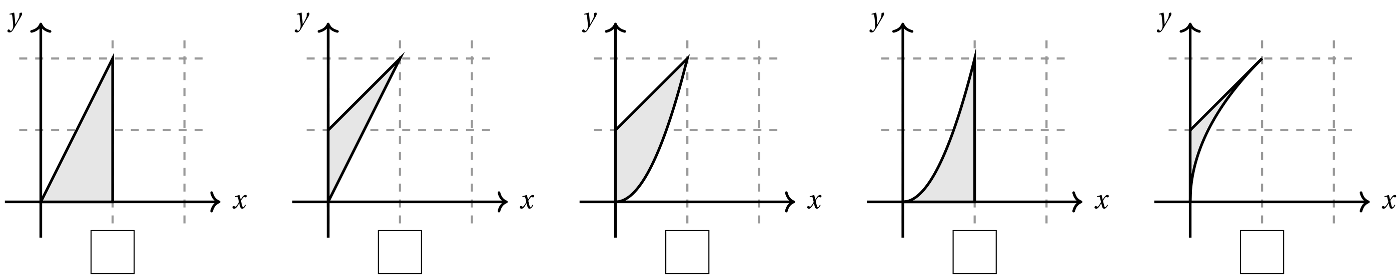
12. Let V be the solid lying below the plane $z = 1$, above the surface $z = -\sqrt{x^2+y^2}$, and inside the cylinder $x^2 + y^2 = 4$. Set up an integral computing the mass of V if the mass density is $\rho(x, y, z) = z + 2$. **(4 points)**

Be sure to fill in your variables of integration in the spaces after the d's.

Mass of $V = \int \int \int \qquad d \quad d \quad d$

13. Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(u, v) = (uv, u + v)$. Let D be the triangle in the uv−plane whose vertices are $(0,0), (1,0), (1,1)$. Let S be the region $T(D)$ in the xy-plane.

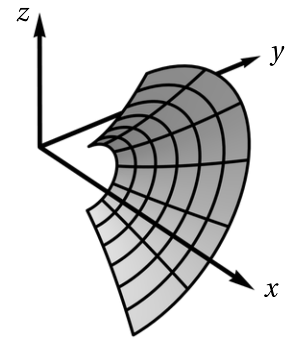
(a) Mark the box below the picture of S; here the dotted grids are made of unit-sized boxes. **(2 points)**



(b) Express $\iint_S y\, dA$ as an integral over D: $\int_0^1 \int_0^u \qquad dv\, du$ **(2 points)**

Scratch Space

14. The curve $y = x^2$ in the xy-plane is revolved about the x-axis in $\mathbb{R}^3$ to produce a surface. Parameterize the **portion of this surface with** $y \geq 0$ **and** $1/2 \leq x \leq 1$ which is shown at right. Be sure to specify the domain D. **(3 points)**



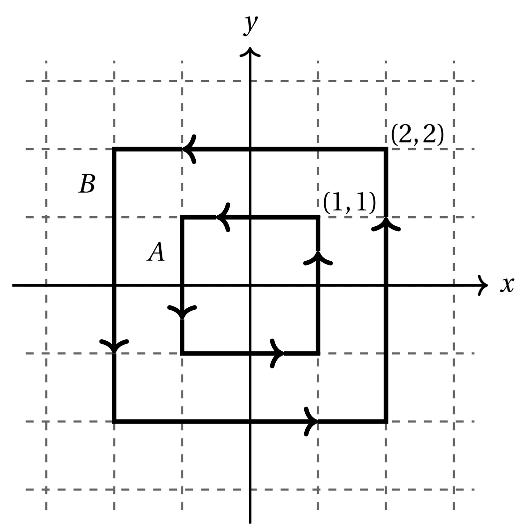
$\mathbf{r}(u, v) = \Big\langle \qquad , \qquad , \qquad \Big\rangle$

$D = \Big\{ (u, v) \;\Big|\; \quad \leq u \leq \quad , \quad \leq v \leq \quad \Big\}$

15. A vector field $\mathbf{F} = \langle P, Q \rangle$ is defined on the plane minus the origin and $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y} + 2$ for all $(x, y) \neq (0,0)$. Let A and B be the two *oriented* curves shown at the right drawn against a grid of unit squares, and suppose $\int_B \mathbf{F} \cdot d\mathbf{r} = 16$. Evaluate the integral:

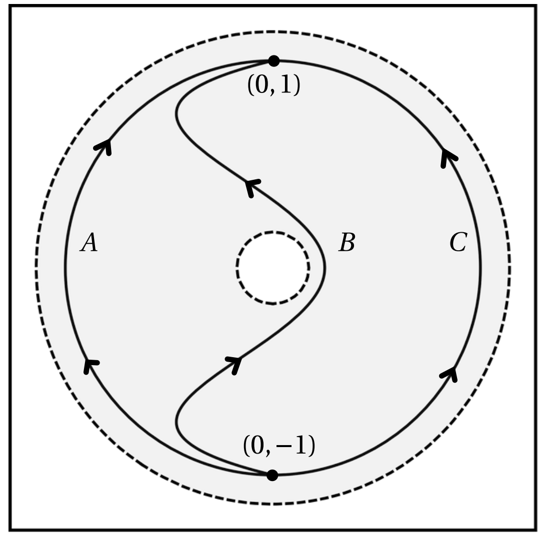
$\int_A \mathbf{F} \cdot d\mathbf{r} =$ | -24 | -16 | -8 | 0 | 8 | 16 | 24 |

(2 points)



Scratch Space

16. The region D defined by $\{0.03 < x^2 + y^2 < 1.3\}$ is shown at right. The curves A, B, C are within this region. Each curve starts at $(0,-1)$ and ends at $(0,1)$. Suppose that $\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is a differentiable vector field defined on D with the properties



$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \qquad \int_A \mathbf{F}\cdot d\mathbf{r} = 2, \quad \text{and} \quad \int_C \mathbf{F}\cdot d\mathbf{r} = -1.$$

(a) The region D is simply connected. **(1 point)**

True	False

(b) $\mathbf{F}$ is conservative. **(1 point)**

Yes	No	Cannot determine

(c) Find $\int_B \mathbf{F}\cdot d\mathbf{r}$. **(1 point)**

−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5	3

17. For this problem, $\mathbf{G} = \langle yz + 2x^2,\ 2xy,\ xy^2 \rangle$ and S is the boundary of the cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$ oriented with the outward pointing normal vectors $\mathbf{n}$. Circle the best response for each of the following.

(a) $\iint_S \mathbf{G}\cdot\mathbf{n}\, dS =$ **(2 points)**

−5	−3	−1	0	1	3	5

(b) $\iint_S (\operatorname{curl}\mathbf{G})\cdot\mathbf{n}\, dS$ is **(1 point)**

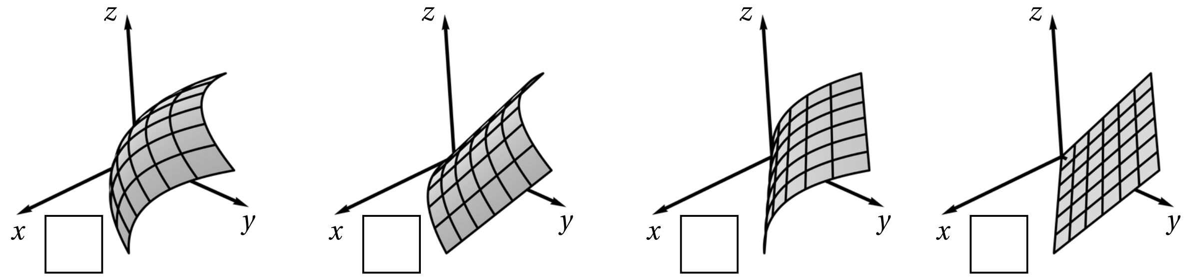
negative	zero	positive

(c) Suppose a charge Q is placed at $\mathbf{p} = \langle 1/2, 1/2, 1/2\rangle$ and let $\mathbf{E} = \dfrac{Q}{4\pi\epsilon_0}\dfrac{1}{|\mathbf{r}-\mathbf{p}|^3}(\mathbf{r}-\mathbf{p})$ for $\mathbf{r} = \langle x, y, z\rangle$ be the resulting electric field. Then $\iint_S \mathbf{E}\cdot\mathbf{n}\, dS =$ ______ **(1 point)**

Scratch Space

18. Consider the surface S parameterized by $\mathbf{r}(u,v) = \langle u,\ u^2+v^2,\ v\rangle$ defined on $D = \{(u,v) \mid 0 \le u \le 1, 0 \le v \le 1\}$ and oriented by the normal vector $\mathbf{n}$ with positive second component.

(a) Mark the box below the picture of S. **(2 points)**



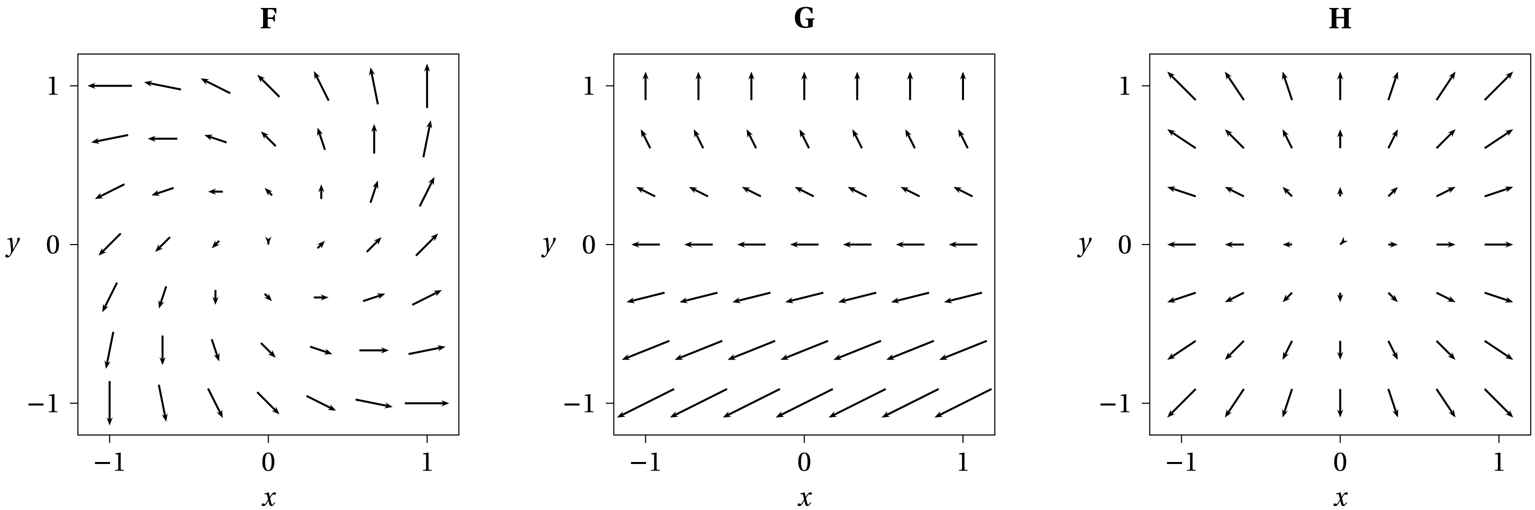
(b) Compute $\iint_S \langle z, 3, -x\rangle \cdot \mathbf{n}\, dS$. **(4 points)**

$$\iint_S \langle z, 3, -x\rangle \cdot \mathbf{n}\, dS =$$

(c) Fill in the integrand so that the surface area of S is: $\int_0^1 \int_0^1 \qquad du\,dv$ **(1 point)**

Scratch Space

19. Three vector fields are shown below, **exactly one** of which is conservative. For each of the following questions, circle the best answer.



(a) The conservative vector field is: | **F** **G** **H** | **(2 points)**

(b) The vector field $\langle y-1,\ y\rangle$ is: | **F** **G** **H** | **(1 point)**

(c) The function $\operatorname{div}\mathbf{H}$ is constant. The value of $\operatorname{div}\mathbf{H}$ at any point is:

| negative zero positive | **(1 point)**

(d) The vector field $\operatorname{curl}\mathbf{F}$ is constant. The value of $\operatorname{curl}\mathbf{F}$ at any point is:

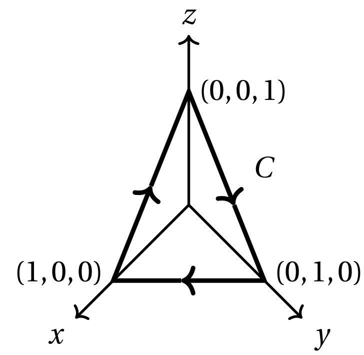
| $\langle 0,\ 0,\ -1\rangle$ $\langle 0,\ 0,\ 0\rangle$ $\langle 0,\ 0,\ 1\rangle$ | **(1 point)**

(e) Let C be the circle $\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{4}$, and $\mathbf{n}$ the outward pointing normal vector in the plane.

The 2D flux $\int_C \mathbf{H}\cdot\mathbf{n}\,ds$ is: | negative zero positive | **(1 point)**

Scratch Space

20. For this problem, $\mathbf{F} = \langle x^2 - 2y,\ y^2 - 2z,\ z^2 - 2x \rangle$ and C is the oriented closed curve made from three straight line segments shown at the right.



(a) Compute curl**F**. **(2 points)**

curl**F** = $\langle \quad , \quad , \quad \rangle$

(b) Compute div**F**. **(1 point)**

div**F** =

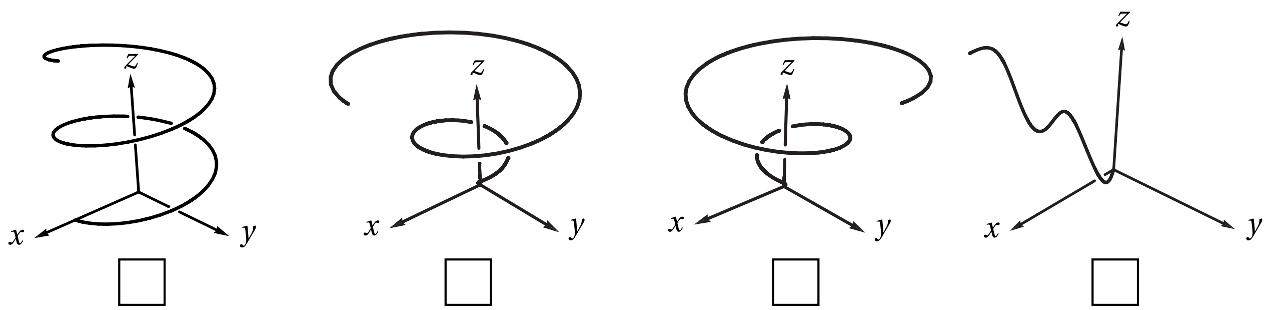
(c) Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. **(4 points)**

$\int_C \mathbf{F} \cdot d\mathbf{r} =$

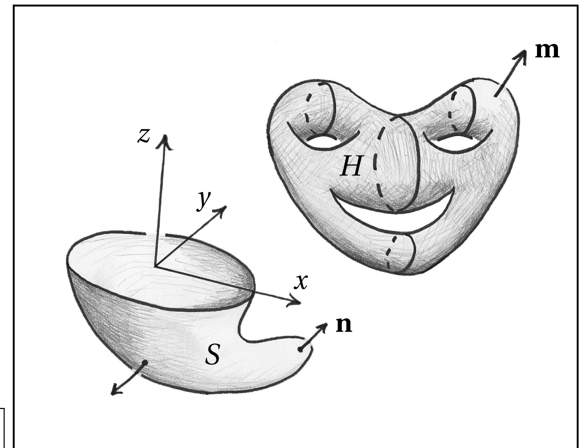
(d) $\int_C \operatorname{div}\mathbf{F}\, ds$ is: negative zero positive **(1 point)**

(e) Is **F** conservative? yes no **(1 point)**

21. Mark the picture of the curve in $\mathbb{R}^3$ parameterized by $\mathbf{r}(t) = \langle\, t\cos t,\ t\sin t,\ t\,\rangle$ for $0 \le t \le 4\pi$. **(2 points)**



22. Let S and H be the surfaces at right; the boundary of S is the unit circle in the xy-plane, and H has no boundary. **1 pt each**



(a) The integral $\iint_H x^2 y^2 + z^2\, dS$ is:

negative   zero   positive

(b) The vector field $\mathbf{F} = \langle\, y+z,\ -x,\ yz\,\rangle$ has $\operatorname{curl}\mathbf{F} = \langle z,\ 1,\ -2\rangle$.

The flux $\iint_S (\operatorname{curl}\mathbf{F}) \cdot \mathbf{n}\, dS$ is:

-5π -4π -3π -2π $-\pi$ 0 π 2π 3π 4π 5π

(c) For $\mathbf{G} = \langle\, x,\ y,\ z\,\rangle$, the flux $\iint_S \mathbf{G}\cdot\mathbf{n}\, dS$ is:

negative   zero   positive

(d) For $\mathbf{E} = \langle z,\ x,\ 2\rangle$, the flux $\iint_S \mathbf{E}\cdot\mathbf{n}\, dS$ is:

-5π -4π -3π -2π $-\pi$ 0 π 2π 3π 4π 5π

Scratch Space