**1.** Let $A = (1,0,1)$, $B = (1,2,0)$, and $C = (2,3,1)$. For each part, circle the best answer. **(1 point each)**

(a) Let $\theta$ be the angle between $\overrightarrow{BA}$ and $\overrightarrow{BC}$. The value of $\theta$ is:

| | | | | |
|---|---|---|---|---|
| $\theta = 0$ | $0 < \theta < \pi/2$ | $\theta = \pi/2$ | $\pi/2 < \theta < \pi$ | $\theta = \pi$ |

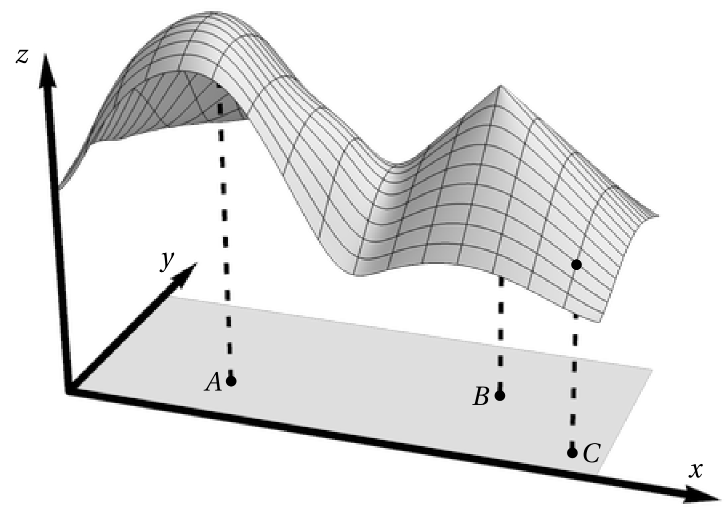
(b) The area of the triangle formed by these points is:

| | | | |
|---|---|---|---|
| $\frac{\sqrt{14}}{2}$ | $\sqrt{14}$ | 7 | 14 |

(c) Let $\ell$ be the line through $B$ and $C$. The distance from $A$ to $\ell$ is:

| | | | |
|---|---|---|---|
| $\frac{\sqrt{14}}{2}$ | $\frac{\sqrt{14}}{\sqrt{3}}$ | $\frac{\sqrt{14}}{\sqrt{5}}$ | $\frac{\sqrt{14}}{\sqrt{15}}$ |

**2.** Consider the function $g\colon \mathbb{R}^2 \to \mathbb{R}$ whose graph is shown at right. Let $A$ and $B$ be the points in $\mathbb{R}^2$ corresponding to the two "peaks" of the graph, and $C$ be the point in $\mathbb{R}^2$ corresponding to the dot on the graph. For each part, circle the answer that is most consistent with the picture. **(1 point each)**




(a) At the point $A$, the function $g$ is:

| | | | |
|---|---|---|---|
| continuous | differentiable | both | neither |

(b) At the point $B$, the function $g$ is:

| | | | |
|---|---|---|---|
| continuous | differentiable | both | neither |

(c) At the point $C$, the function $\frac{\partial g}{\partial x}$ is:

| | | |
|---|---|---|
| negative | zero | positive |

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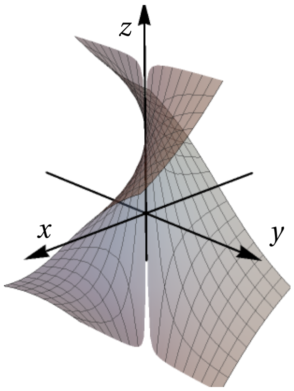
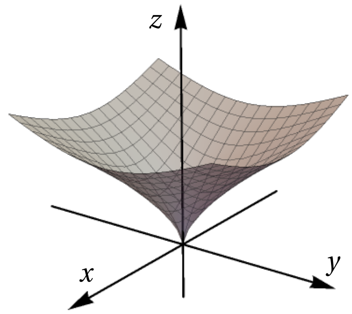
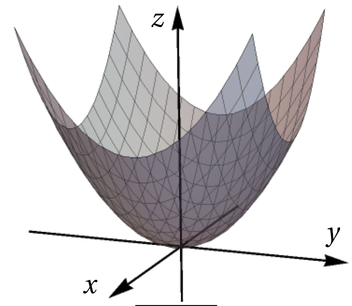
**Scratch Space**

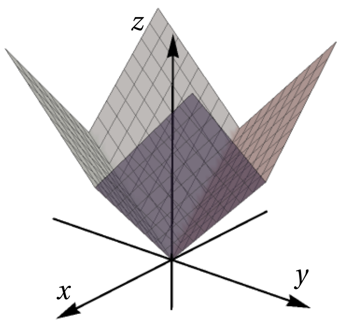
**3.** For each of the given functions, label the box below the picture corresponding to its graph. **(1 point each)**

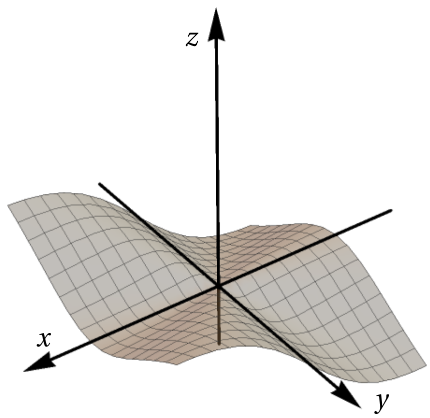
$(A)\quad |x| + |y| \qquad (B)\quad x^2 + y^2 \qquad (C)\quad \dfrac{xy^2}{x^2 + y^2}$

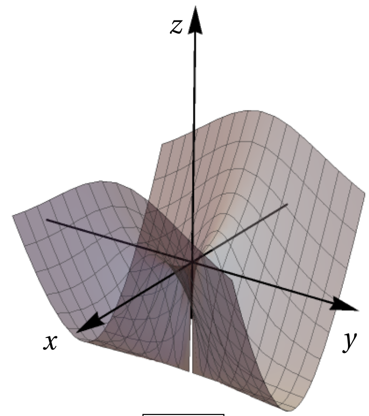



**4.** It is raining on a hill whose height is given by the function $h(x, y) = 20 - 2x^2 - 3y^2$. Assume that water always flows downhill, in the direction where the height of the hill decreases most quickly. At the point $(1, 1)$ what is the direction (in the xy-plane) in which the water will flow? **(2 points)**

At $(1, 1)$ the rain water flows in direction of $\langle \qquad , \qquad \rangle$

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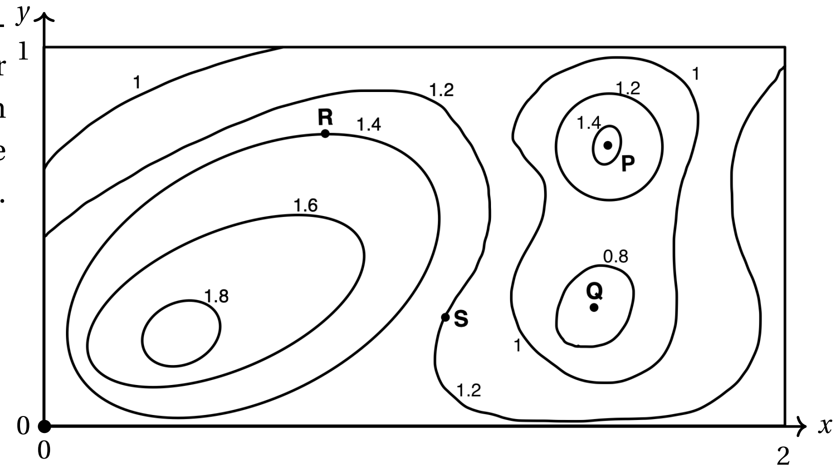
**Scratch Space**

**5.** A rectangular garbage container with dimensions 2 meter high by 2 meter wide by 1 meter deep is partially filled with trash. The function $f(x, y)$ describes the height (in meters) of the trash; a contour map of $f$ is shown to the right. For each part below, circle the best answer.




(a) Classify the behavior at the given points. **(1 point each)**

| | | | | |
|---|---|---|---|---|
| At $P$: | $f$ has a local min | $f$ has a local max | $f$ has a saddle point | $P$ is not a critical point |
| At $Q$: | $f$ has a local min | $f$ has a local max | $f$ has a saddle point | $Q$ is not a critical point |
| At $R$: | $f$ has a local min | $f$ has a local max | $f$ has a saddle point | $R$ is not a critical point |

(b) $\nabla f(R) \approx$ $\langle 0, -3.5\rangle$ $\langle 0, -1.2\rangle$ $\langle 0, -0.2\rangle$ $\langle 0, 0\rangle$ $\langle 0, 0.2\rangle$ $\langle 0, 1.2\rangle$ $\langle 0, 3.5\rangle$ **(1 point)**

(c) Let $\mathbf{u}$ be a unit vector in the direction of $\overrightarrow{RS}$. The directional derivative $D_{\mathbf{u}} f(R)$ is:

negative zero positive **(2 points)**

(d) The volume of trash in the container ($m^3$) is $\approx$ 1.1 2.5 3.8 5.1 7.2 **(2 points)**

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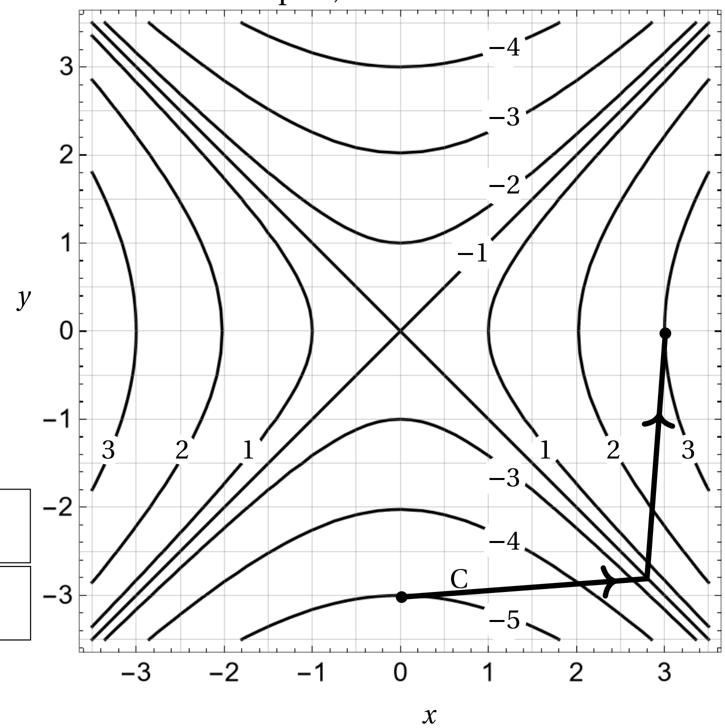
**Scratch Space**

**6.** The contour plot of a differentiable function $f$ is shown below. For each part, circle the best answer.

(a) Estimate $\int_C f\, ds$: **(2 points)**

| | | | | | | |
|---|---|---|---|---|---|---|
| $-9$ | $-5.5$ | $-0.6$ | $0$ | $0.6$ | $5.5$ | $9$ |




(b) Estimate $\int_C \nabla f \cdot d\mathbf{r}$:

| | | | | | | |
|---|---|---|---|---|---|---|
| $-16$ | $-8$ | $-4$ | $0$ | $4$ | $8$ | $16$ |

**(2 points)**

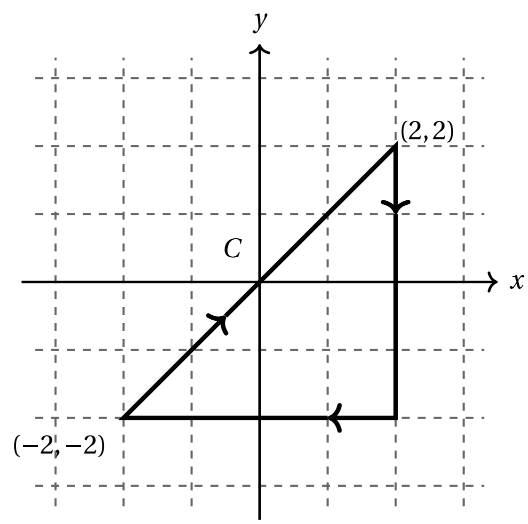
(c) Find the points on the curve $x^2+(y-2)^2=1$ where $f$ has max/min values. **(2 points)**

Max value = ☐ at the point(s) ☐

Min value = ☐ at the point(s) ☐

(d) What is the absolute maximum value of $f$ on the region $D=\{x^2+y^2<1\}$? Write *DNE* if none exists.

Max value on $D$ = ☐ **(1 point)**




**7.** Let $\mathbf{F}=\left\langle 3x^2y-y,\ x^3+2x+\sqrt{1+y^4}\right\rangle$. For $C$, the curve shown at right, compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$:

| | | | | | | |
|---|---|---|---|---|---|---|
| $-24$ | $-16$ | $-8$ | $0$ | $8$ | $16$ | $24$ |

**(2 points)**

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**Scratch Space**

**8.** Determine the limits in the problems below. Be sure to *explain your reasoning* for full credit. If a limit does not exist, write "DNE" in the box provided. **(2 points each)**

(a) Determine $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy+x^2}{x^2+y^2}$.

$$\lim_{(x,y)\to(0,0)} \frac{xy+x^2}{x^2+y^2} =$$

(b) Determine $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{2x^2-x^3y}$.

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{2x^2-x^3y} =$$

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**Scratch Space**

**9.** The function $f(x,y)$ describes the temperature (°C) in a region $R$ in the plane, so that $f(x,y)$ is the measured temperature at position $(x,y)$. Some measured values of $f$ and its rates of change are given in the following table. Assuming that $f$ is differentiable, use this data to approximate the temperature at $(1.5, 3.1)$. **(2 points)**

| $(x,y)$ | $f(x,y)$ | $f_x(x,y)$ | $f_y(x,y)$ |
|---|---|---|---|
| $(1,3)$ | 4 | 2 | 3 |
| $(0.5,0.1)$ | −5 | −1 | −6 |

Temperature at $(1.5, 3.1)$ is ≈

**10.** Suppose $f:\mathbb{R}^2 \to \mathbb{R}$ is a differentiable function of $x$ and $y$ where $x(s,t) = t\cos(2s) - e^{2t}$ and $y(s,t) = t\sin(2s) + e^{t\sin(t)}$.

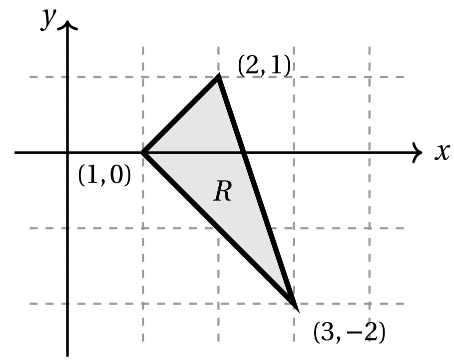
Let $g(s,t) = f(x(s,t), y(s,t))$. Use the table of values on the right, to calculate $g_s\left(\frac{\pi}{4}, \pi\right)$. **(5 points)**

| | $g$ | $f$ | $f_x$ | $f_y$ |
|---|---|---|---|---|
| $(\pi/4, \pi)$ | 2 | −1 | 3 | 5 |
| $(-e^{2\pi}, \pi+1)$ | 4 | 2 | −3 | −2 |

$$g_s\left(\frac{\pi}{4}, \pi\right) =$$

**Scratch Space**

**11.** Using the transformation $T(u,v) = \langle u+v+1, u-v\rangle$, rewrite the integral $\iint_R x\,dA$ as an iterated integral over a subset $S$ in the $uv$-plane with $T(S) = R$. *Do not evalute the integral.* **(5 points)**



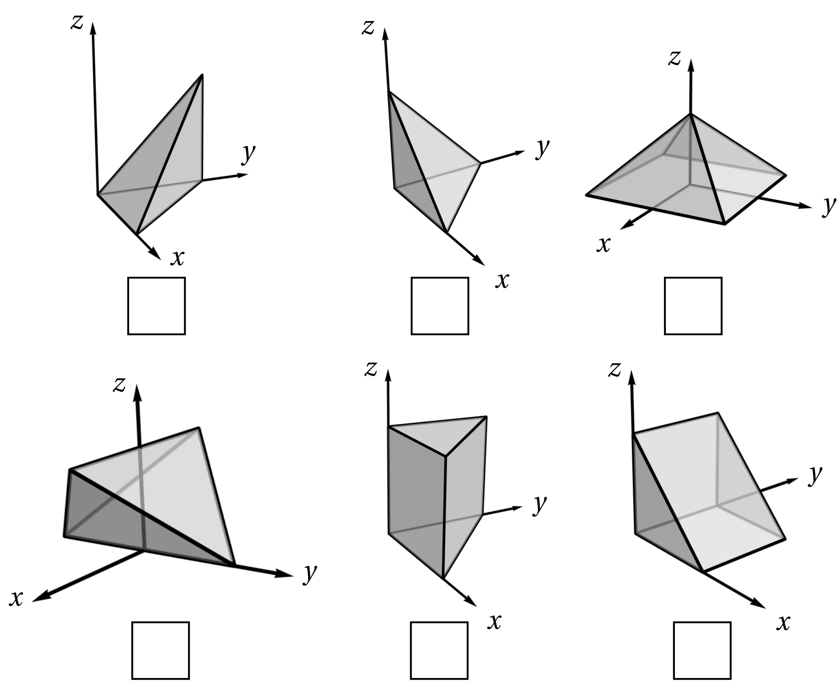

$$\iint_R x\,dA = \boxed{\int \int \qquad du\,dv}$$

**Note:** The order of integration is already determined.

**12.** For each of the given integrals, label the box below the picture of the corresponding region of integration.
**(2 points each)**

(A) $\displaystyle\int_0^1\int_0^1\int_0^{1-z} f(x,y,z)\,dx\,dy\,dz$

(B) $\displaystyle\int_0^1\int_{-1+z}^{1-z}\int_{-1+z}^{1-z} g(x,y,z)\,dx\,dy\,dz$

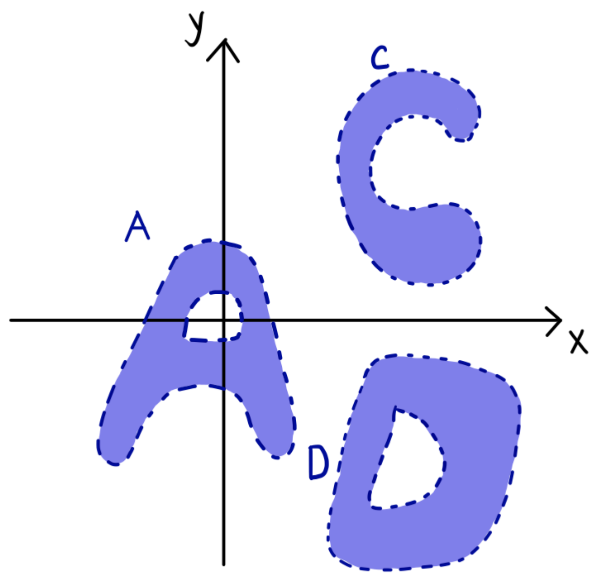


**13.** Consider a 2D vector field $\mathbf{F}$, whose domain consists of those points of $\mathbb{R}^2$ which are not the origin.

Let $\mathbf{F} = \langle P, Q \rangle$.

Assume that $P_y = Q_x$ on the domain of definition of $\mathbf{F}$, and assume further that $\mathbf{F}$ is not conservative on its domain.

Given are the (interiors of the) three regions $A, C, D$ sketched in the picture at right. From this information, which of the following statements are true? Mark all the true statements. **(3 points)**




☐ For every closed continuous curve $\gamma$ contained in $A$, the line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ must be zero.

☐ For every closed continuous curve $\gamma$ contained in $C$, the line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ must be zero.

☐ There exists some closed continuous curve $\gamma$ contained in $C$, such that the line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ must be non-zero.

☐ For every closed continuous curve $\gamma$ contained in $D$, the line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ must be zero.

☐ There exists some closed continuous curve $\gamma$ contained in $D$, such that the line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ must be non-zero.

☐ No such vector field exists.

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**Scratch Space**

**14.** Let $\mathbf{F} = (1 + x + yz)\mathbf{i} + 2y\mathbf{j} + (z + yx)\mathbf{k}$.

(a) Compute curl($\mathbf{F}$). **(1 point)**

| curl($\mathbf{F}$) = $\langle$ , , $\rangle$ . |
|---|

(b) Let $S$ be the portion of the cone $z = \sqrt{x^2 + y^2}$ with $z \leq 2$, with outward pointing normal vector. Compute the flux of curl($\mathbf{F}$) through $S$. **(5 points)**

| flux = |
|---|

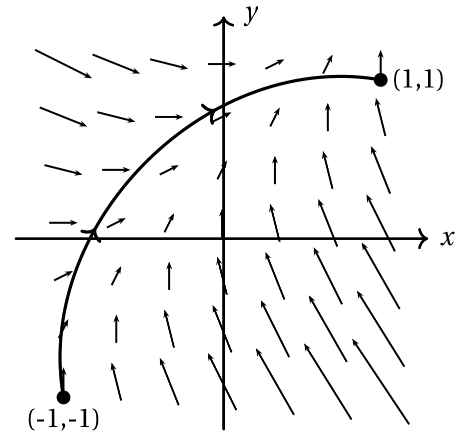
(c) Let $E$ be the sphere $x^2 + y^2 + z^2 = 1$ with outward pointing normal. Compute the flux of $\mathbf{F}$ through $E$. **(3 points)**

| flux = |
|---|

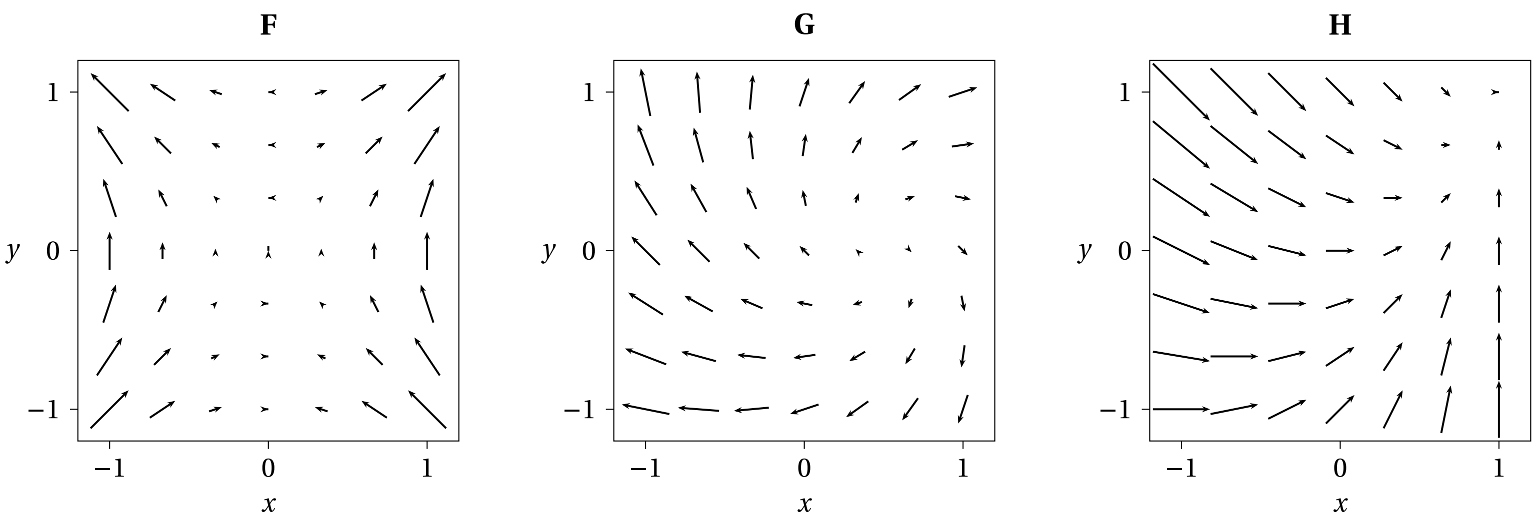
(d) Is $\mathbf{F}$ conservative? | Yes No | **(1 point)**

**15.** A conservative force field **F** is shown at right. Compute the work done by **F** to move a particle from $(-1,-1)$ to $(1,1)$ along the indicated path. For scale, $\mathbf{F}(0,0) = \langle 0, 0.2\rangle$.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $-1$ | $-\frac{2}{5}$ | $-\frac{2\sqrt{2}}{5}$ | $0$ | $\frac{2\sqrt{2}}{5}$ | $\frac{2}{5}$ | $1$ | **(2 points)** |




**16.** Three vector fields are shown below, **exactly one** of which is conservative. For each of the following questions, circle the best answer.




(a) The conservative field is: **F** **G** **H** **(2 points)**

(b) The vector field $(1-x)\mathbf{i} + (x-y)\mathbf{j}$ is: **F** **G** **H** **(1 point)**

(c) The vector field curl(**G**) is constant. The value of curl(**G**) at any point is:

$\langle 0,0,-1\rangle$ $\langle 0,0,0\rangle$ $\langle 0,0,1\rangle$ **(1 point)**

(d) **Exactly one** of these vector fields has nonconstant divergence. Circle it: **F** **G** **H** **(1 point)**