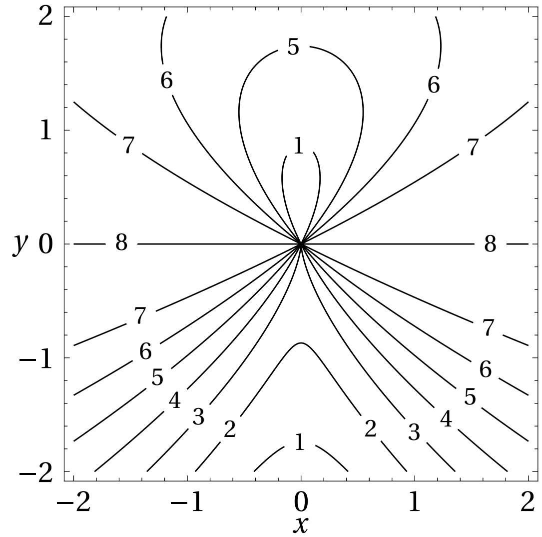
1. Consider the function $f(x, y)$ whose contour map is shown at right, where the value of f on each level curve is indicated by the number along it. For each part, give the answer that is **most consistent** with the given data. **For (a) and (b) be sure to explain your reasoning in the space provided.** If the limit does not exist, write “DNE” in the answer box.



(a) Determine $\lim_{x\to 0} f(x,0)$. **(2 points)**

$\lim_{x\to 0} f(x,0) =$

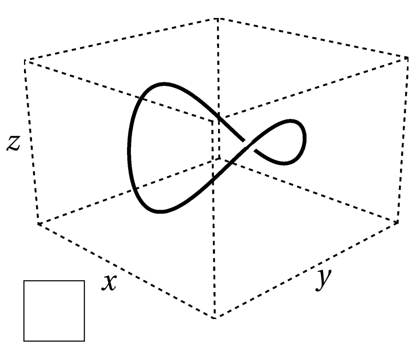
(a) Determine $\lim_{(x,y)\to(0,0)} f(x,y)$. **(2 points)**

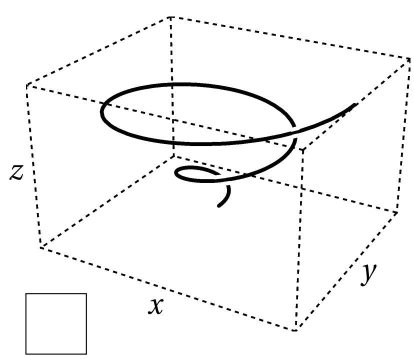
$\lim_{(x,y)\to(0,0)} f(x,y) =$

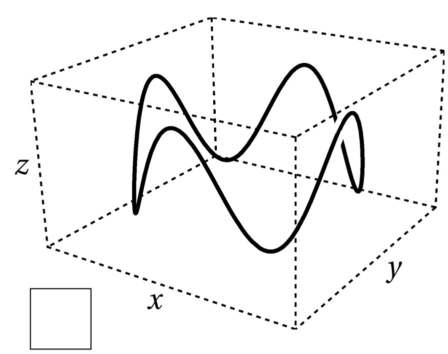
(c) Determine $\lim_{(x,y)\to(1,-1)} f(x,y)$. **(1 point)**

$\lim_{(x,y)\to(1,-1)} f(x,y) =$

2. Mark the box next to the curve that is parameterized by $\mathbf{r}(t) = \langle 2\cos(t),\ 2\sin(t),\ \cos(4t)\rangle$ for $0 \le t \le 2\pi$. **(2 points)**





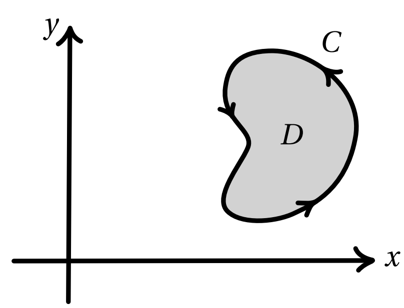


3. Suppose $f\colon \mathbb{R}^2 \to \mathbb{R}$ has the table of values and partial derivatives shown at right. For $x(s,t) = s + 2t$ and $y(s,t) = s^2 - t$, let $F(s,t) = f\big(x(s,t), y(s,t)\big)$ be their composition with f. Compute $\dfrac{\partial F}{\partial s}(2,1)$. **(4 points)**

(x,y)	$f(x,y)$	$\dfrac{\partial f}{\partial x}$	$\dfrac{\partial f}{\partial y}$
$(2,1)$	0	7	6
$(2,-1)$	-12	7	-1
$(3,3)$	19	-8	5
$(4,3)$	7	3	2

$\dfrac{\partial F}{\partial s}(2,1) =$

4. Consider the region D in the plane bounded by the curve C as shown at right. For each part, circle the best answer. **(1 point each)**



(a) For $\mathbf{F}(x,y) = \langle\, x+1,\ y^2 \,\rangle$, the integral $\displaystyle\int_C \mathbf{F}\cdot dr$ is

negative zero positive

(b) The integral $\displaystyle\int_C \big(y\,dx + 2dy\big)$ is

negative zero positive

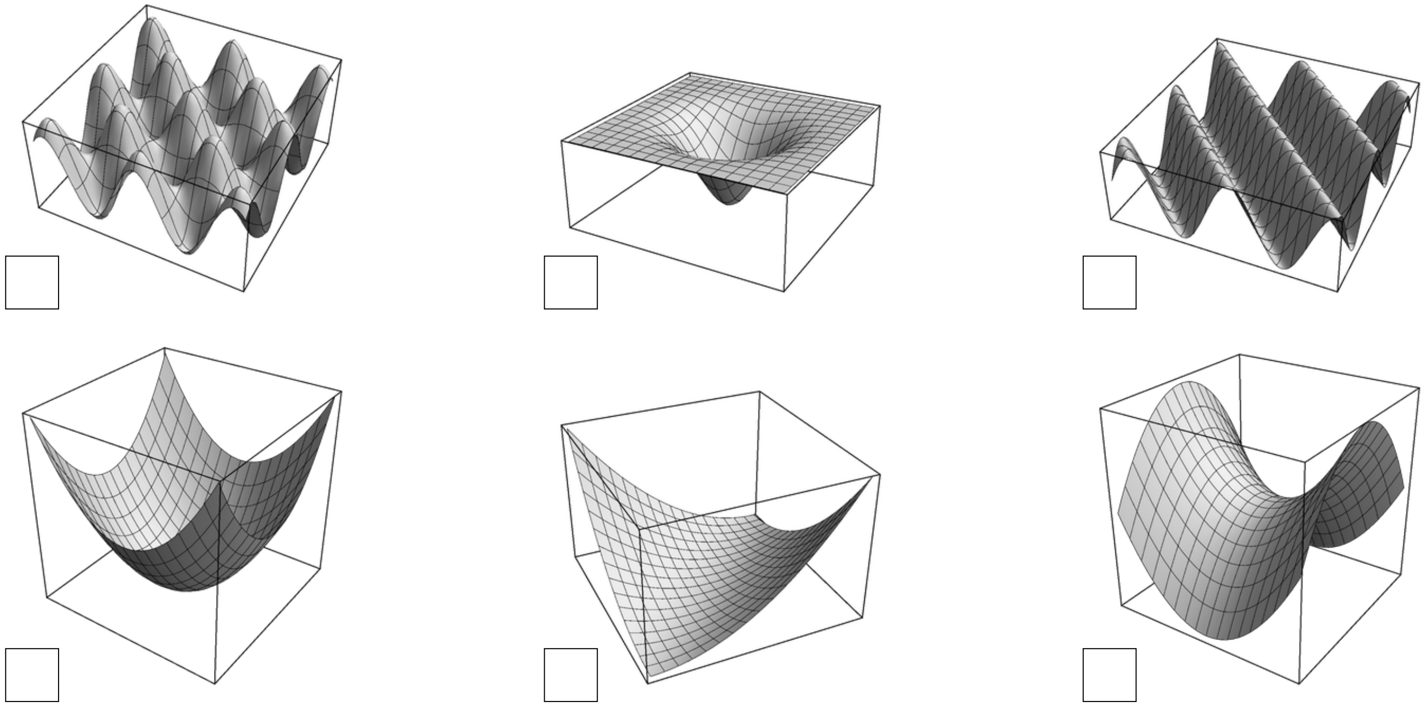
(c) The integral $\displaystyle\iint_D (y-x)\; dA$ is

negative zero positive

Scratch Space

5. For each function label its graph from among the options below: **(1 point each)**

(A) $x^2 - y^2$ (B) $\cos(x+y)$ (C) $1 - e^{-(x^2+y^2)}$



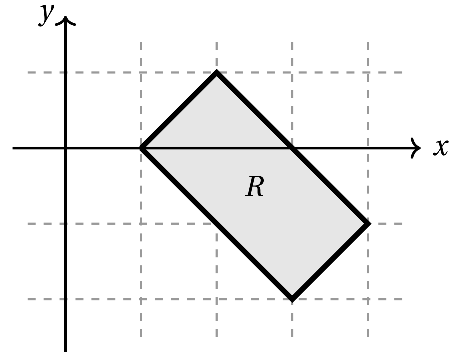
6. A rectangular metallic plate R is placed in the plane with vertices at $(-2,-1)$, $(-2,1)$, $(2,-1)$, and $(2,1)$. The density (in g/cm^2) of the plate, $\rho(x,y)$, at various points is shown in the table, where x and y are measured in cm. Circle the best estimate for the mass of the plate. **(2 points)**

$\rho(x,y)$		$x = -1$	$x = 1$
y	$1/2$	4	7
y	$-1/2$	1	3

Mass of $R \approx$ | 0 | 4 | 15 | 30 | 46 | 60 | 78 | grams.

Scratch Space

7. Let R be the rectangle whose vertices are $(1,0)$, $(2,1)$, $(3,-2)$, and $(4,-1)$ shown at the right.



(a) Find a transformation $T(u,v)$ from the uv–plane to the xy–plane with $T(S) = R$, where $S = \{(u,v) \mid 0 \le u \le 1, 0 \le v \le 1\}$ is the unit square. **(4 points)**

$T(u,v) = (\qquad\qquad , \qquad\qquad)$

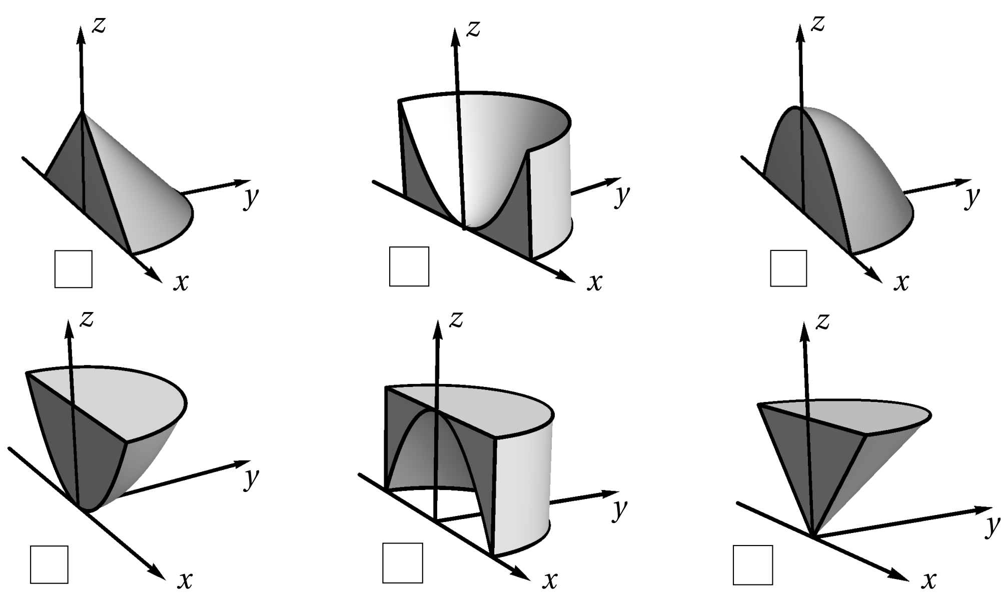
(b) Use your answer in (a) to fill in the integrand below to evaluate $\iint_R \cos(x)\, dA$ by a change of coordinates. **(2 points)**

$$\iint_R \cos(x)\, dA = \int_0^1 \int_0^1 \qquad\qquad du\, dv$$

8. For each of the integrals below, label the solid corresponding to the region of integration. **(2 points each)**

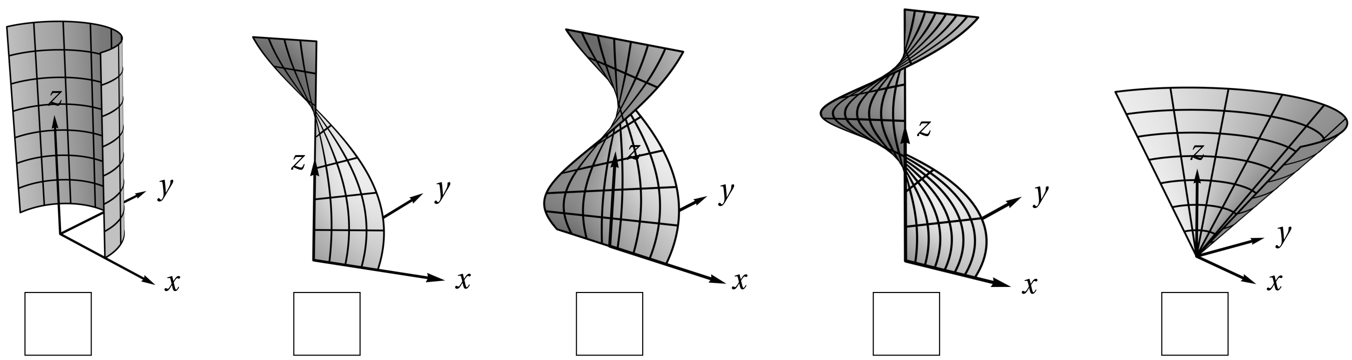
(A) $\displaystyle\int_0^{\pi} \int_0^1 \int_0^{r^2} f(r,\theta,z)\, r\, dz\, dr\, d\theta$

(B) $\displaystyle\int_0^{\pi} \int_0^1 \int_0^{\sqrt{z}} f(r,\theta,z)\, r\, dr\, dz\, d\theta$



9. Let S be the surface in $\mathbb{R}^3$ parameterized by $\mathbf{r}(u, v) = \langle u\cos(v),\ u\sin(v),\ v\rangle$ for $0 \le u \le 1$ and $0 \le v \le \pi$.

(a) Check the box below the correct picture of S. **(2 points)**



(b) Fill in the integrand below so that the integral computes $\iint_S y\, dS$. **(4 points)**

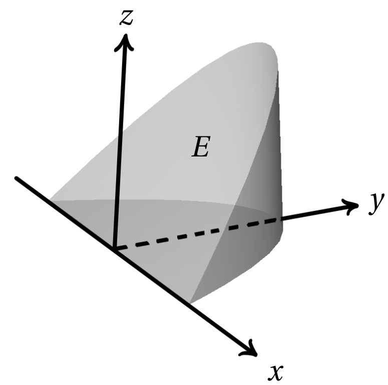
$$\iint_S y\, dS = \int_0^{\pi} \int_0^{1} \qquad\qquad du\, dv$$

10. Let R be the region in the positive octant that lies above the cone $x^2 + y^2 = z^2$ and below the plane $z = 5$. Suppose R is made of material of density $\rho(x, y, z) = x$. Fill in the limits and integrand so that the integral computes the mass of R using spherical coordinates. Be sure to follow the provided order of integration. **(5 points)**

$$\text{mass} = \int \qquad \int \qquad \int \qquad d\rho\, d\theta\, d\phi$$

Scratch Space

11. Consider the region E shown at right, which is bounded by the xy-plane, the plane $z - y = 0$ and the surface $x^2 + y = 1$.



(a) Fill in the limits and integrand of the triple integral below so that it computes the volume of E. Be sure to follow the provided order of integration. **(4 points)**

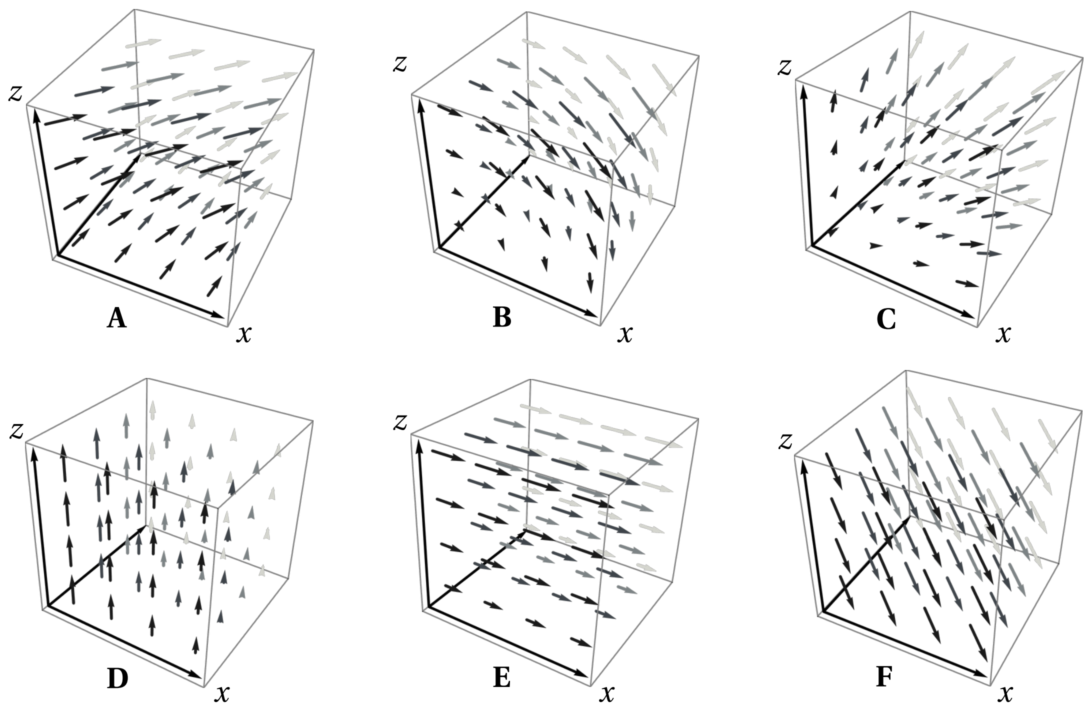
$$\text{Vol}(E) = \int \quad \int \quad \int \qquad dx\, dy\, dz$$

(b) Let S be the curved portion of the boundary of E where $x^2 + y = 1$. Parameterize S by $\mathbf{r}\colon D \to S$, where the domain D is a rectangle. **(4 points)**

$$D = \{ \qquad \}$$

$$\mathbf{r}(u, v) = \langle \qquad , \qquad , \qquad \rangle$$

12. Here are plots of six vector fields on the box where $0 \le x \le 1$, $0 \le y \le 1$, and $0 \le z \le 1$. For each part, circle the best answer. **(1 point each)**



(a) The vector field given by $\langle z,\ 1,\ 0\rangle$ is: **A B C D E F**

(b) Exactly one of these vector fields has nonzero divergence. It is: **A B C D E F**

For this vector field, the divergence is generally: negative positive

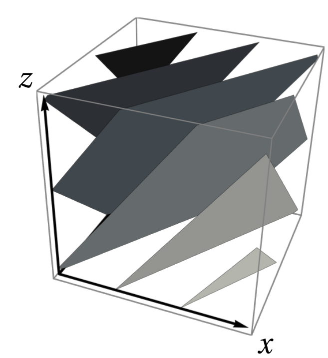
(c) The vector field **D** is conservative: true false

(d) Exactly one of the vector fields is constant, that is, independent of position. It is: **A B C D E F**

(e) The vector field curl **B** is constant. The value of curl **B** is:

$\mathbf{i}$ $-\mathbf{i}$ $\mathbf{j}$ $-\mathbf{j}$ $\mathbf{k}$ $-\mathbf{k}$ $\mathbf{0}$

(f) The vector field that is the gradient of a function f whose level sets are shown at right is: **A B C D E F**



13. Let R be the solid region in $\mathbb{R}^3$ bounded by the cylinder $x^2+y^2=1$ and the planes $z=0$ and $z=1$. Divide the boundary ∂R into three parts: the bottom B where $z=0$, the top T where $z=1$, and the curved surface S where $x^2+y^2=1$. Orient all of these surfaces by the normal vectors that point out of R. Consider the vector field $\mathbf{F}=\langle 2x,\ 0,\ 1-z\rangle$.

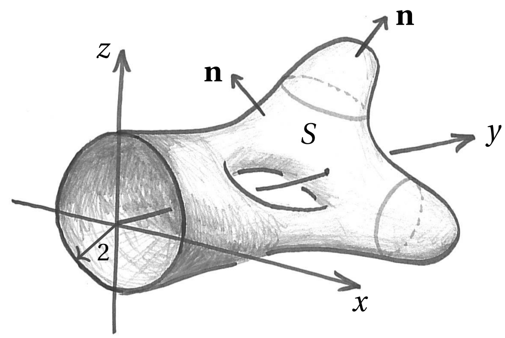
(a) Compute the flux of $\mathbf{F}$ through each of S, T, and B. (Hint: $\int_0^{2\pi}\sin^2 t\,dt=\int_0^{2\pi}\cos^2 t\,dt=\pi$.) **(6 points)**

$\iint_S \mathbf{F}\cdot\mathbf{n}\,dS =$	$\iint_T \mathbf{F}\cdot\mathbf{n}\,dS =$	$\iint_B \mathbf{F}\cdot\mathbf{n}\,dS =$

(b) Use the Divergence Theorem to check your computation for the flux through ∂R in (a–c). **(2 points)**

$\iint_{\partial R} \mathbf{F}\cdot\mathbf{n}\,dS =$

14. The surface S shown at right has boundary the circle C of radius 2 in the xz-plane.



(a) Consider the vector field $\mathbf{F} = \langle z,\ y,\ -x \rangle$ on $\mathbb{R}^3$. With respect to the normal vector field $\mathbf{n}$ indicated, compute the flux of $\operatorname{curl} \mathbf{F}$ through S. **(5 points)**

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} =$$

(a) Consider the vector field $\mathbf{G} = \langle 3z + e^{\sin x},\ e^{\cos y},\ \sin(e^z) \rangle$. With $C = \partial S$ oriented **counter-clockwise** from our viewpoint, compute $\int_C \mathbf{G} \cdot d\mathbf{r}$. **(4 points)**

$$\int_C \mathbf{G} \cdot d\mathbf{r} =$$

15. Consider the vector fields $\mathbf{A} = \langle 2x,\ z,\ y \rangle$ and $\mathbf{B} = \langle x,\ 0,\ y \rangle$ and $\mathbf{C} = \langle y,\ 0,\ z \rangle$. **(1 point each)**

(a) Circle the unique vector field that is conservative:

A	**B**	*C*

(b) Suppose **F** is your answer in (a) and W is any curve starting at $(2, 0, -1)$ and ending at $(1, 1, 1)$.

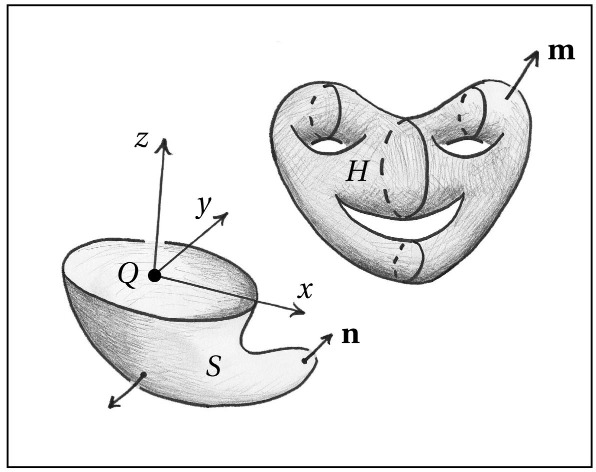
Circle the value of $\int_W \mathbf{F} \cdot d\mathbf{r}$:

−5	−4	−3	−2	−1	0	1	2	3	4	5

(c) Circle the unique vector field that is curl **G** for some vector field **G**:

A	**B**	*C*

16. Let S and H be the surfaces at right; the boundary of S is the unit circle in the xy-plane, and H has no boundary. Suppose there is a **negative** charge Q placed at the origin and let **E** be the resulting electrical field. For each part, circle the correct answer. **(1 point each)**



(a) The flux $\iint_S \mathbf{E} \cdot \mathbf{n}\, dS$ is:

negative	zero	positive

(b) The flux $\iint_H \mathbf{E} \cdot \mathbf{m}\, dS$ is:

negative	zero	positive

(c) Consider the sphere S where $x^2 + y^2 + z^2 = 100$. The flux with respect to the outward normals **u** is:

$\iint_S \mathbf{E} \cdot \mathbf{u}\, dS =$

(d) The integral $\iint_H x^2 y^2 + z^2\, dS$ is:

negative	zero	positive

Scratch Space

Extra Credit: (5 points)

Consider the solid region $E = \{-2 \le x \le 6 \text{ and } -2 \le y \le 2 \text{ and } -2 \le z \le 2 \text{ and } x^2 + y^2 \ge 1 \text{ and } (y-4)^2 + z^2 \ge 1\}$ inside $\mathbb{R}^3$.

(a) Draw an accurate picture of E.

(b) Give a vector field $\mathbf{F}$ on E where $\operatorname{curl} \mathbf{F} = 0$ but $\mathbf{F}$ is not conservative.

(c) Find a second vector field $\mathbf{G}$ on E where $\operatorname{curl} \mathbf{G} = 0$ and $\mathbf{G}$ is not conservative where $\mathbf{G} \neq a\mathbf{F} + \nabla h$ for all a in $\mathbb{R}$ and differentiable functions $h\colon E \to \mathbb{R}$.

Scratch page. Feel free to detach from the rest of the exam. It does not need to be turned in.