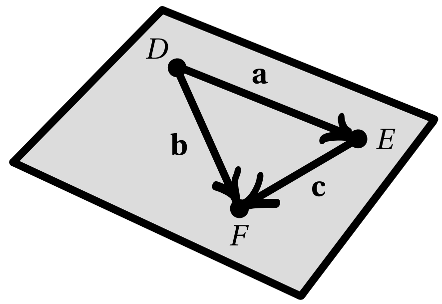
1. (a) Consider the points $A = (0,1,2)$, $B = (2,2,3)$, and $C = (-1,3,4)$. Compute the vectors $\mathbf{v} = \overrightarrow{AB}$ and $\mathbf{w} = \overrightarrow{AC}$. **(2 points)**

$\mathbf{v} = \langle \quad , \quad , \quad \rangle$ $\mathbf{w} = \langle \quad , \quad , \quad \rangle$

(b) Consider the points $D = (0,2,1)$, $E = (1,4,1)$, and $F = (3,8,2)$ and the vectors $\mathbf{a} = \langle 1,2,0\rangle$, $\mathbf{b} = \langle 3,6,1\rangle$, and $\mathbf{c} = \langle 2,4,1\rangle$ as shown at right. Find a normal vector $\mathbf{n}$ to the plane containing the points D, E, and F. **(3 points)**



$\mathbf{n} = \langle \quad , \quad , \quad \rangle$

(c) Let $P = (2,-1,1)$ and $\mathbf{u} = \langle 3,2,4\rangle$. Find a linear equation for the plane that contains P and has normal vector $\mathbf{u}$. **(2 points)**

Equation: $\square\, x + \square\, y + \square\, z = \square$

(d) For two vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$, which of the following does $|\mathbf{v} \times \mathbf{w}|$ measure? Circle your answer. **(1 point)**

- The length of $\mathbf{v} - \mathbf{w}$.
- The area of the parallelogram determined by $\mathbf{v}$ and $\mathbf{w}$.
- The volume of the parallelepiped determined by $\mathbf{v}$, $\mathbf{w}$, and $\mathbf{v} \times \mathbf{w}$.

2. (a) Find the midpoint M of the straight-line segment between the points $P = (1, 3, -2)$ and $Q = (3, -3, 4)$. **(2 points)**

$M = (\quad , \quad , \quad)$

(b) Let L be the line parametrized by $\mathbf{r}(t) = \mathbf{a} + t\mathbf{b}$ for $\mathbf{a} = \langle 1, 0, 2 \rangle$ and $\mathbf{b} = \langle 2, -1, 1 \rangle$. Find the point Q of intersection of the line L with the plane whose equation is $3x - 2y + z = 14$. **(3 points)**

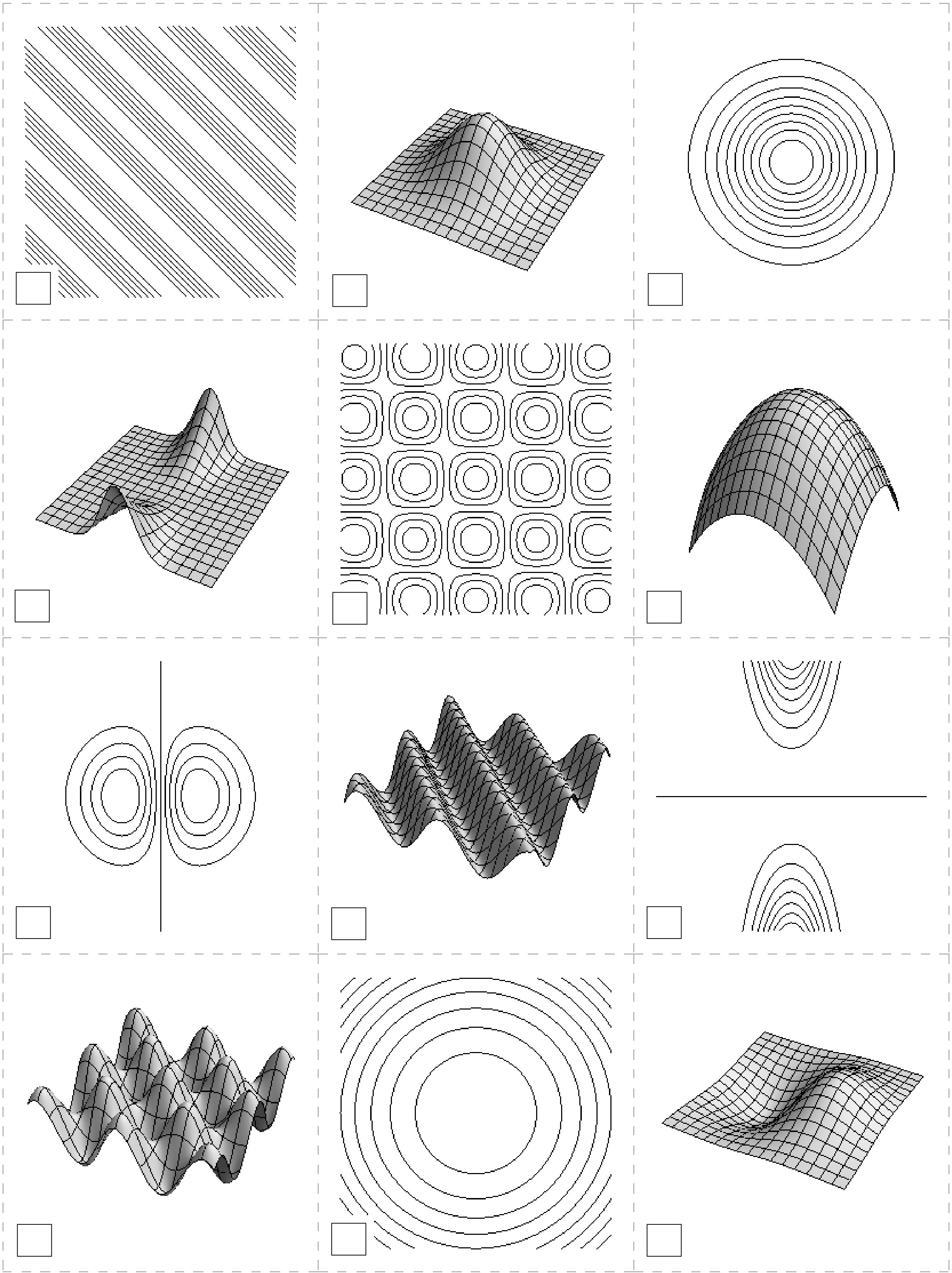
$Q = (\quad , \quad , \quad)$

3. Consider the function $f(x, y) = \dfrac{xy - x^2 y}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$. Evaluate $\lim_{(x,y)\to(0,0)} f(x, y)$ or explain why it does not exist. **(5 points)**

4. For each function

(a) $-x^2 - y^2$ (b) $\cos(x + y)$ (c) $y^2 e^{-x^2}$

label its graph and its level set diagram from among the options below. Here each level set diagram consists of level sets $\{f(x, y) = c_i\}$ drawn for evenly spaced c_i. **(9 points)**



5. (a) Compute the partial derivatives of $f(x, y) = x^3 + 2xy + y$. **(2 points)**

$\dfrac{\partial f}{\partial x} =$

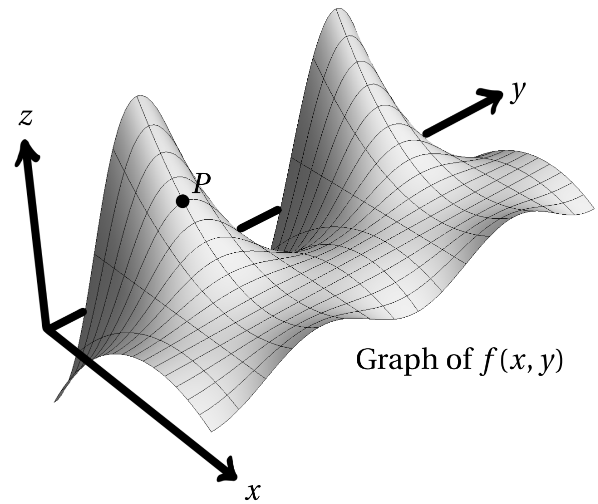
$\dfrac{\partial f}{\partial y} =$

(b) Consider the graph of $f\colon \mathbb{R}^2 \to \mathbb{R}$ shown at right. If P is the point $\big(a, b, f(a,b)\big)$, find the sign of each of the quantities below. Circle your answers. **(1 point each)**

$f_x(a,b)$: positive negative 0

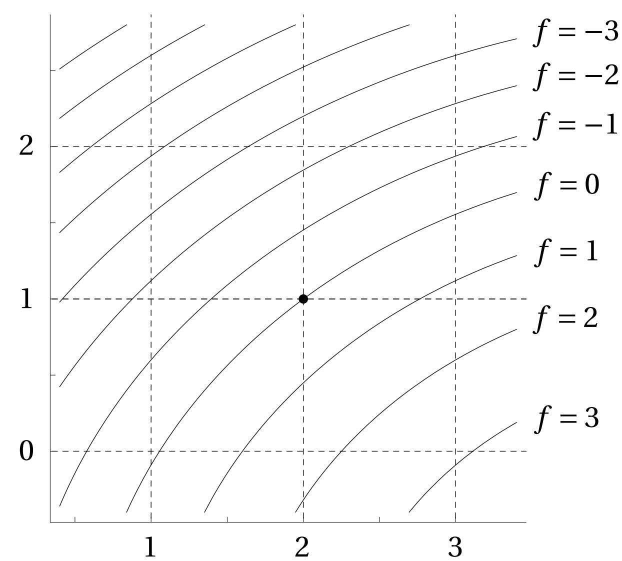
$f_y(a,b)$: positive negative 0

$f_{xx}(a,b)$: positive negative 0



Graph of $f(x, y)$

(c) Use the contour plot of $f(x, y)$ shown at right to estimate $f_x(2,1)$ and $f_y(2,1)$. For each, circle the number below that is closest to your estimate. **(4 points)**



$f_x(2,1)$:	−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5	3
$f_y(2,1)$:	−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2	2.5	3

6. Suppose a function $f\colon \mathbb{R}^2 \to \mathbb{R}$ has $f(1,2) = 5$, $\dfrac{\partial f}{\partial x}(1,2) = -2$, and $\dfrac{\partial f}{\partial y}(1,2) = 3$. Use linear approximation to estimate $f(1.1, 1.9)$. **(3 points)**

$f(1.1, 1.9) \approx$

7. Suppose

$$f(x,y)\colon \mathbb{R}^2 \to \mathbb{R}, \quad x(s,t)\colon \mathbb{R}^2 \to \mathbb{R}, \quad \text{and} \quad y(s,t)\colon \mathbb{R}^2 \to \mathbb{R}$$

are functions. Let $F(s,t) = f\big(x(s,t), y(s,t)\big)$ be their composition.

(a) Write the formula for $\dfrac{\partial F}{\partial s}$ using the Chain Rule. **(1 points)**

$\dfrac{\partial F}{\partial s} =$

(b) Suppose $x(s,t) = 2s + t$ and $y(s,t) = s^2 t - 1$ and that $f(x,y)$ has the table of values and partial derivatives shown at right. Compute $\dfrac{\partial F}{\partial s}(2,1)$. **(4 points)**

(x,y)	$f(x,y)$	$\dfrac{\partial f}{\partial x}$	$\dfrac{\partial f}{\partial y}$
$(2,3)$	0	3	6
$(2,1)$	2	-2	-1
$(2,4)$	3	4	6
$(5,3)$	1	3	5

$\dfrac{\partial F}{\partial s}(2,1) =$

Extra credit problem: Let $E\colon \mathbb{R}^2 \to \mathbb{R}$ be given by $E(x, y) = 2x + y^2$. Find a $\delta > 0$ so that $|E(\mathbf{h})| < 0.01$ for all $\mathbf{h} = (x, y)$ with $|\mathbf{h}| < \delta$. Carefully justify why the δ you provide is good enough. **(2 points)**

Scratch space below and on back